Global Register Allocation

(Slides from Andrew Myers)

Main idea

- Want to replace temporary variables with some fixed set of registers
- First: need to know which variables are live after each instruction
  - Two simultaneously live variables cannot be allocated to the same register

Register allocation

- For every node n in CFG, we have out[n]
  - Set of temporaries live out of n
- Two variables *interfere* if
  - both initially live (ie: function args), or
  - both appear in out[n] for any n
- How to assign registers to variables?

Interference graph

- Nodes of the graph = variables
- Edges connect variables that interfere with one another
- Nodes will be assigned a color corresponding to the register assigned to the variable
- Two colors can't be next to one another in the graph

Interference graph

Instructions	Live vars
b = a + 2	
c = b * b	
b = c + 1	
return b * a	

Interference graph

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c = b * b	
b = c + 1	
	b,a
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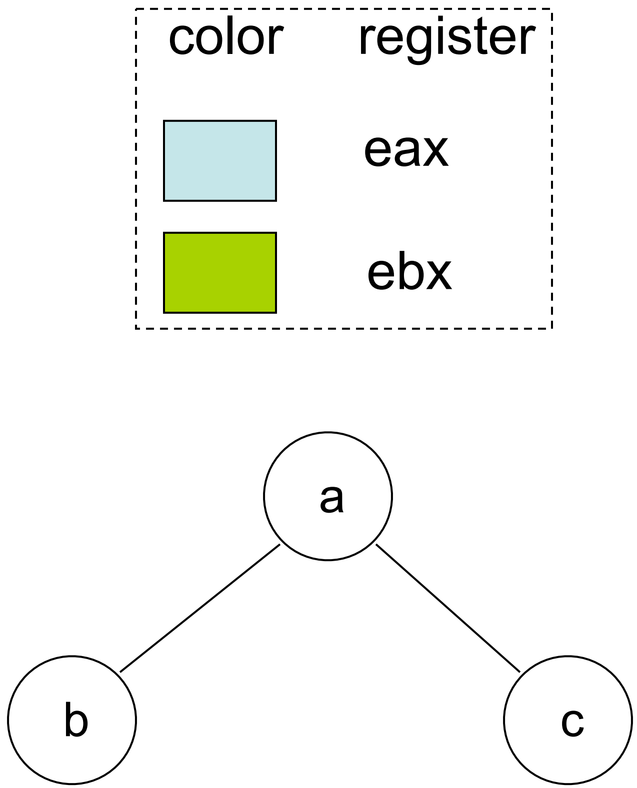

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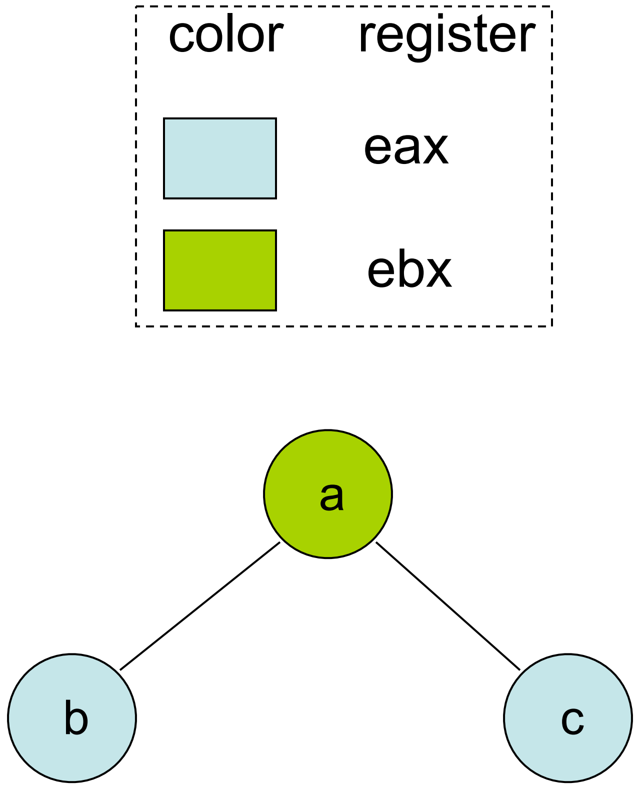


Graph coloring

- Questions:
  - Can we efficiently find a coloring of the graph whenever possible?
  - Can we efficiently find the optimum coloring of the graph?
  - How do we choose registers to avoid move instructions?
  - What do we do when there aren't enough colors (registers) to color the graph?

Coloring a graph

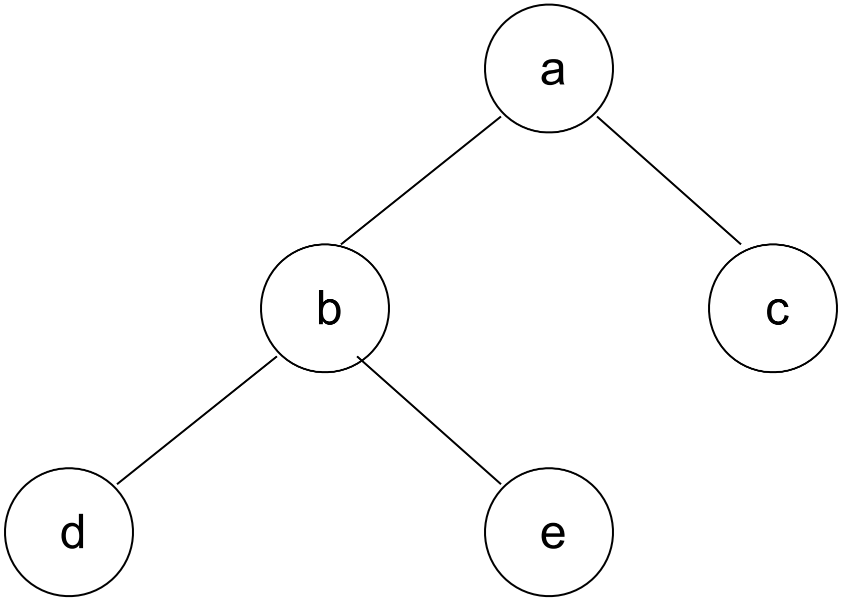
- Kempe's algorithm [1879] for finding a K-coloring of a graph
- Assume K=3
- Step 1 (simplify): find a node with at most K-1 edges and cut it out of the graph. (Remember this node on a stack for later stages.)

Coloring a graph

- Once a coloring is found for the simpler graph, we can always color the node we saved on the stack
- Step 2 (color): when the simplified subgraph has been colored, add back the node on the top of the stack and assign it a color not taken by one of the adjacent nodes

Coloring

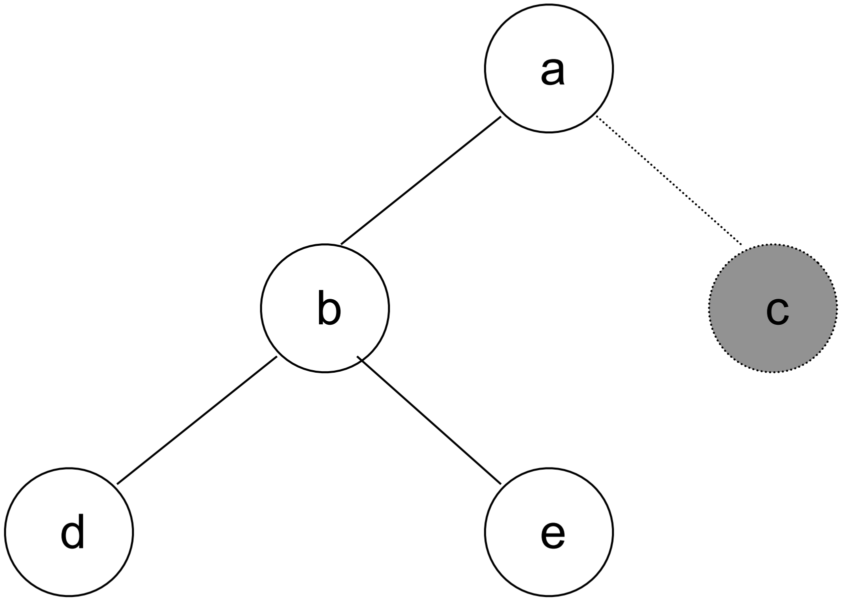
color	register
(light blue)	eax
(green)	ebx



stack:

Coloring

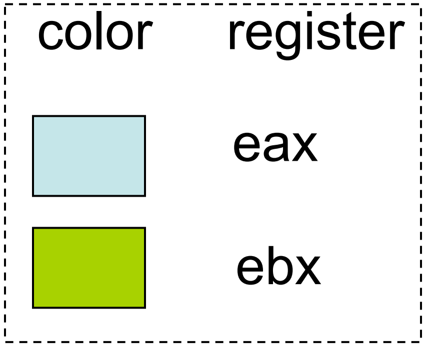
color	register
(light blue)	eax
(green)	ebx

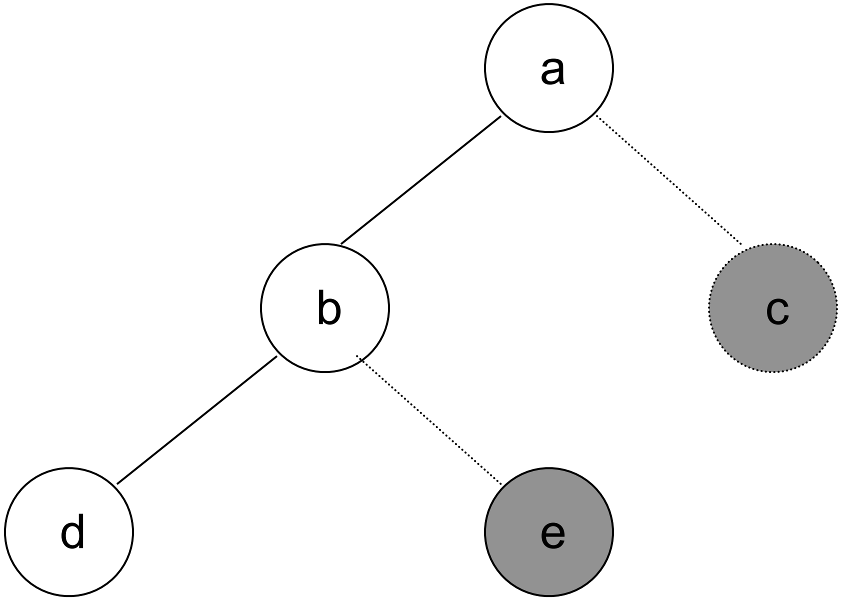


stack:

c

Coloring





stack:

e
c

Coloring

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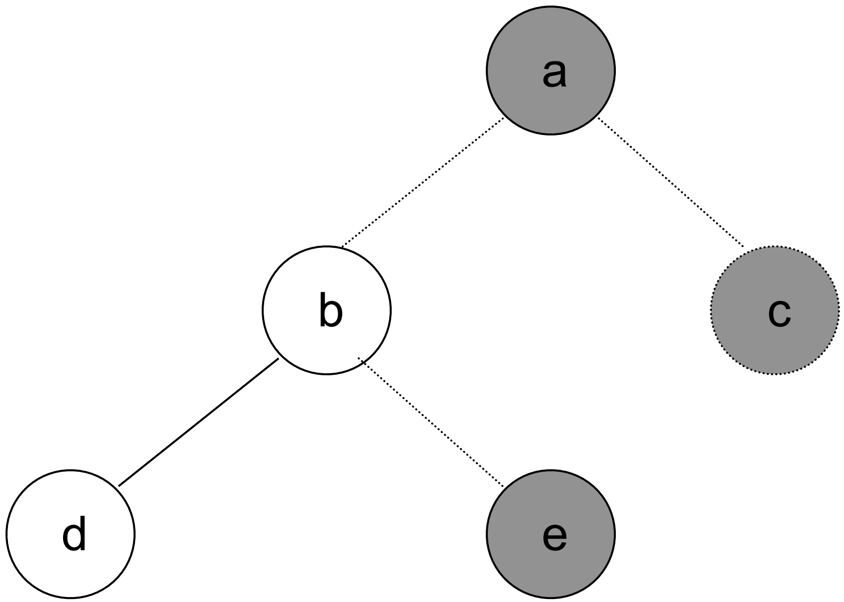
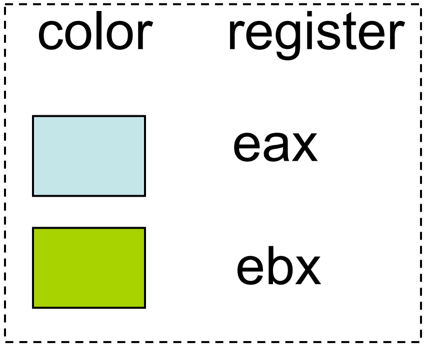
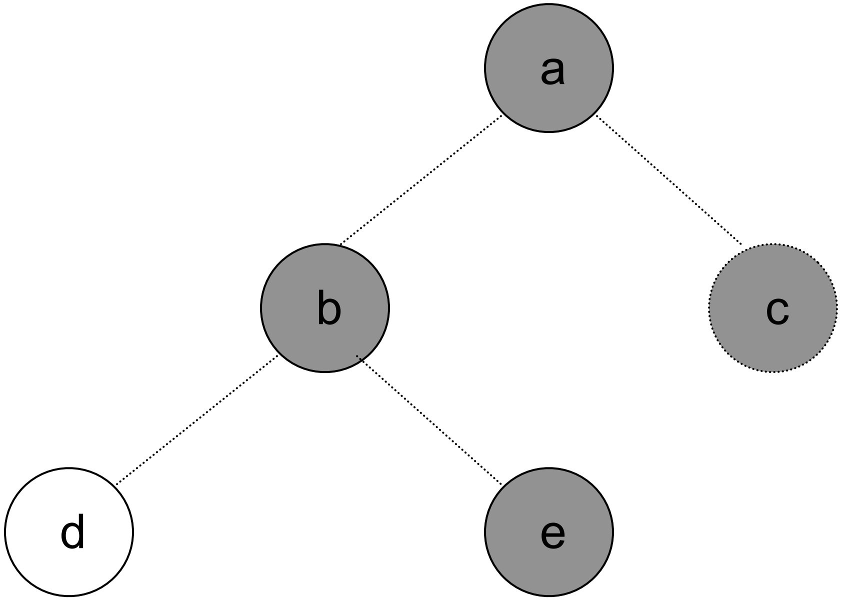


stack:

a
e
c

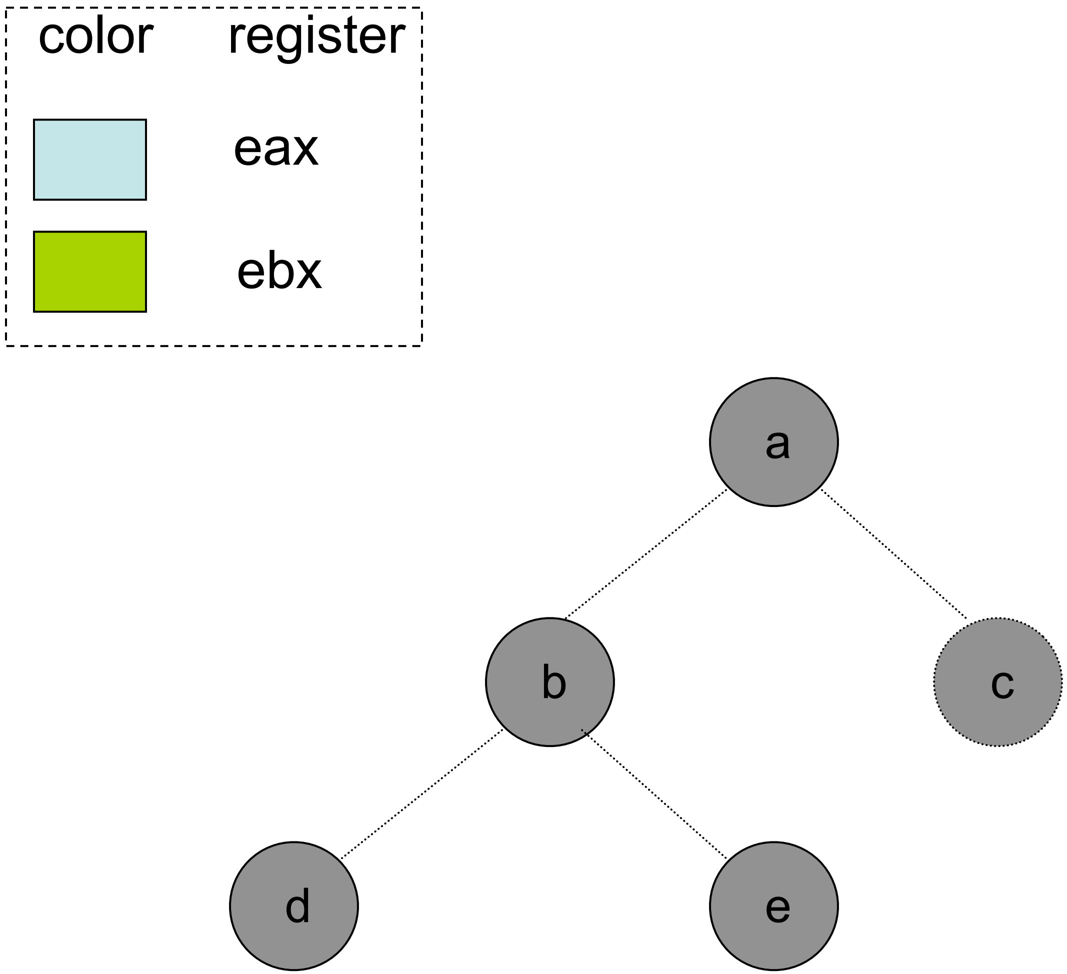
Coloring





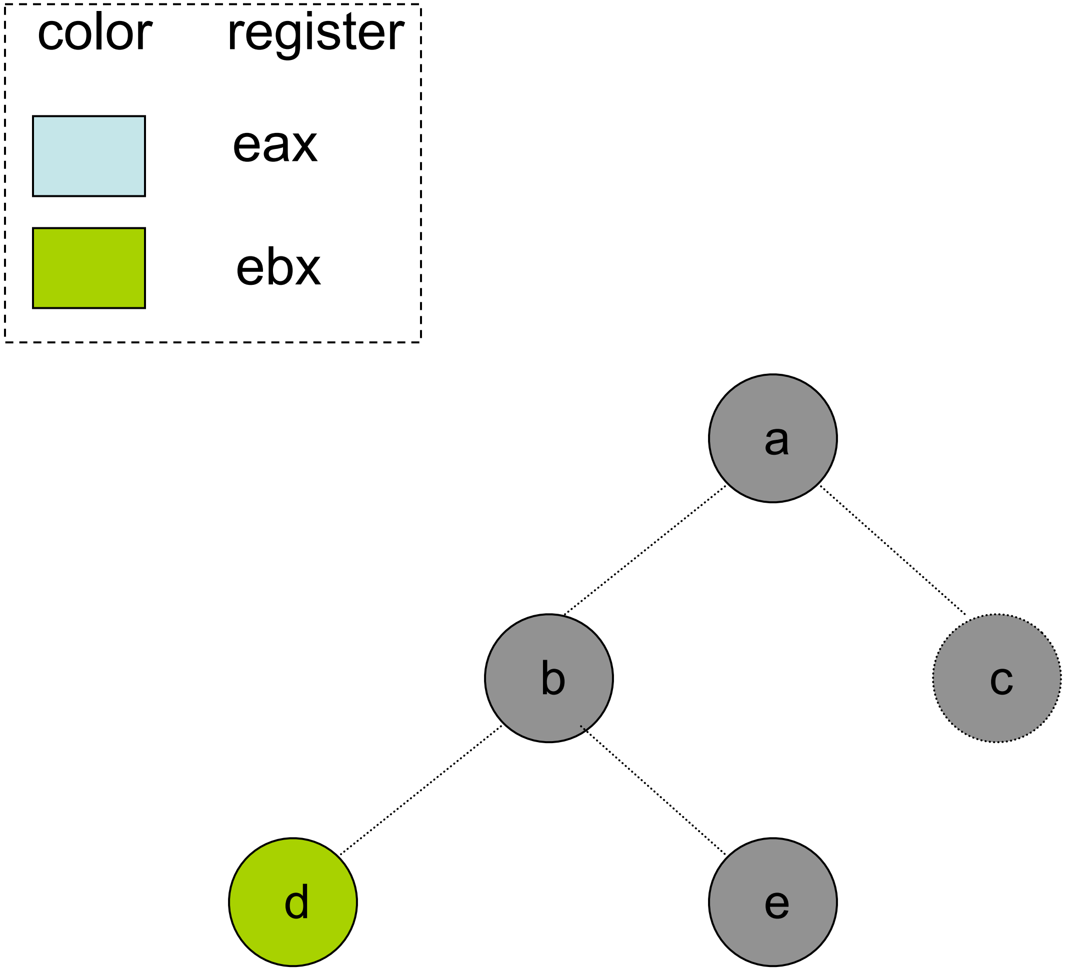
stack:
b
a
e
c

Coloring



stack:
d
b
a
e
c

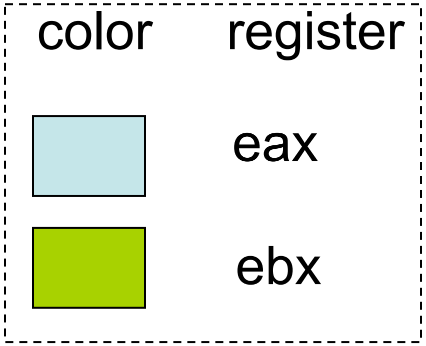
Coloring

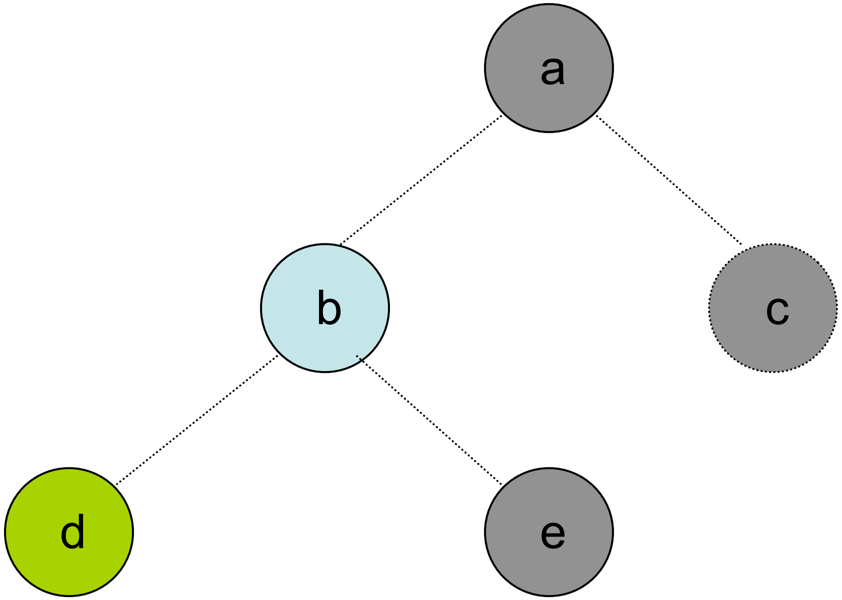


stack:

b
a
e
c

Coloring

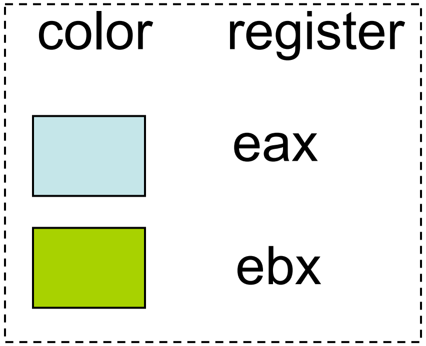


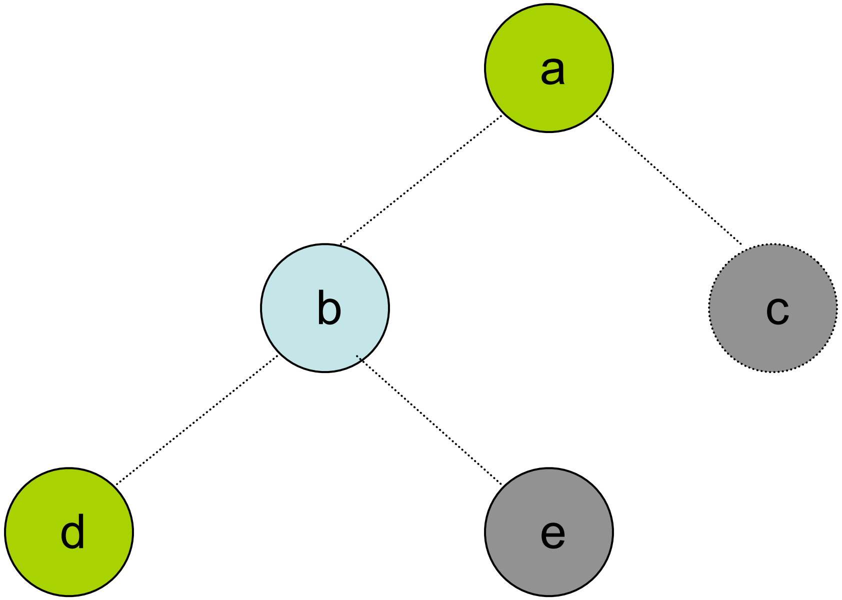


stack:

a
e
c

Coloring

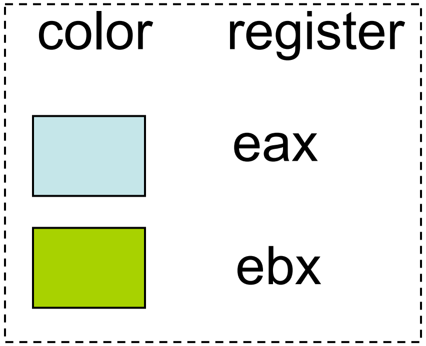


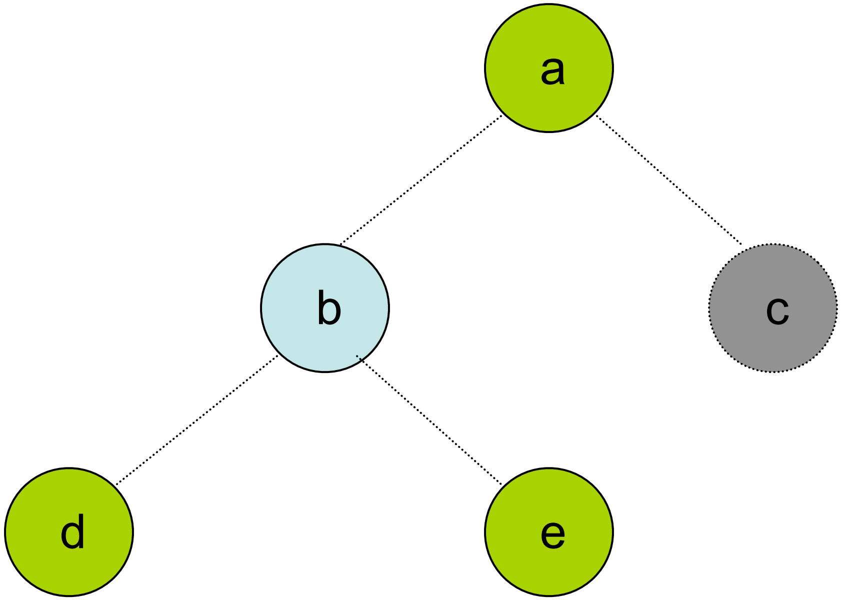


stack:

e
c

Coloring

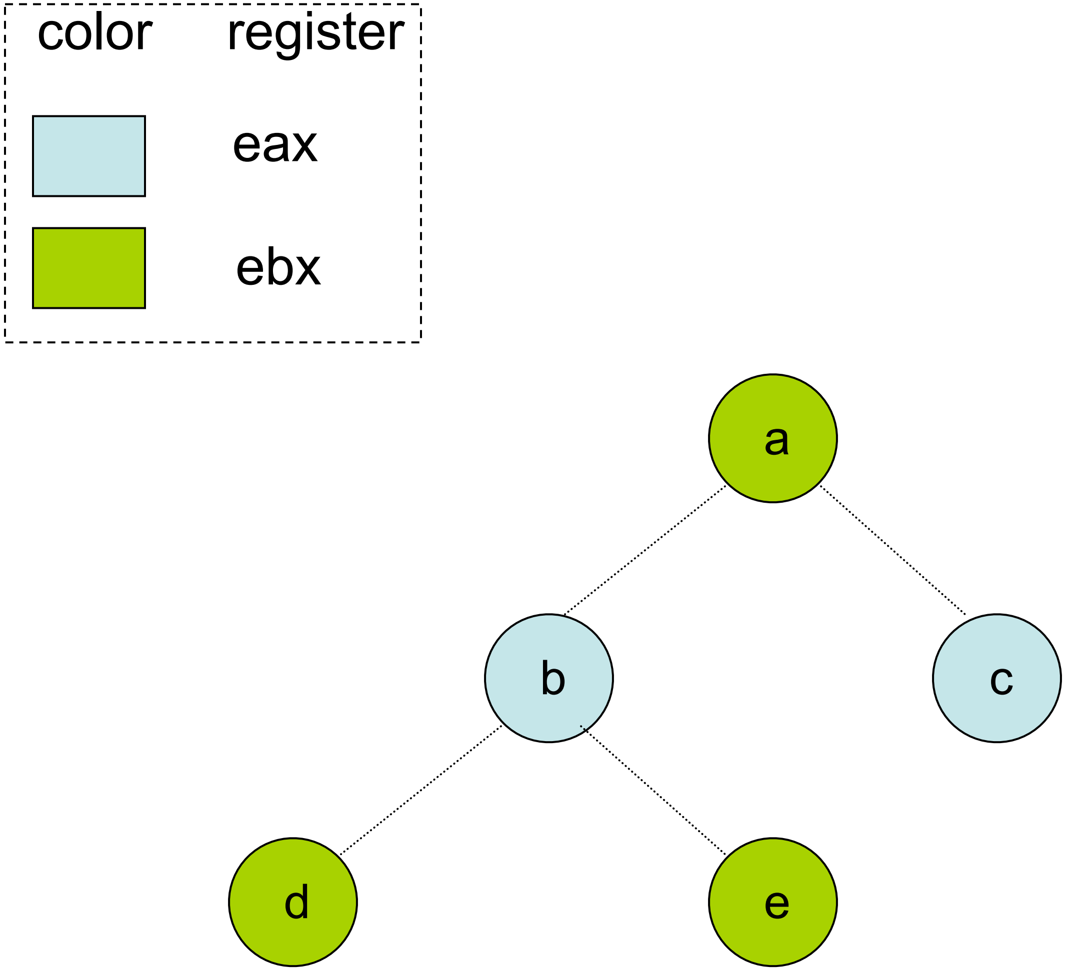




stack:

c

Coloring

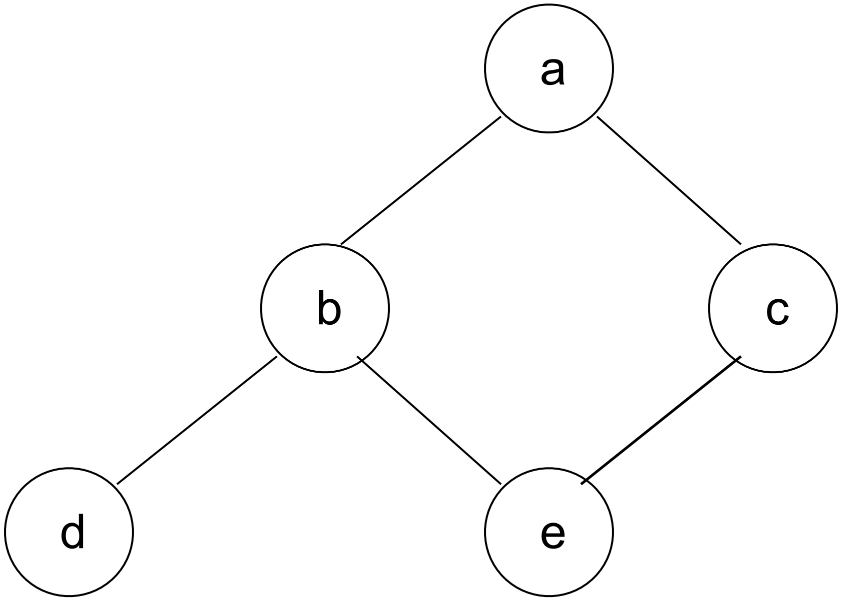


stack:

Failure

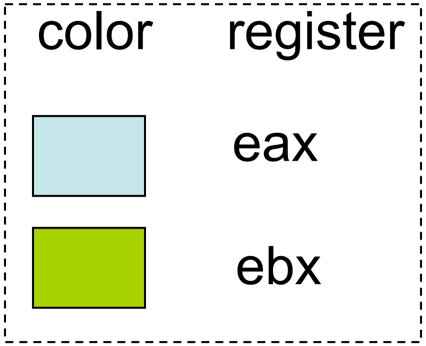
- If the graph cannot be colored, it will eventually be simplified to graph in which every node has at least K neighbors
- Sometimes, the graph is still K-colorable!
- Finding a K-coloring in all situations is an NP-complete problem
  - We will have to approximate to make register allocators fast enough

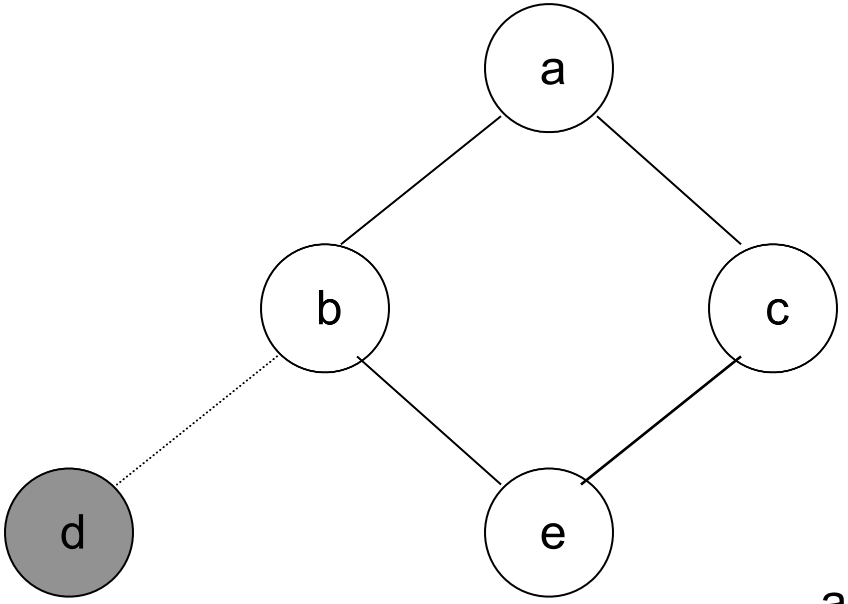
Coloring



stack:

Coloring



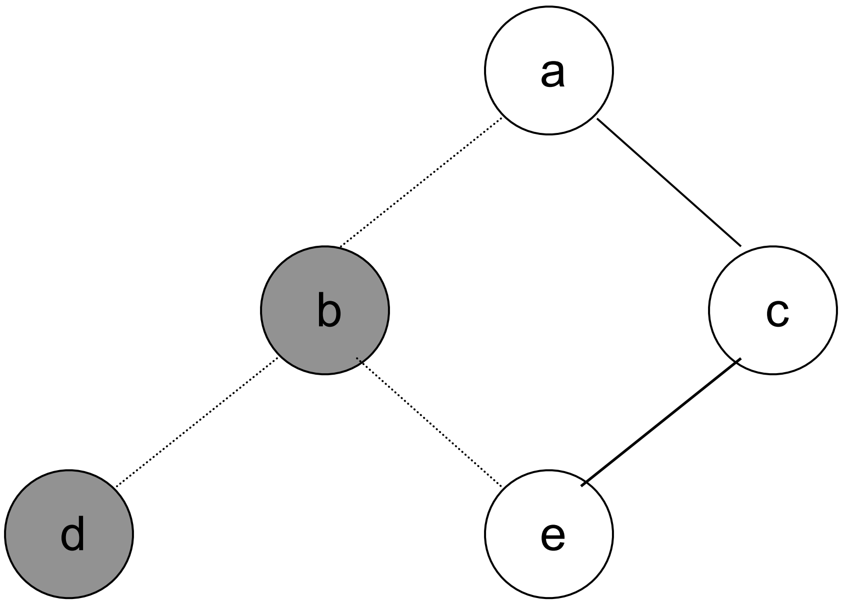


stack:
d

all nodes have
2 neighbours!

Coloring

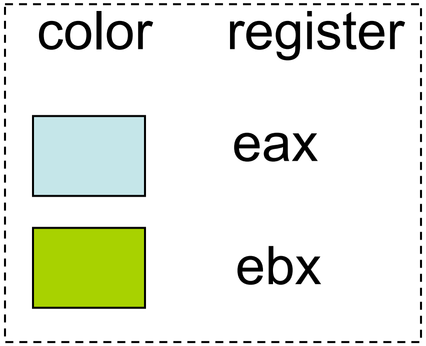
color	register
(light blue)	eax
(green)	ebx

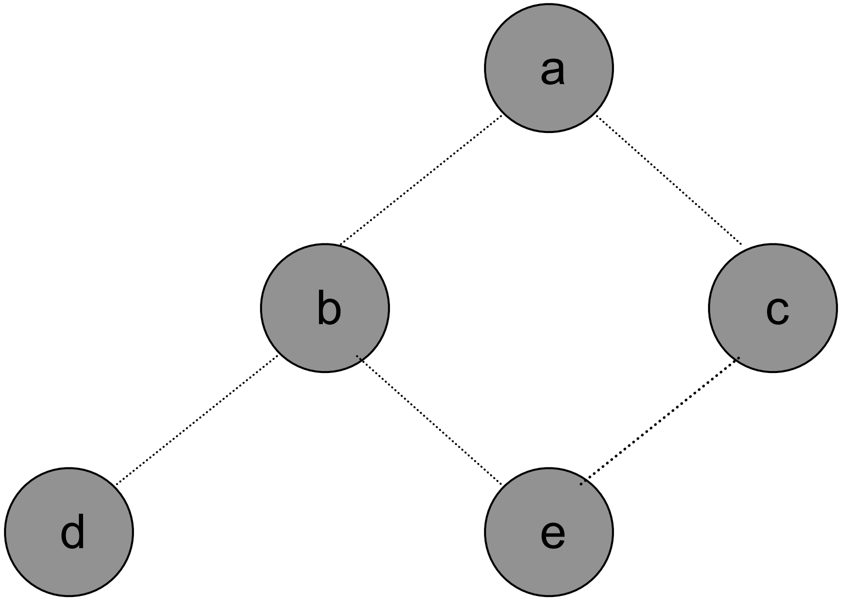


stack:

b
d

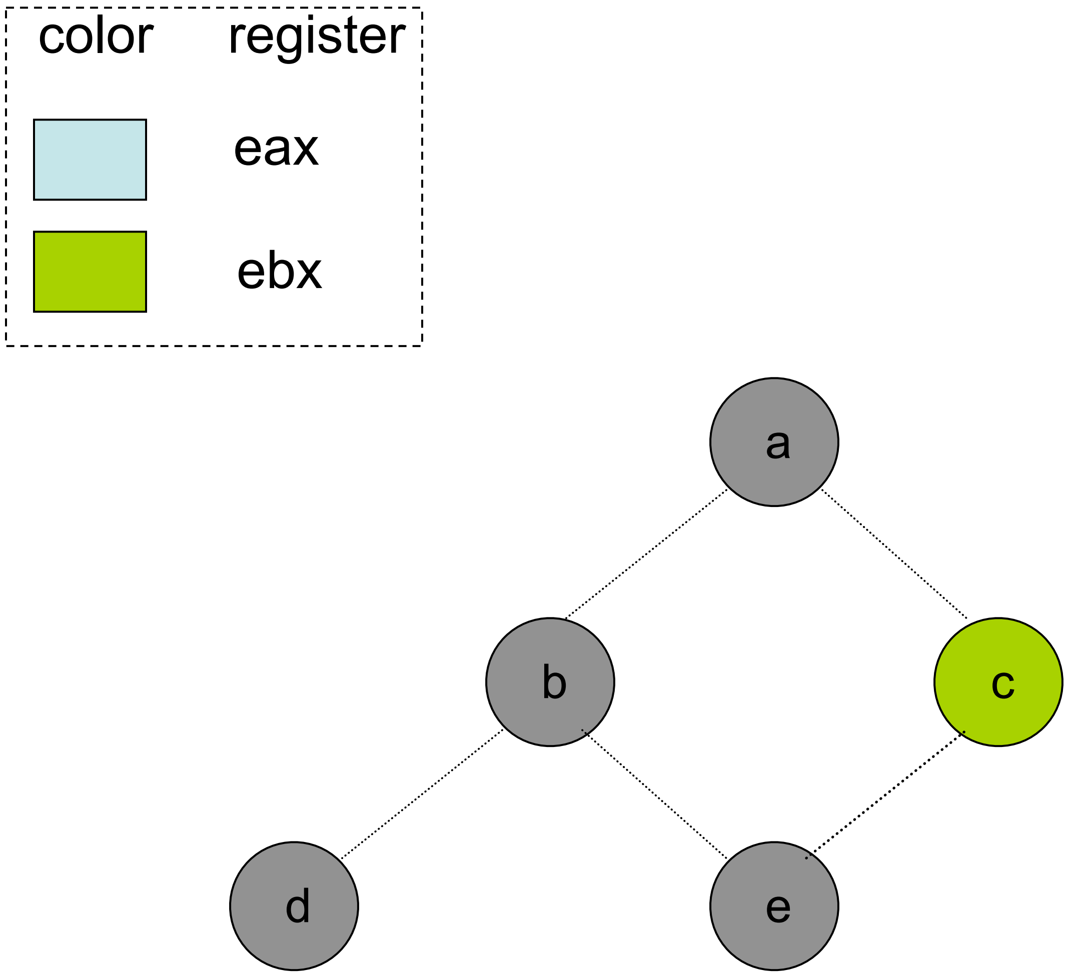
Coloring





stack:
c
e
a
b
d

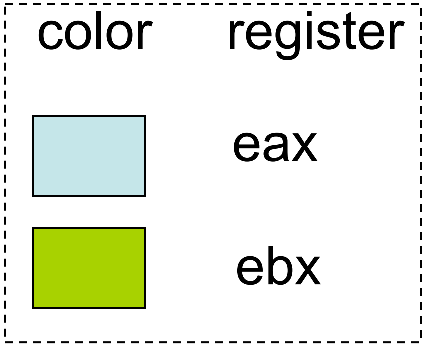
Coloring

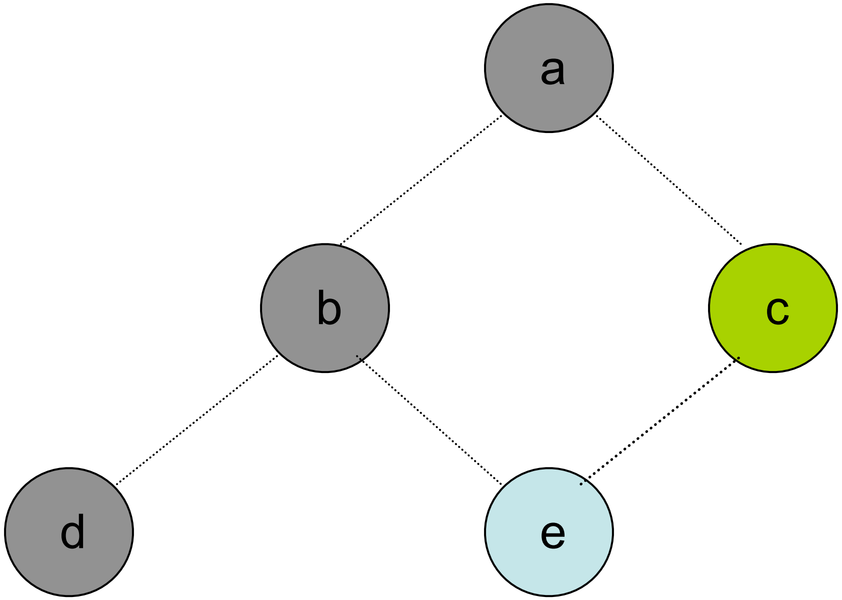


stack:

e
a
b
d

Coloring

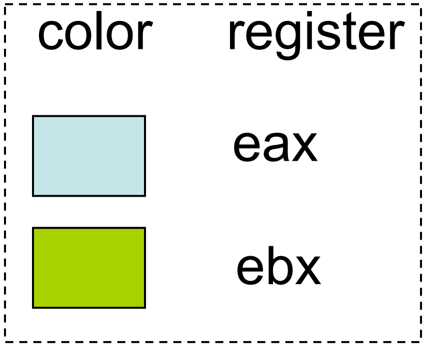


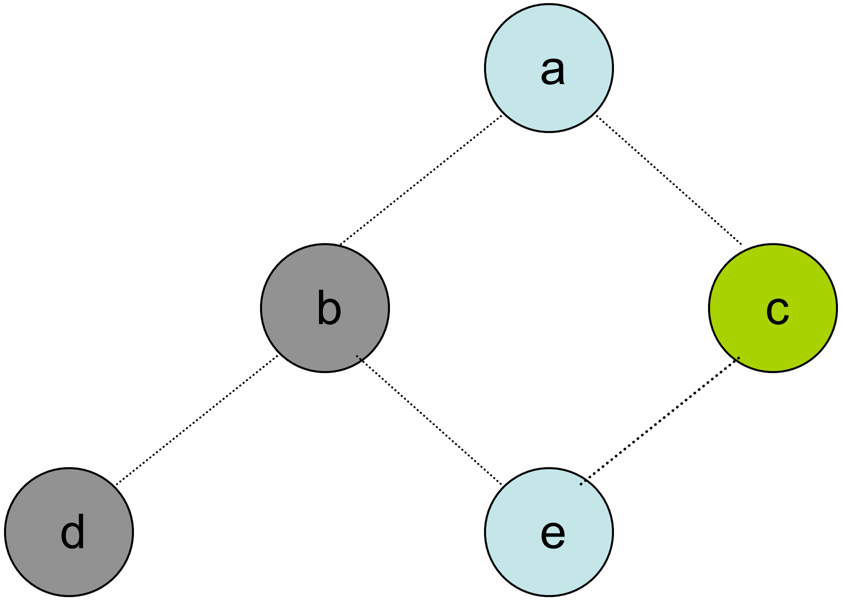


stack:

a
b
d

Coloring

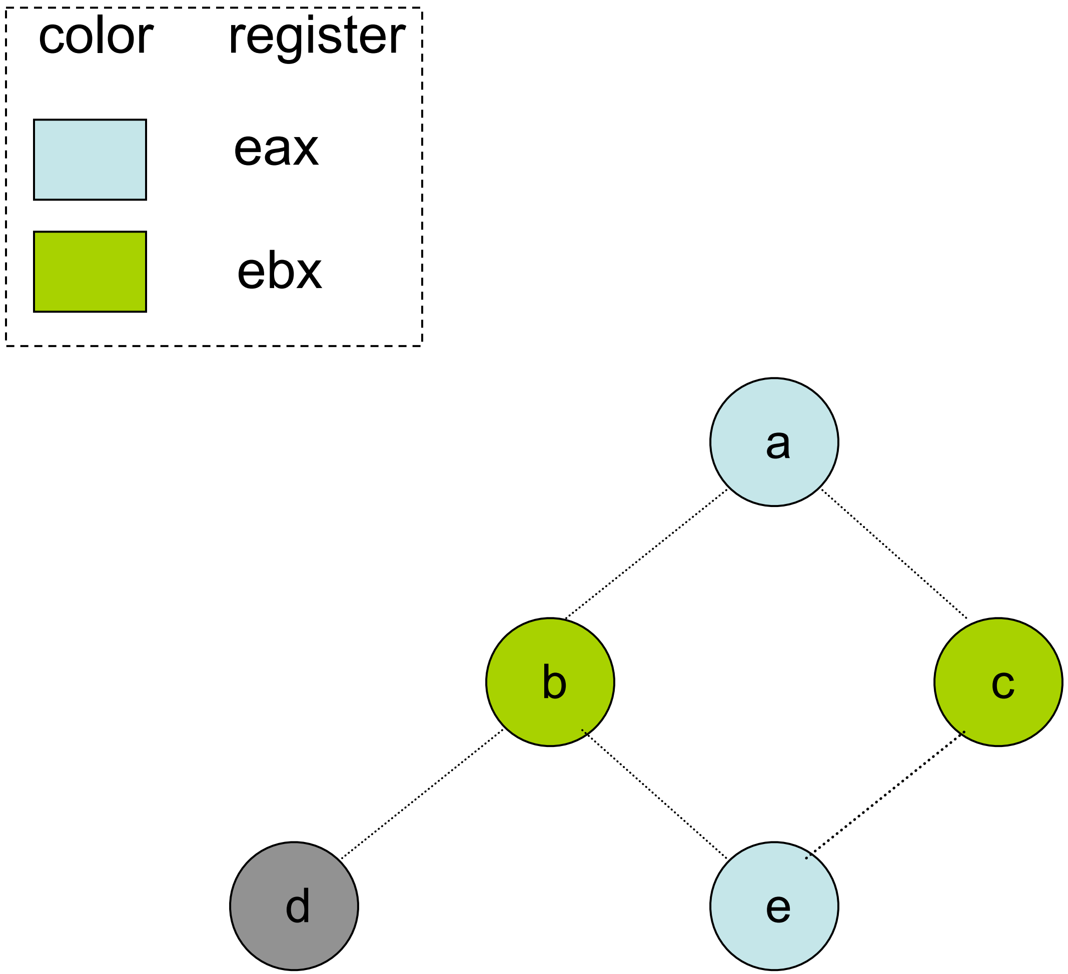




stack:

b
d

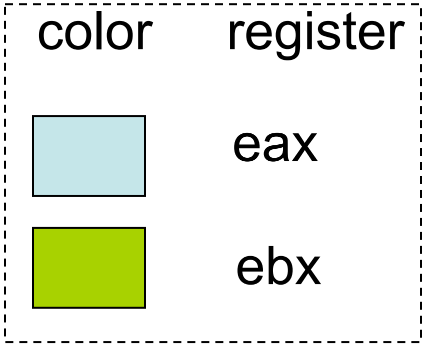
Coloring

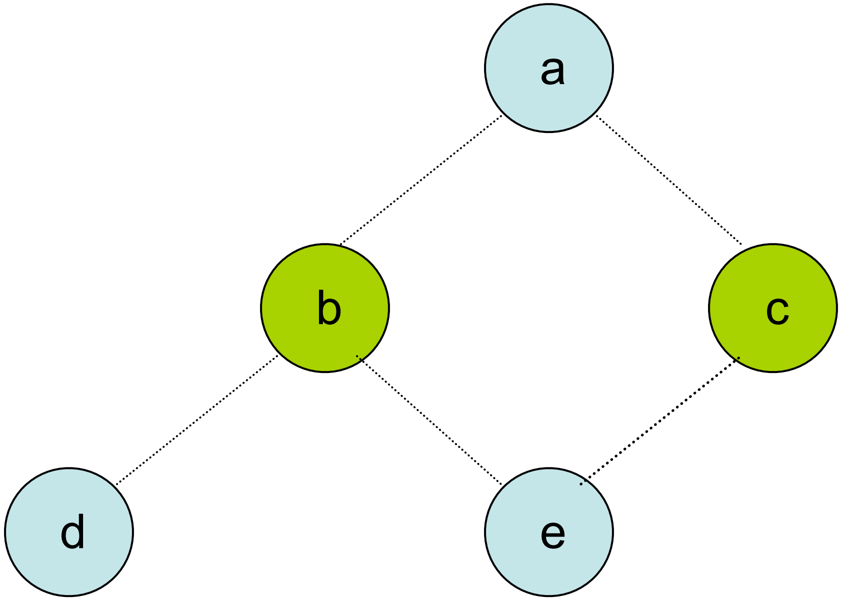


stack:

d

Coloring

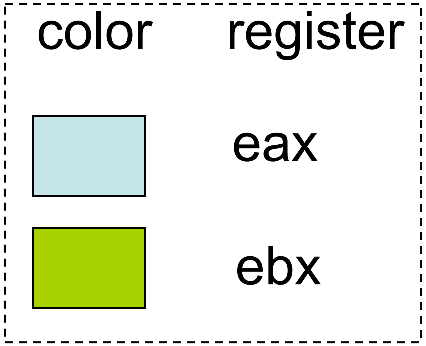




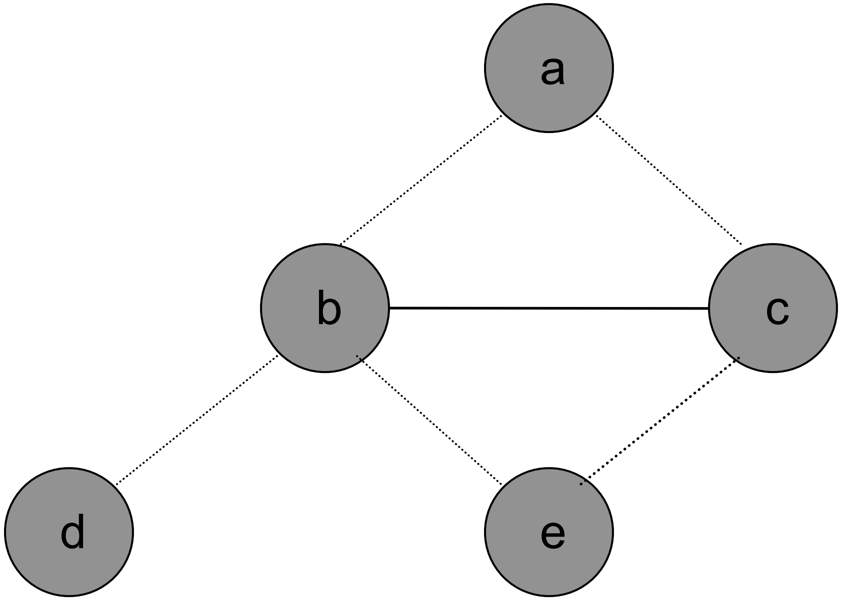
stack:

We got lucky!

Coloring



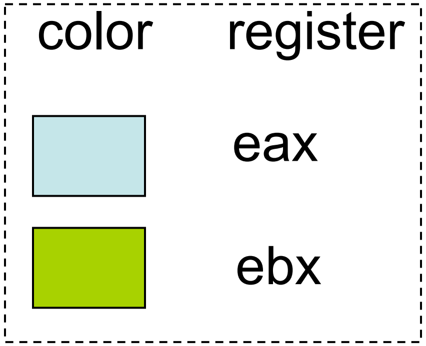
Some graphs can't be colored
in K colors:



stack:

c
b
e
a
d

Coloring



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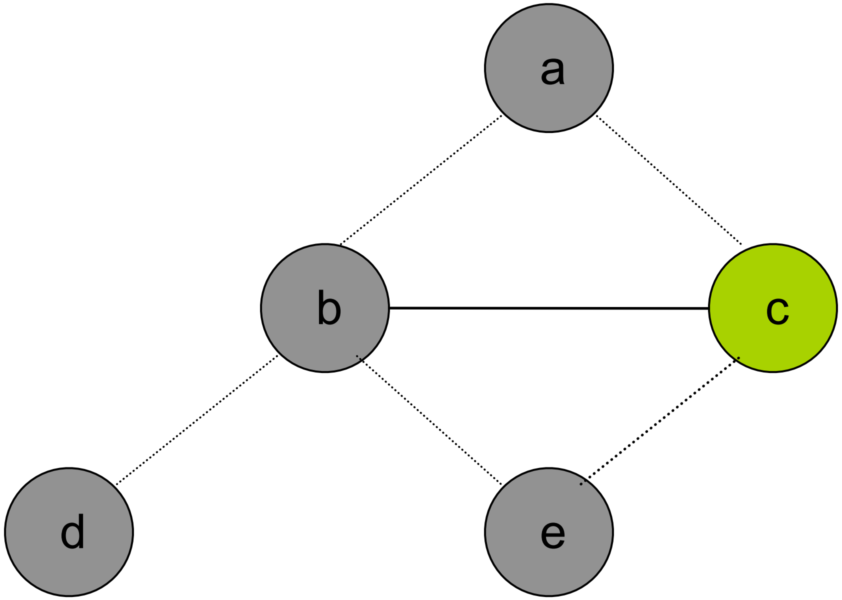
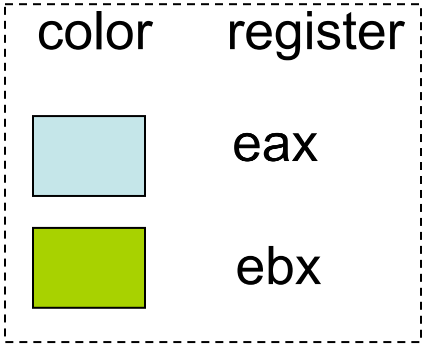


stack:

b
e
a
d

Coloring



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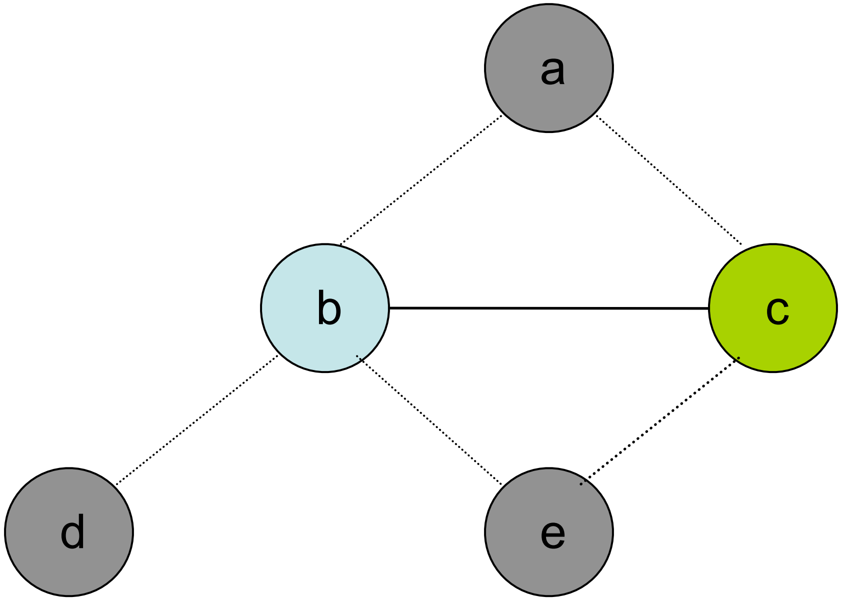
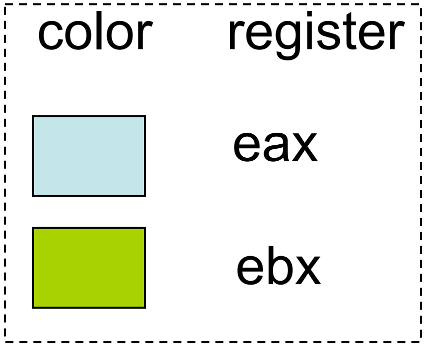


stack:

e
a
d

Coloring



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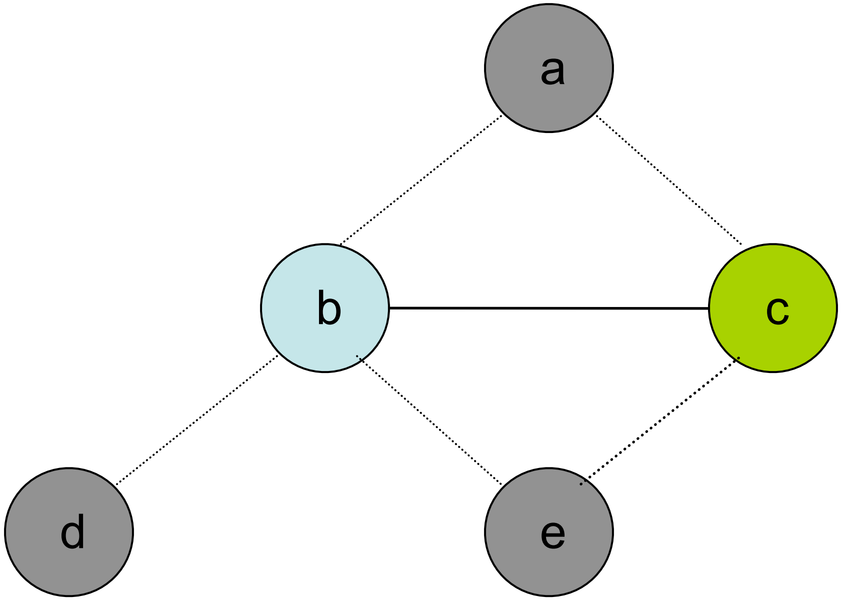


stack:

e
a
d

no colors left for e!

Spilling

- Step 3 (spilling): once all nodes have K or more neighbors, pick a node for spilling
  - Storage on the stack
- There are many heuristics that can be used to pick a node
  - not in an inner loop

Spilling code

- We need to generate extra instructions to load variables from stack and store them
- These instructions use registers themselves. What to do?
  - Stupid approach: always keep extra registers handy for shuffling data in and out: what a waste!
  - Better approach: rewrite code introducing a new temporary; rerun liveness analysis and register allocation
    - Intuition: you were not able to assign a single register to the variable that was spilled but there may be a free register available at each spot where you need to use the value of that variable

Rewriting code

- Consider: add t1 t2
  - Suppose t2 is selected for spilling and assigned to stack location [ebp-24]
  - Invent new temporary t35 for just this instruction and rewrite:
    - mov t35, [ebp – 24];
    - add t1, t35
  - Advantage: t35 has a very short live range and is much less likely to interfere.
  - Rerun the algorithm; fewer variables will spill

Precolored Nodes

- Some variables are pre-assigned to registers
  - Eg: mul on x86/pentium
    - uses eax; defines eax, edx
  - Eg: call on x86/pentium
    - Defines (trashes) caller-save registers eax, ecx, edx
- Treat these registers as special temporaries; before beginning, add them to the graph with their colors

Precolored Nodes

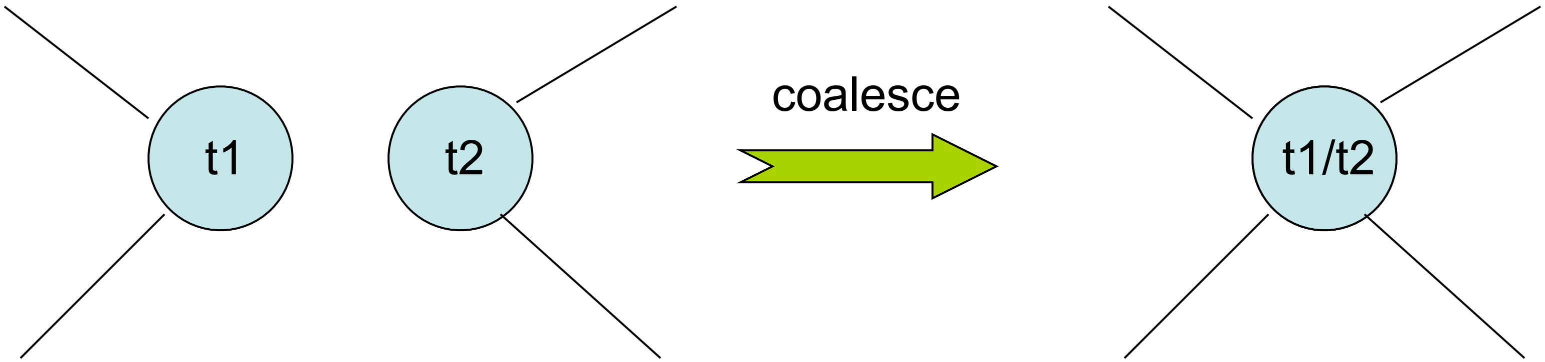
- Can’t simplify a graph by removing a precolored node
- Precolored nodes are the starting point of the coloring process
- Once simplified down to colored nodes start adding back the other nodes as before

Optimizing Moves

- Code generation produces a lot of extra move instructions
  - mov t1, t2
  - If we can assign t1 and t2 to the same register, we do not have to execute the mov
  - Idea: if t1 and t2 are not connected in the interference graph, we coalesce into a single variable

Coalescing

- Problem: coalescing can increase the number of interference edges and make a graph uncolorable



- Solution 1 (Briggs): avoid creation of high-degree (>= K) nodes
- Solution 2 (George): a can be coalesced with b if every neighbour t of a:
  - already interferes with b, or
  - has low-degree (< K)

Simplify & Coalesce

- Step 1 (simplify): simplify as much as possible without removing nodes that are the source or destination of a move (move-related nodes)
- Step 2 (coalesce): coalesce move-related nodes provided low-degree node results
- Step 3 (freeze): if neither steps 1 or 2 apply, freeze a move instruction: registers involved are marked not move-related and try step 1 again

Overall Algorithm