Name: ____________________________________________

Purdue Email: ______________________________________

Please sign the following:

I affirm that the answers given on this test are mine and mine alone. I did not receive help from any person or material (other than those explicitly allowed).

X ________________________________

| Part 1:       | /30 |
| Part 2:       | /30 |
| Part 3:       | /15 |
| Part 4:       | /25 |
| Total:        | /100 |
Part 1: Dataflow analysis (30 pts)

In this problem, you will design a dataflow analysis that performs a sign analysis. A sign analysis determines at each program point whether a variable is definitely positive (+), definitely negative (−), definitely zero (0), or could be anything (?).

Problem 1 (10 pts): Provide a lattice for this dataflow analysis. A few things to keep in mind: (i) don’t forget that having no information is different from a variable being “anything”; (ii) if a variable is 0, it also counts as both + and −; (iii) ? means that a variable might be positive or might be negative.

The lattice for this analysis is given above. Note that 0 is below + and −, because it counts as both. If we merge information coming from two branches, with one saying that x is 0 and the other saying that x is −, we want the result to be −.
Problem 2 (1 pts): Should this analysis be forward or backward?

This is very similar to a constant propagation analysis; we want it to be forward. Whether a variable is + or - depends on what has happened to the variable in the past.

Problem 3 (1 pts): Does this analysis use meet or join at merge points?

We should use join (think about the discussion of merging in the lattice solution).

Problem 4: For each statement, define the transfer function. Each subproblem shows one or more tables (except part (D)). The column headers give the value of $y_{in}$ and the row headers (if the exist) give the value of $z_{in}$. For each subproblem, fill in the table with the value of $x_{out}$ (i.e., the value that $x$ takes after the statement is processed) given the input values.

A) $x := y \times z$ (4.5 pts)

<table>
<thead>
<tr>
<th></th>
<th>+</th>
<th>-</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

B) $x := y - z$ (4.5 pts)

<table>
<thead>
<tr>
<th></th>
<th>+</th>
<th>-</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>?</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

C) $x := y \times y$ (2 pts)

<table>
<thead>
<tr>
<th></th>
<th>+</th>
<th>-</th>
<th>0</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Note that ? * ? = + because $y \times y$ is either 0 or +, which is best approximated by +.

D) if (x < 0) then {...} else {...} (1 pt)

For this subproblem, give the values of $x$ along the true branch and the false branch:
$x_{true}: -$ 

$x_{false}: +$ (we don’t use ? because $x_{false}$ is either 0 or +, which is best approximated with +.
Problem 5 (6 pts): For the following CFG, fill in the boxes with the value of $x$ and $y$ that you get after performing your dataflow analysis (assume that $x$ and $y$ are arguments to the function, and hence could have any value at the beginning).
Part 2: Dependence analysis and loop optimization (30 pts)

For the next four problems, consider the following loop:

for (int i = 1; i < 6; i++) {
    for (int j = 1; j < 6; j++) {
        A[2i][j] = A[i+1][j-1];
    }
}

Problem 1 (10 pts): Below is the iteration space graph for the loop nest (i along the x-axis, j along the y-axis). Draw the dependence arrows that arise from analyzing the loop.

Problem 2 (2 pts): List the distance vectors that arise from this loop nest.

(0,1), (1,1), (2,1)

Problem 3 (2 pts): List the direction vectors that arise from this loop nest.

(0,+), (+, +).

Problem 4 (2 pts): Can the loops in this nest be interchanged? Why or why not?

Yes—none of the dependences get violated if the i and j loops are interchanged.
Problem 5 (2 pts): Give an example of a direction vector which prevents loop interchange.

\((-,+,-)\)

For the next two problems, consider the following loop:

```
for (int i = 0; i < N; i++) {
}
```

Problem 6 (2 pts): Give an informal (1-2 sentence) argument for why there would be no loop-carried dependences in this loop.

\(A[2i+1]\) will only write to odd locations, while \(A[4i-2]\) will only read from even locations.

Problem 7 (8 pts): Use the GCD test to prove there are no dependences in this loop (show your work!)

For there to be a dependence in the loop, there must be an \(i\) and \(i'\) such that:

\[2i + 1 = 4i' - 2\]

Rearranging, we get:

\[4i' - 2i = 3\]

The GCD test tells us this equation will have a solution provided \(gcd(4, 2)\) divides 3. However, \(gcd(4, 2) = 2\), which does not divide 3. Hence, there is no solution to the equation, and there is no dependence.

Problem 8 (2 pts): Why might the GCD test say that there is a dependence in a loop even when there isn’t?

The GCD test does not consider loop bounds. The solution to the equation might require variable values that fall outside the bounds of the loop.
Part 3: Pointer analysis (15 pts)

Problem 1 (2 pts): What is the difference between a weak update and a strong update when doing pointer analysis?

Not part of 468 exam material. But the difference is that a weak update does not remove any existing information, while a strong update does.

Problem 2 (5 pts): Why does \( *x = y \) require a weak update when doing flow-sensitive pointer analysis?

Not part of 468 exam material. \( *x = y \) requires a weak update because we don’t know what \( x \) points to. Hence, we don’t know which variable’s points-to set needs to be updated.

Problem 3 (8 pts): Consider a flow-sensitive analysis where all strong updates have been replaced with weak updates. Will this analysis produce different results from a flow-insensitive analysis like Andersen’s? Why or why not? (Hint: consider applying both analyses to a function with no branching control flow—\( i.e., \) a single basic block).

Not part of 468 exam material. This analysis can produce different results, however.
Part 4: Review (25 pts)

Problem 1 (5 pts): Give the grammar for a language that cannot be parsed by an LL(1) parser, but can be parsed by an LL(2) parser.

\[
A \rightarrow xyA \\
A \rightarrow xx
\]

This can’t be parsed by an LL(1) parser because at the predict stage we wouldn’t be able to distinguish between the two rules with just one character of lookahead.

Problem 2 (10 pts): Fill in the CFSM for the following grammar (label lines with their transitions, fill in any missing boxes):

\[
S \rightarrow Aab$
A \rightarrow aA$
A \rightarrow c
\]
Consider the problem of allocating temporaries to registers for the following piece of code:

\[
\begin{align*}
T1 &= A + B; \\
T2 &= C + D; \\
T3 &= T2 + E; \\
T4 &= T1 + T2; \\
T5 &= F + T2; \\
T6 &= T1 + T4; \\
T7 &= T6 + T5;
\end{align*}
\]

**Problem 3 (5 pts):** How many registers do you need to allocate temporaries to registers without spilling?

We begin by performing a liveness analysis on the code. Each entry shows which variables are live *after* the statement is executed.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Live Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T1 = A + B; )</td>
<td>( T1 )</td>
</tr>
<tr>
<td>( T2 = C + D; )</td>
<td>( T1, T2 )</td>
</tr>
<tr>
<td>( T3 = T2 + E; )</td>
<td>( T1, T2 )</td>
</tr>
<tr>
<td>( T4 = T1 + T2; )</td>
<td>( T1, T2, T4 )</td>
</tr>
<tr>
<td>( T5 = F + T2; )</td>
<td>( T1, T4, T5 )</td>
</tr>
<tr>
<td>( T6 = T1 + T4; )</td>
<td>( T6, T5 )</td>
</tr>
<tr>
<td>( T7 = T6 + T5; )</td>
<td></td>
</tr>
</tbody>
</table>

We see that the most number of temporaries that are live at a time is 3, so it would take three registers to perform register allocation without spilling.

**Problem 4 (5 pts):** Draw the interference graph for the code.

Not part of 468 exam material. You can draw this graph by creating one node per temporary, and drawing lines between temporaries that are live at the same time.