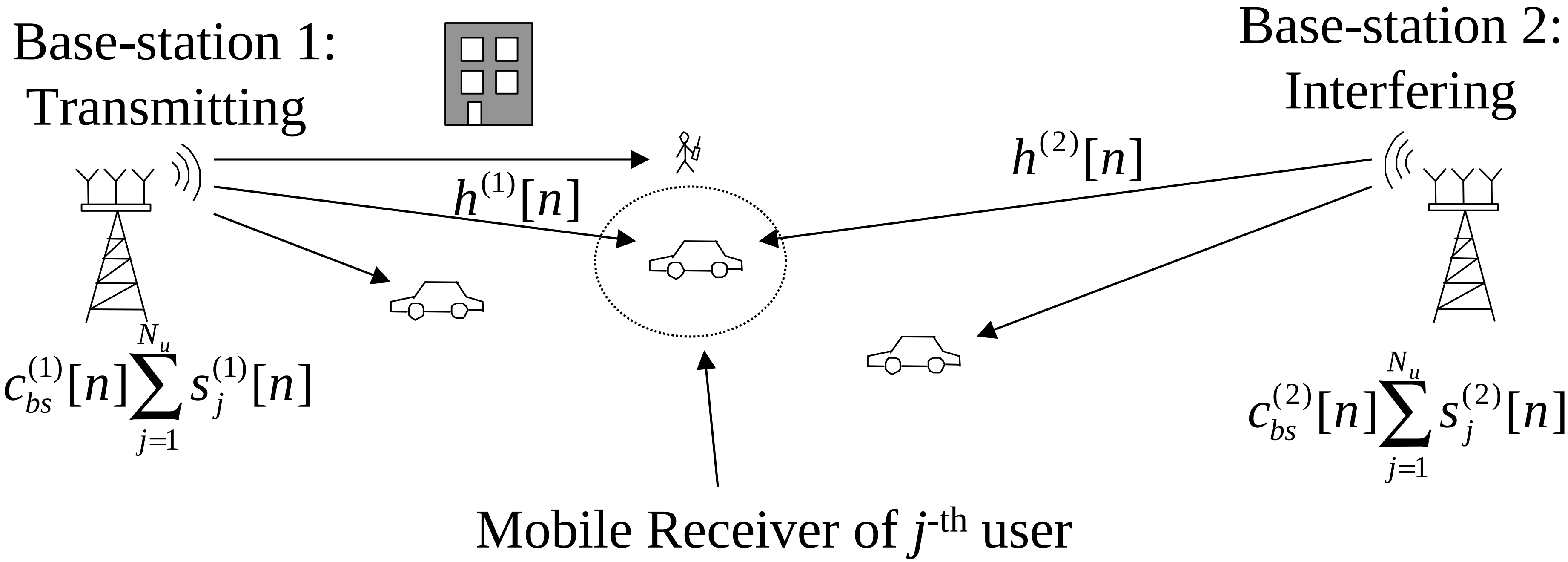
INTRO: Linear Equalizers for CDMA Downlink



- Down-link: bottle-neck in future cellular / PCS systems
  - higher demand for data
  - internet usage is download oriented
- CDMA cell capacity limited by Multi-Access Interference (MAI)
- How can we effectively reduce MAI?
- Are multiple antennas at the mobile justified?

IN A NUTSHELL

Downlink specific linear equalizers

Features:

- Equalizer restores "chip sequence", then followed by despreading with channel code times base station dependent long code (scrambling code)
- The equalizer is independent of the user's channel code
- Only need to know spreading code of desired user
- Equalizer is valid over coherence time of channel (don't need to design a new equalizer every symbol)

We examine "performance upper bound" assuming:

- perfect synchronization and knowledge of the channel
- unchanging channel

Base-station transmits "synchronous sum signal" at chip rate

$$s[n] = c_{bs}[n] \sum_{j=1}^{N_u} \sum_{m=0}^{N_s - 1} b_j[m] c_j[n - N_c m]$$

Base-station dependent long-code

BPSK information symbols, j-th user

Length N_c Walsh-Hadamard Channel code, j-th user

- While power due to any one user is small, combined signal is above the noise floor
- Some users may be assigned multiple codes (or use OVSF codes) to increase data rate

CDMA downlink - Problem

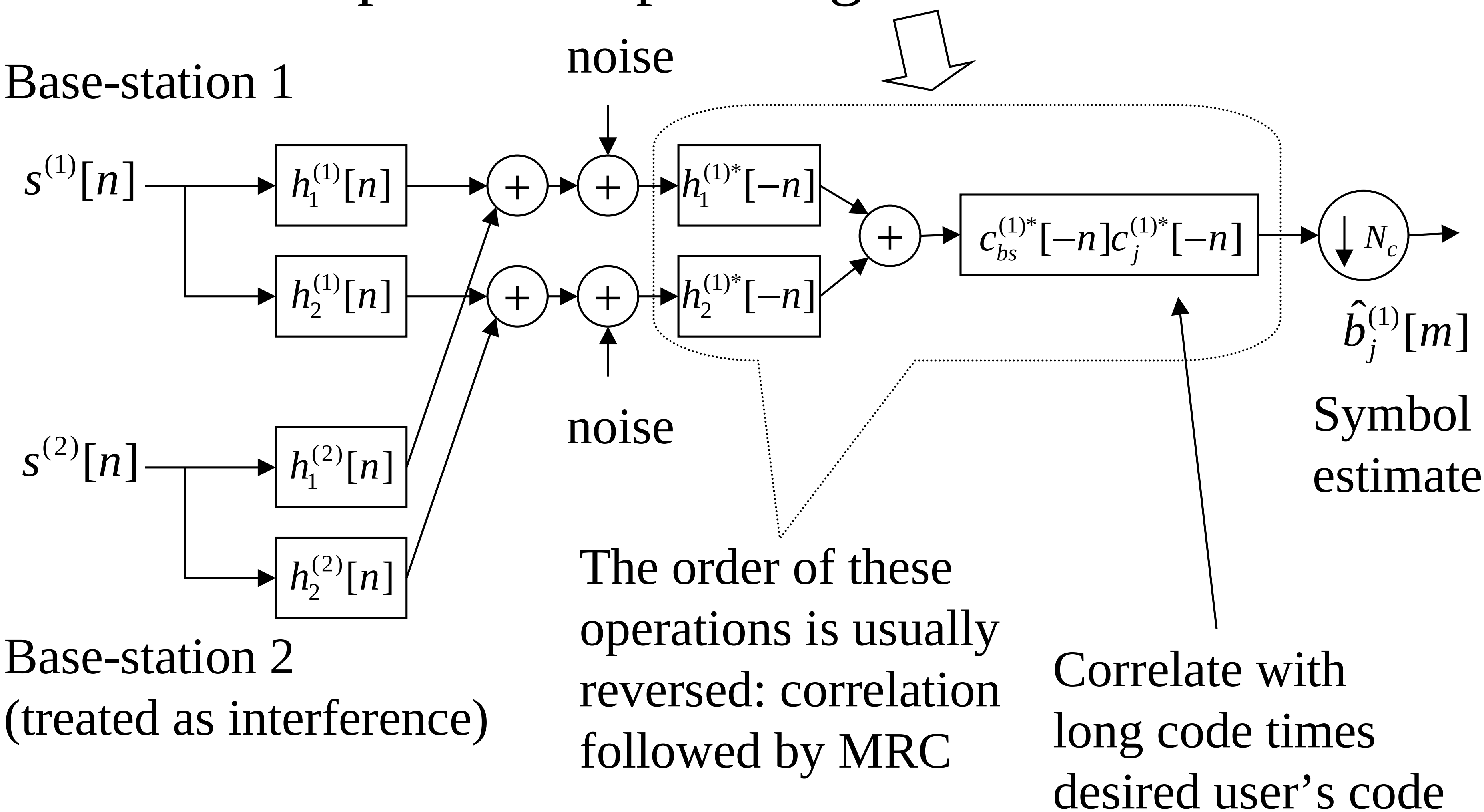
- Walsh-Hadamard channel codes are used in current and 3G standards - *Orthogonal*
  - allows separation of desired user's signal from the rest
- However, orthogonality of channel codes is lost when fading is frequency-selective
- Walsh-Hadamard channel codes have bad correlation properties
  - auto-correlation: self interference (ICI)
  - cross-correlation: multiple access interference (MAI)

Problem with RAKE Receiver

- Traditional RAKE receiver encounters strong interference from multiple delayed arrivals due to frequency selective multi-path
- Situation is even worse when at edge-of-cell: other Base-station's signals have significant interfering energy
- Multi-access interference (MAI) limits number of users
- MMSE equalizers can overcome problems
  - suppress interference
  - allow more users (or data) per cell (capacity)

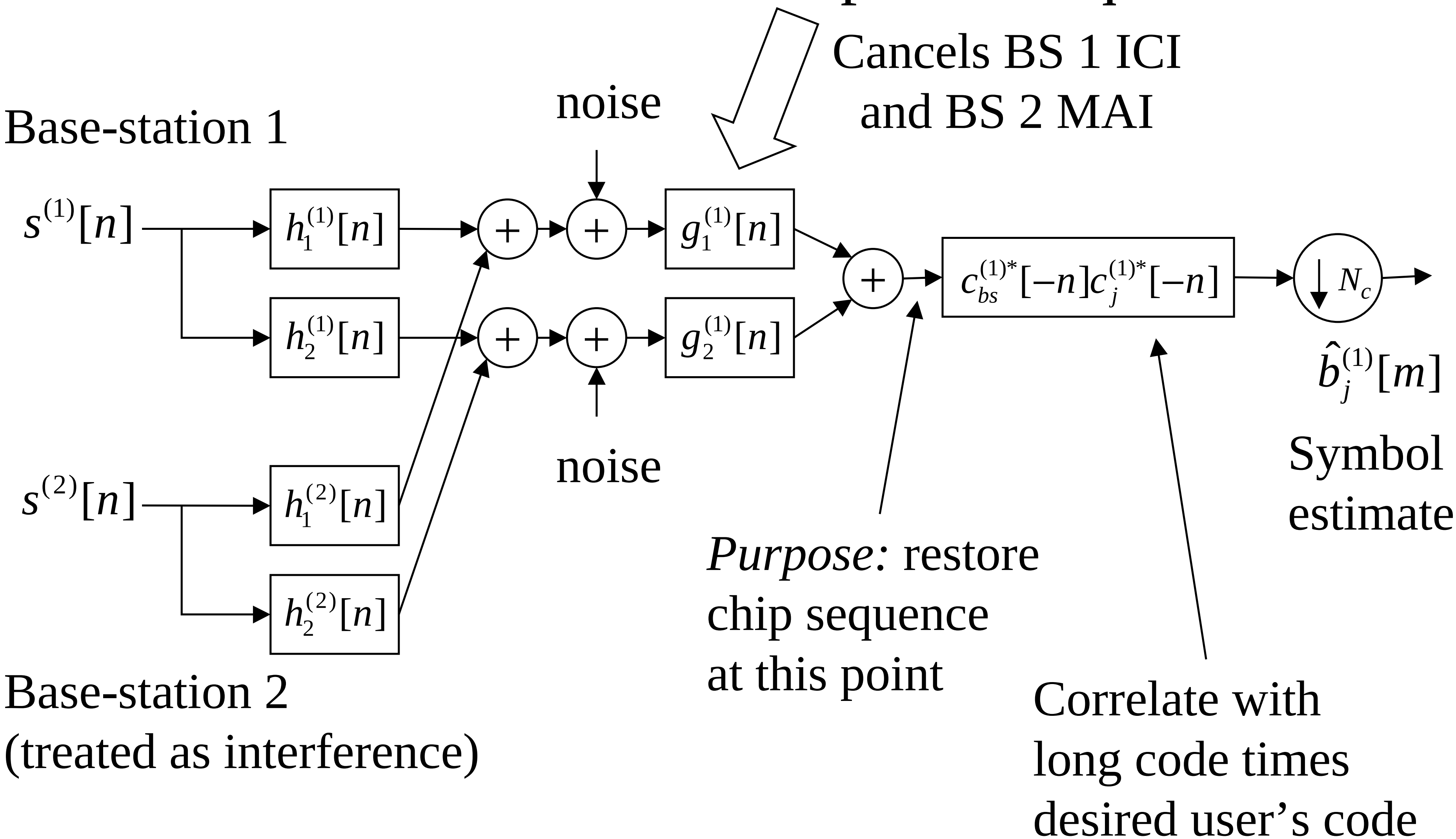
CONVENTIONAL APPROACH

Multi-channel RAKE Receiver:
Multi-path Incorporating Matched Filter



PROPOSED RECEIVER

Multi-channel MMSE Chip-level Equalizer



MMSE Criterion:

Minimize $\underset{\mathbf{g}^{(1)}}{\ }$ $E\{|\mathbf{g}^{(1)H}\mathbf{y}[n]-s^{(1)}[n-D]|^2\}$

Note: $E\{\mathbf{s}^{(k)}[n]\mathbf{s}^{(k)H}[n]\}=\mathbf{I}$
due to I.I.D. long code assumption

Solution:

$$\mathbf{g}^{(1)}=\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H+\mathbf{R}_{\eta\eta}\right)^{-1}\mathbf{H}^{(1)}\boldsymbol{\delta}_D$$

where $\tilde{\mathbf{H}}=\left[\mathbf{H}^{(1)} \quad \vdots \quad \mathbf{H}^{(2)}\right]$

$$\mathbf{R}_{\eta\eta}=E\{\boldsymbol{\eta}[n]\boldsymbol{\eta}^H[n]\}$$ Noise covariance matrix

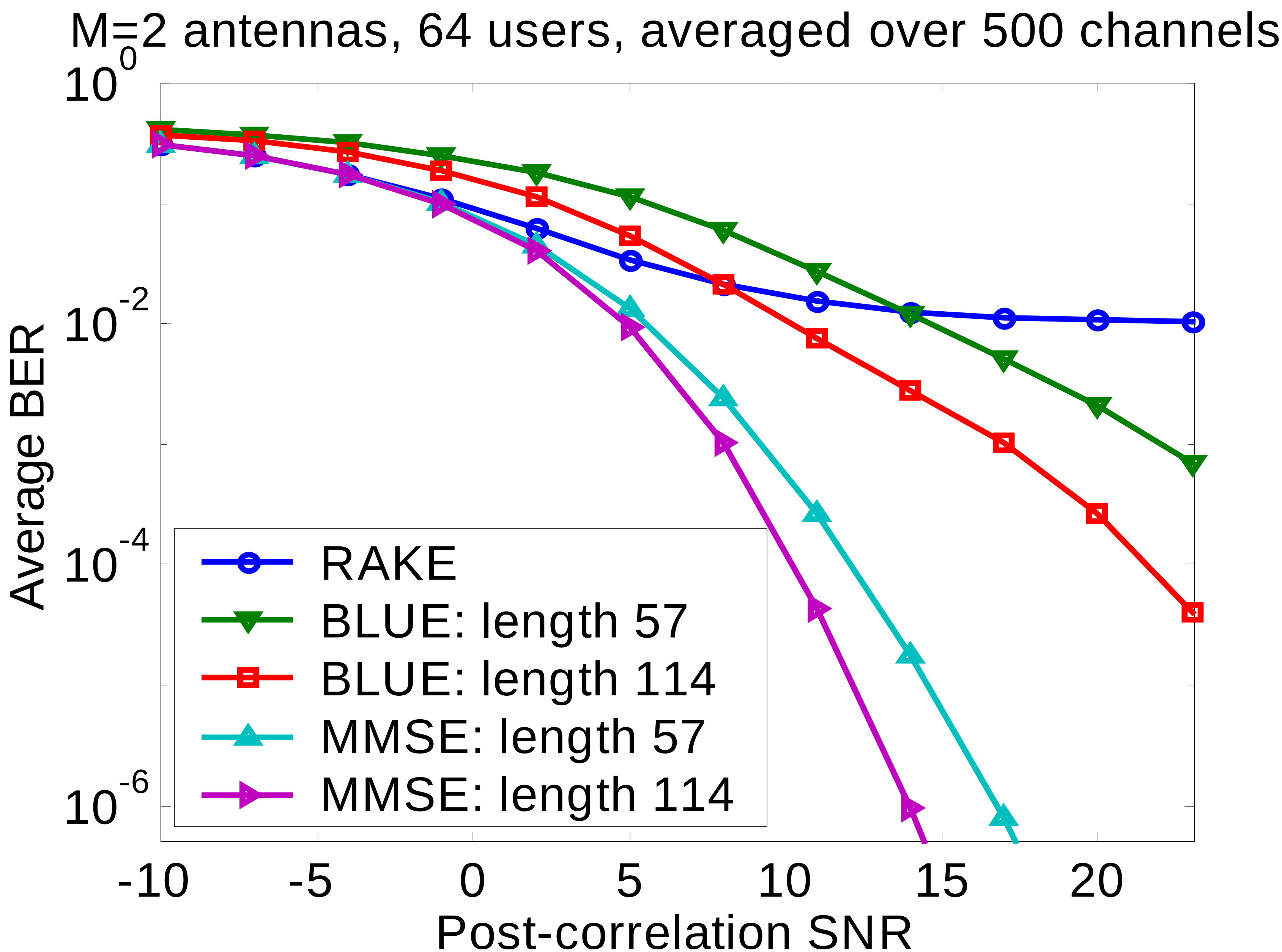
$$\boldsymbol{\delta}_D=\left[0 \quad \cdots \quad 0 \quad 1 \quad 0 \quad \cdots \quad 0\right]^T$$

(D+1)-th position

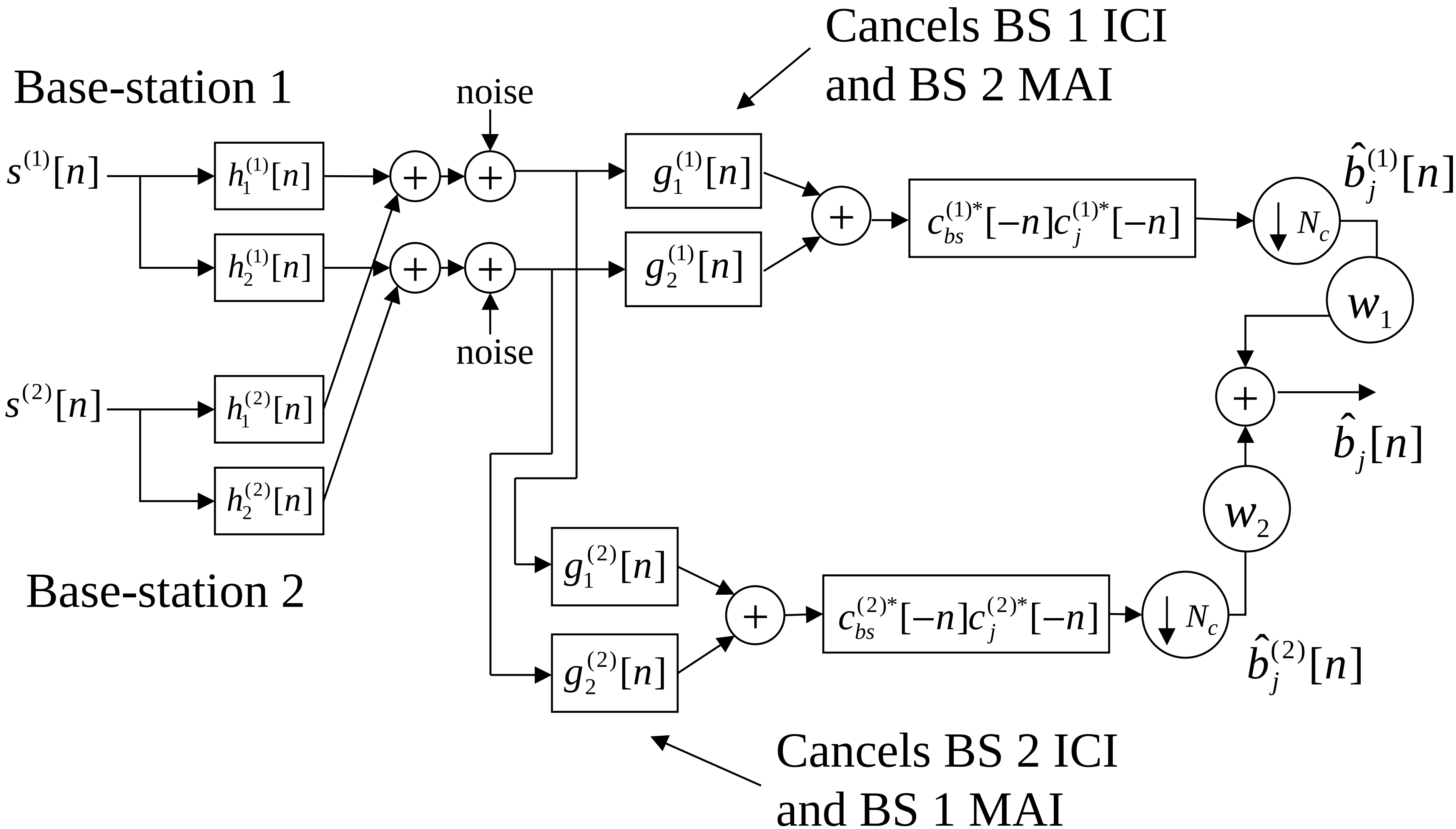
SIMULATIONS

- Saturated Cells - 64 equal power users
- 3.6864 MHz chip rate (T_c = .27 μs)
- BPSK data symbols
- Walsh-Hadamard channel codes, length 64
- Optimal delay D chosen
- QPSK scrambling code, length 32768 (IS-95)
- Square root raised cosine chip waveform, β=.22
- Chip matched filter used at receiver
- BER performance averaged over many channels

Average BER Performance, RAKE vs BLUE (ZF) vs MMSE
Equalizer Length = 57 or 114 Chips



Soft-hand-off: Combines Symbol Outputs of Two "Parallel" Equalizers



Soft Hand-off Operation: Derivation of weights

$$\hat{b}_1^{(1)}[m] = a_1 b_1[m] + \gamma_1$$

$$\hat{b}_1^{(2)}[m] = a_2 b_1[m] + \gamma_2$$

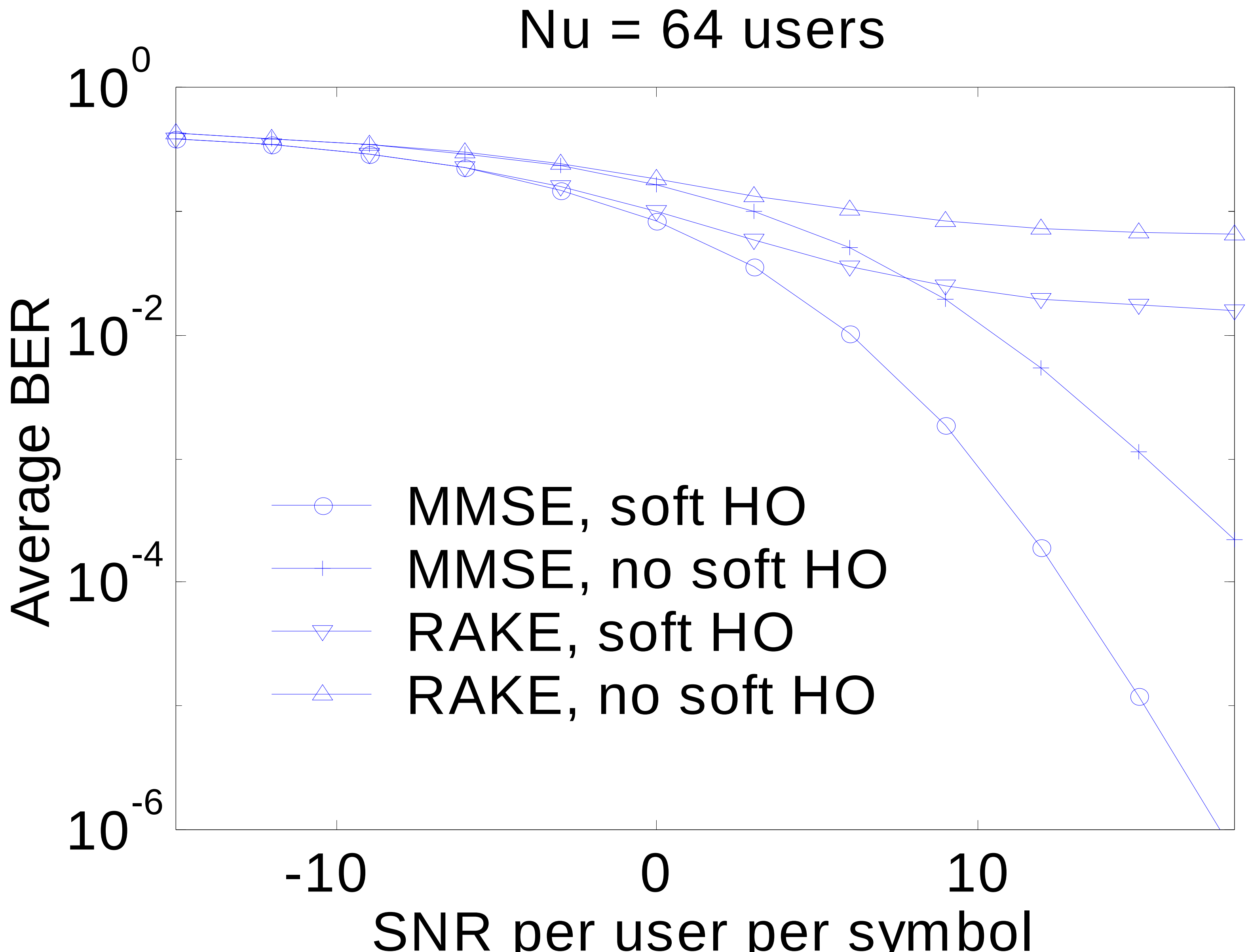
- **a_1, a_2 non-random constants**
- **γ_1 , γ_2 independent with E{|γ_1|^2}, E{|γ_2|^2} known**

Combine $$\hat{b}_1[m] = w_1 \hat{b}_1^{(1)}[m] + w_2 \hat{b}_1^{(2)}[m]$$

where w_1, w_2 **chosen to minimize**

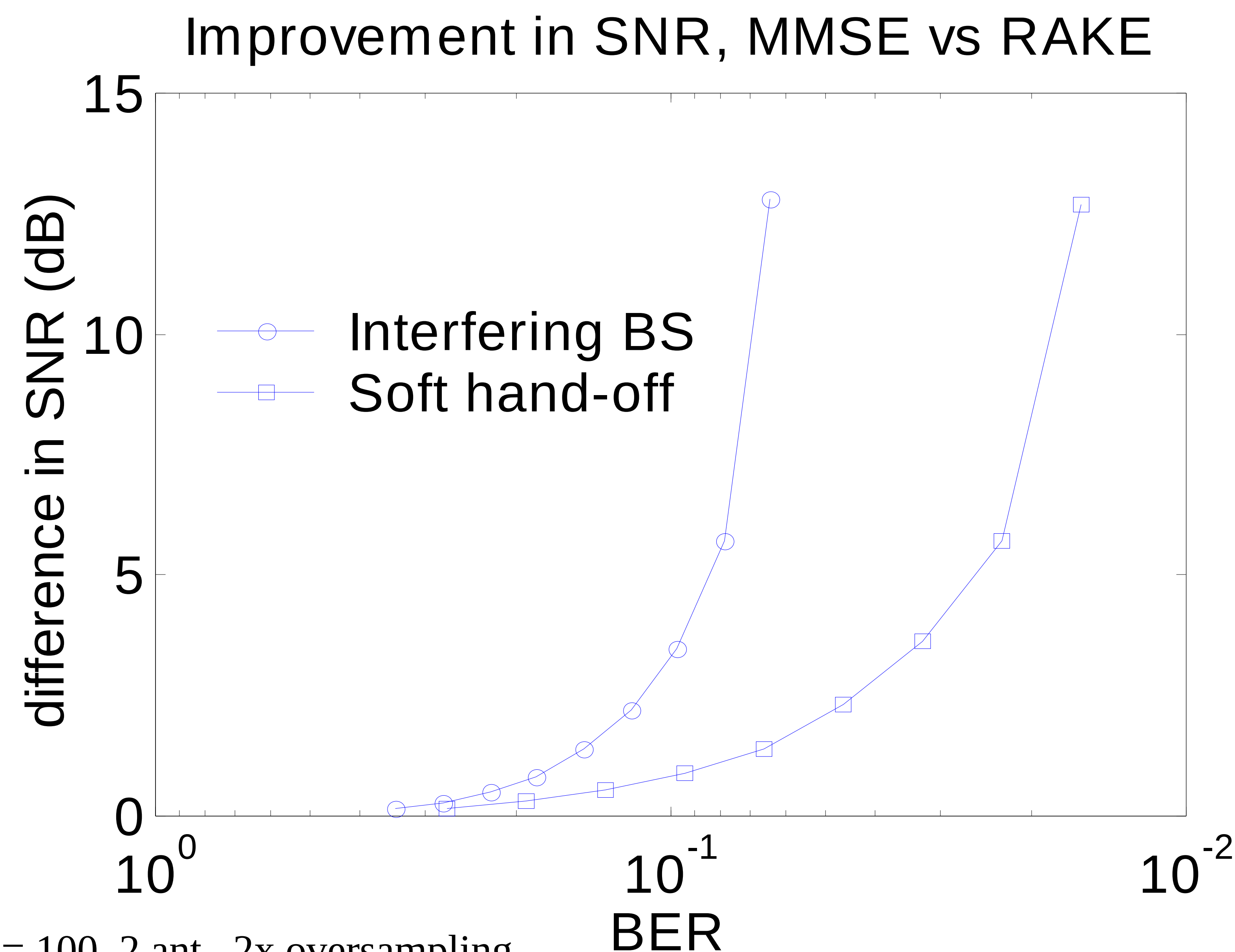
$$E\{|\hat{b}_1[m] - b_1[m]|^2\}$$

Soft-hand-off Simulation



N_g = 100, 2 ant., 2x oversampling

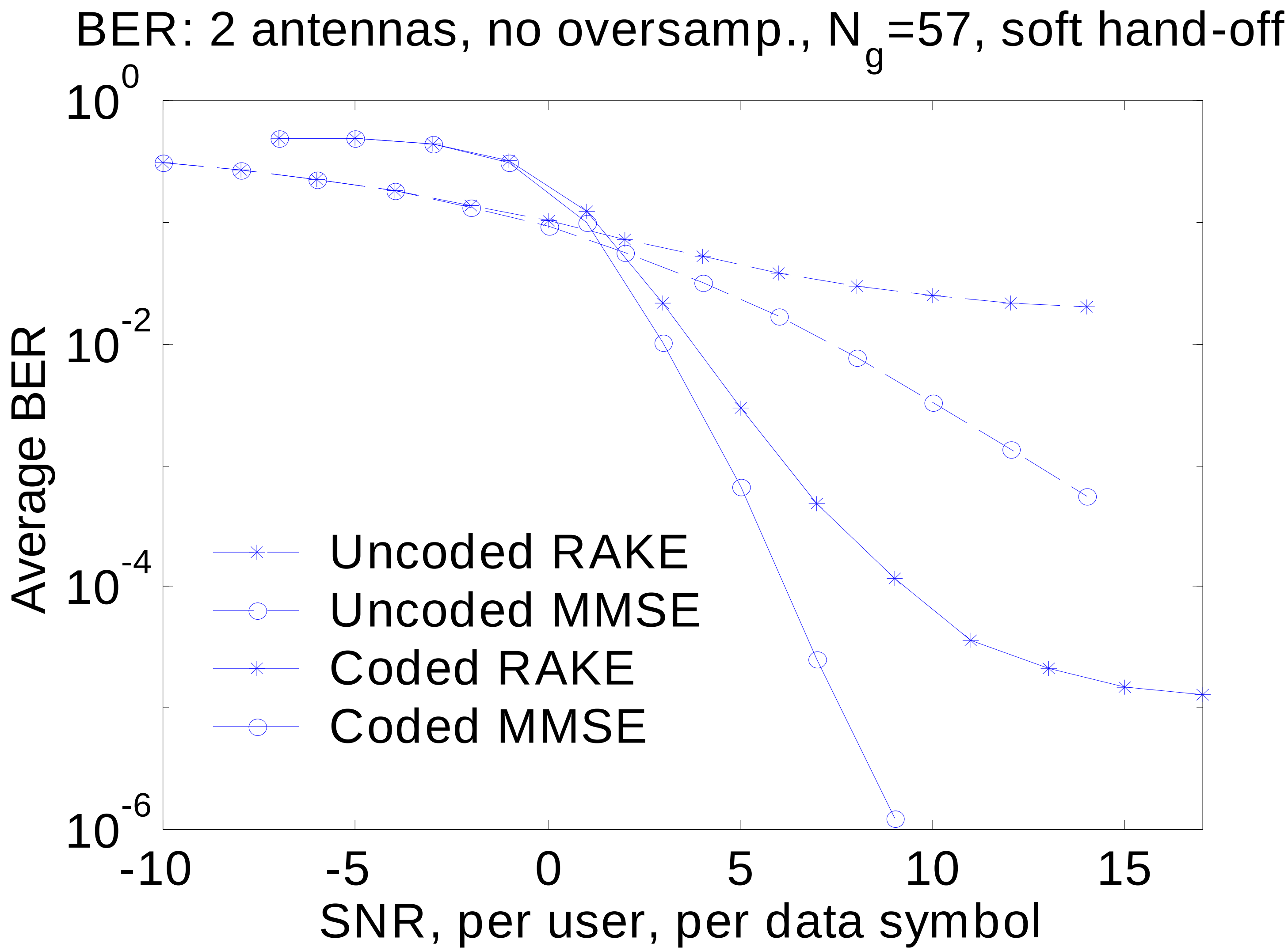
Soft-hand-off Simulation (cont.)



N_g = 100, 2 ant., 2x oversampling

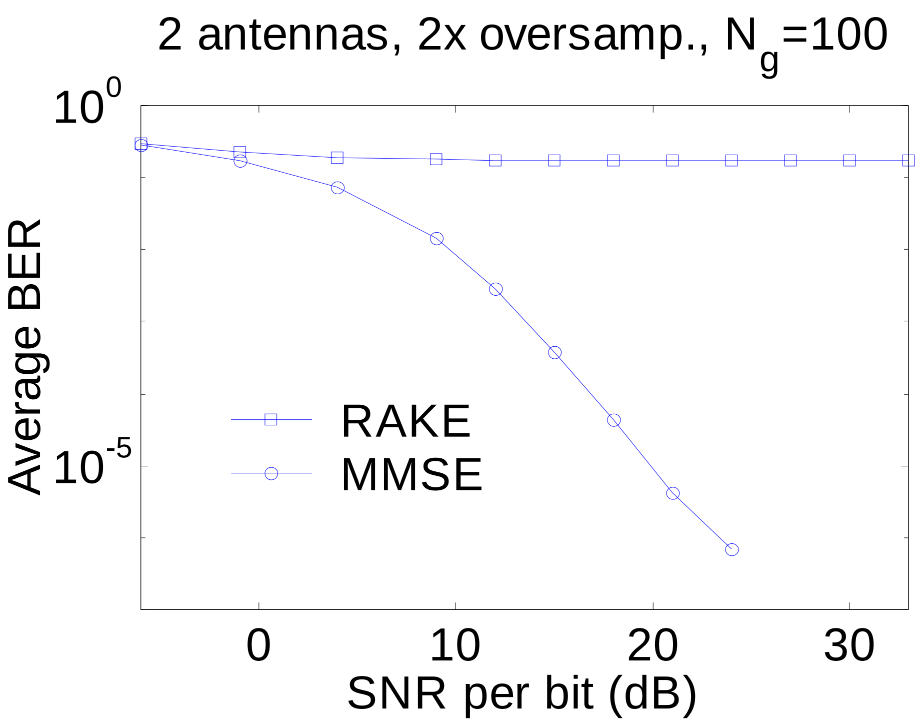
CODED BER RESULTS

Rate 1/2 Conv. Code, K=7

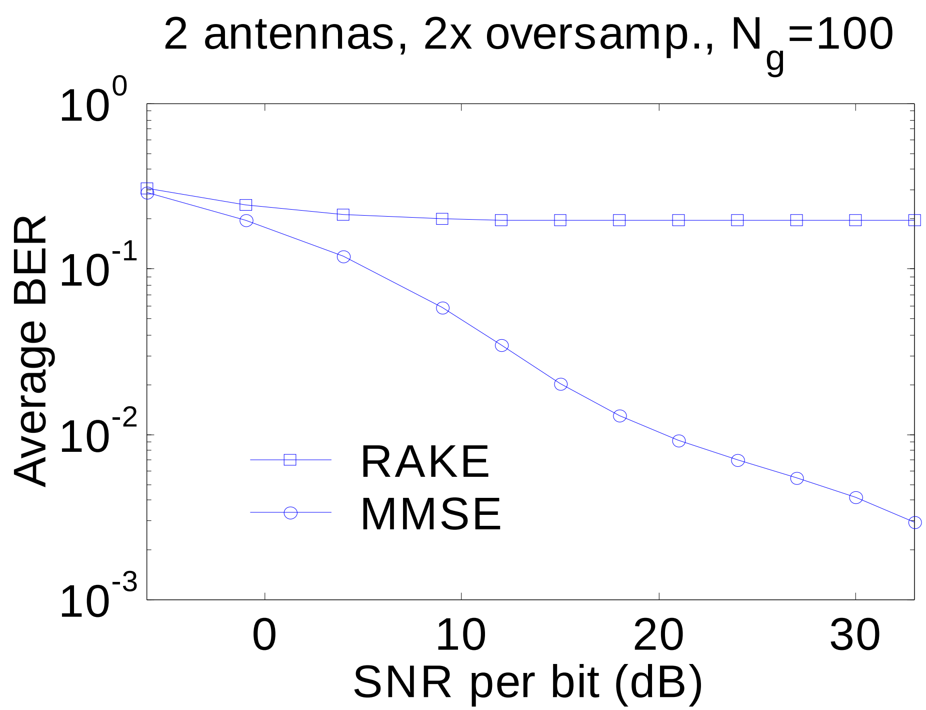


Uncoded 16 QAM Results

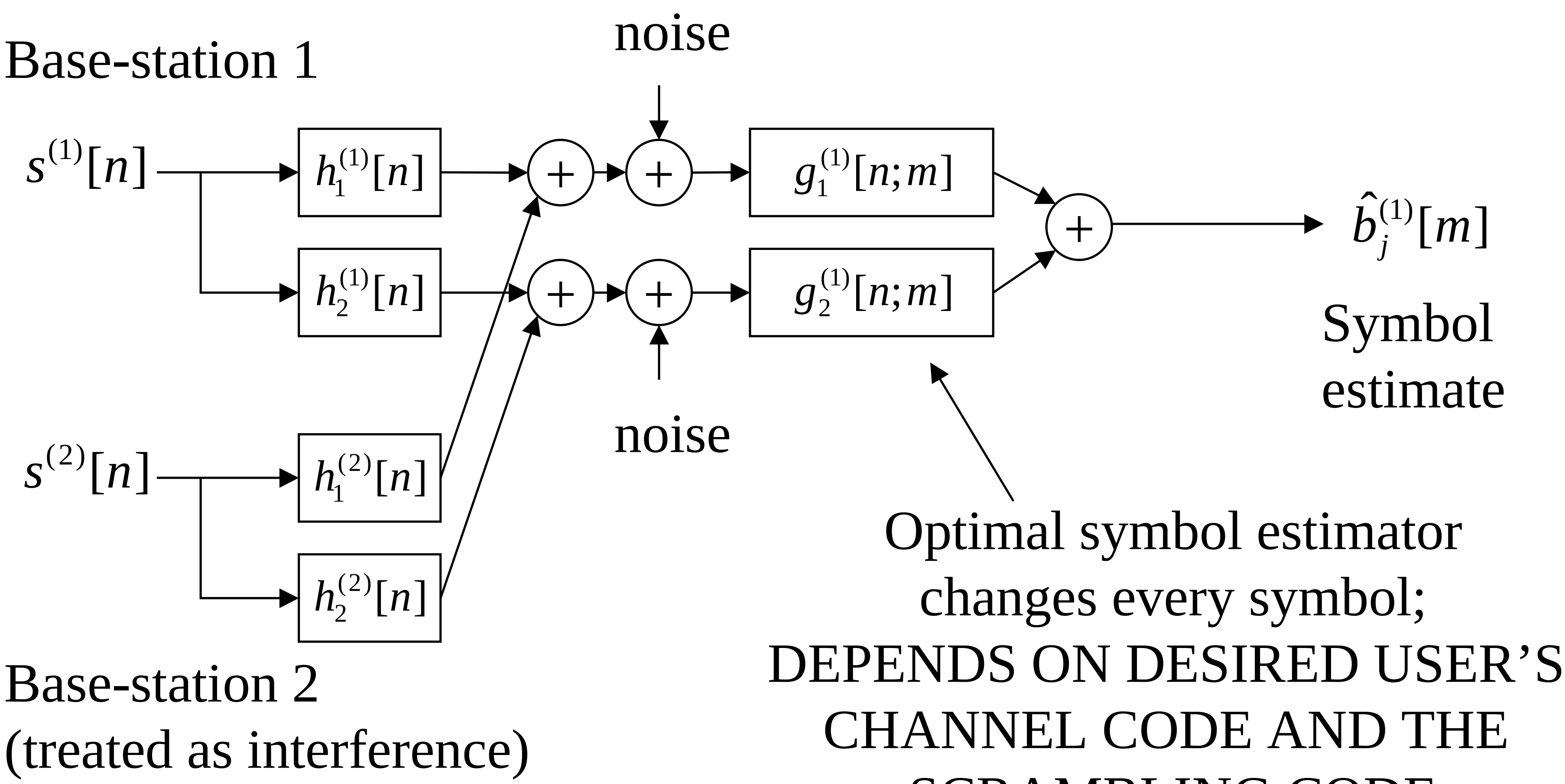
1 Base-station



2 Base-stations (soft hand-off)



MMSE SYMBOL-LEVEL EQUALIZER



MMSE Criterion for j^{th} user's symbol:

$$\underset{\mathbf{g}_m^{(1)}}{\text{Minimize}} \quad E\{|\mathbf{g}_m^{(1)H}\mathbf{y}[n]-b_j^{(1)}[m]|^2\}$$

Solution:

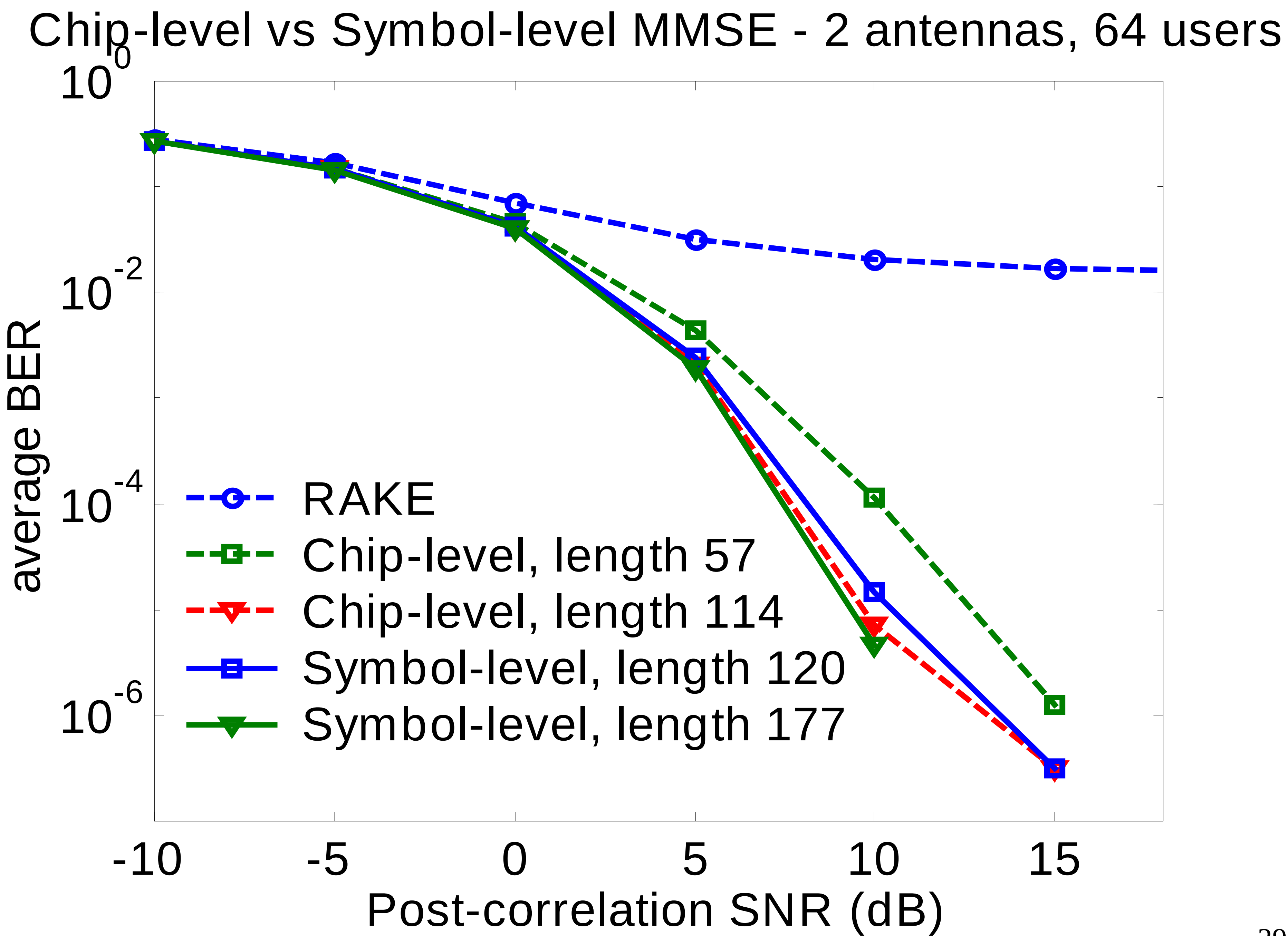
Note: $\mathbf{R}_{SS}=E\{\mathbf{s}^{(k)}[n]\mathbf{s}^{(k)H}[n]\}\approx\mathbf{I}$
with many users

$$\mathbf{g}_m^{(1)}=\left(\tilde{\mathbf{H}}\mathbf{R}_{SS}\tilde{\mathbf{H}}^H+\mathbf{R}_{\eta\eta}\right)^{-1}\mathbf{H}^{(1)}\tilde{\mathbf{c}}_j[m]$$

where

$$\tilde{\mathbf{c}}_j[m]=\begin{bmatrix}0 & \cdots & 0 & \mathbf{c}_j[n;m]^T & 0 & \cdots & 0\end{bmatrix}^T$$

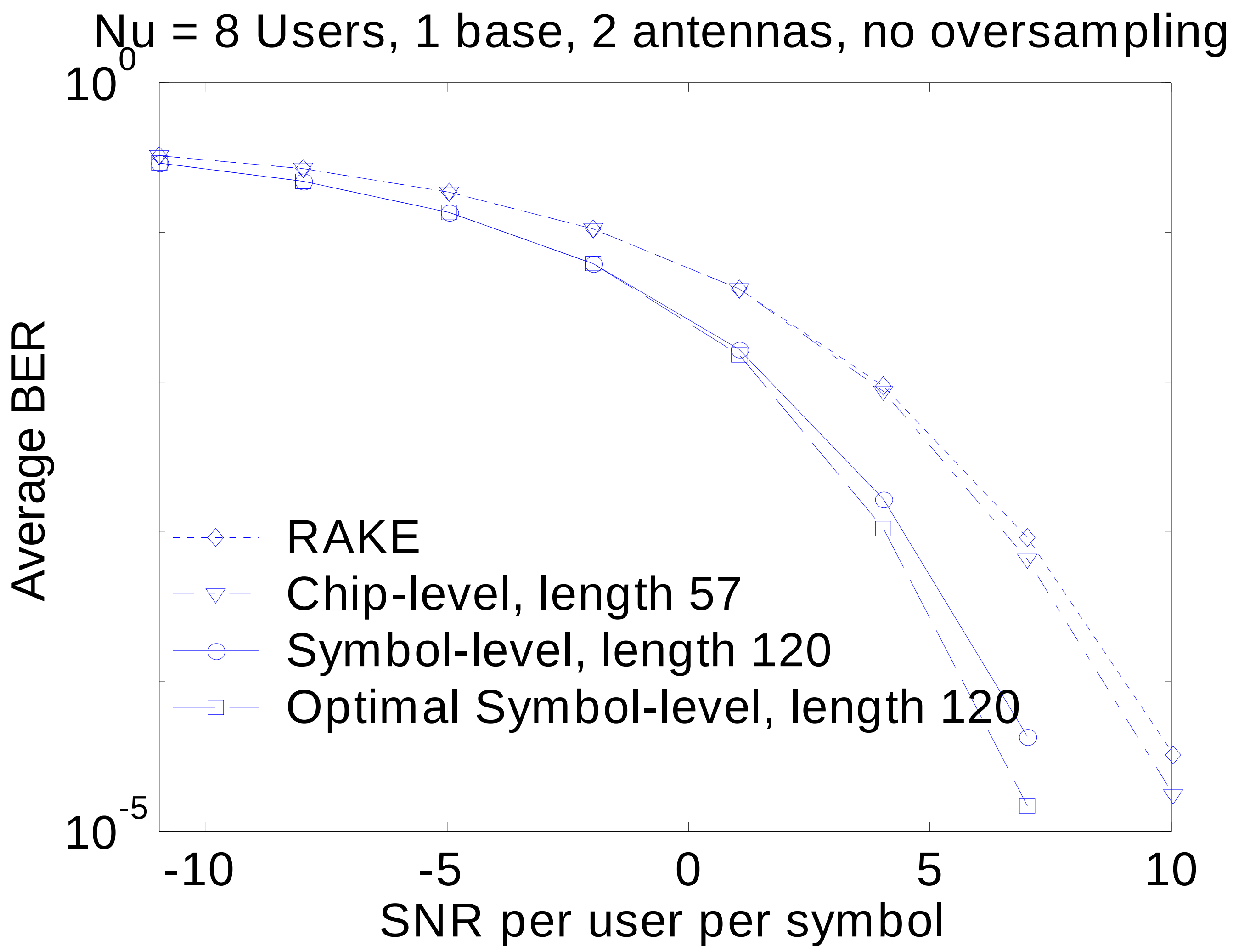
Length N_c channel code of jth user times
long code in m^{th} symbol interval



Chip-level vs Symbol-level MMSE - 2 antennas, 64 users
10^0
10^-2
10^-4
10^-6
average BER
RAKE
Chip-level, length 57
Chip-level, length 114
Symbol-level, length 120
Symbol-level, length 177
-10
-5
0
5
10
15
Post-correlation SNR (dB)

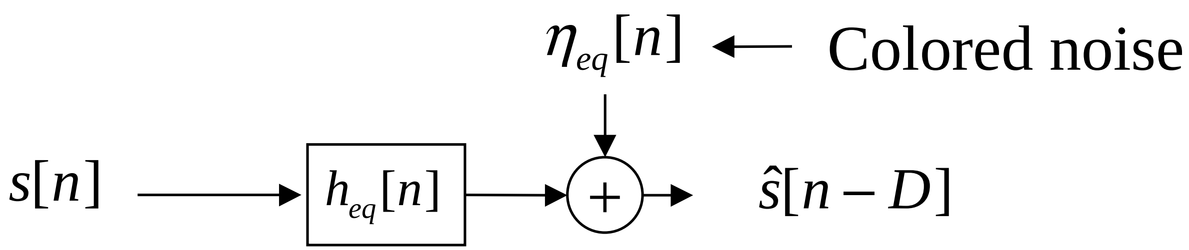
Optimal Symbol-level

Use actual $\mathbf{R}_{SS}$ **in** $\mathbf{g}_m^{(1)} = \left(\tilde{\mathbf{H}}\mathbf{R}_{SS}\tilde{\mathbf{H}}^H + \mathbf{R}_{\eta\eta}\right)^{-1}\mathbf{H}^{(1)}\tilde{\mathbf{c}}_j[m]$

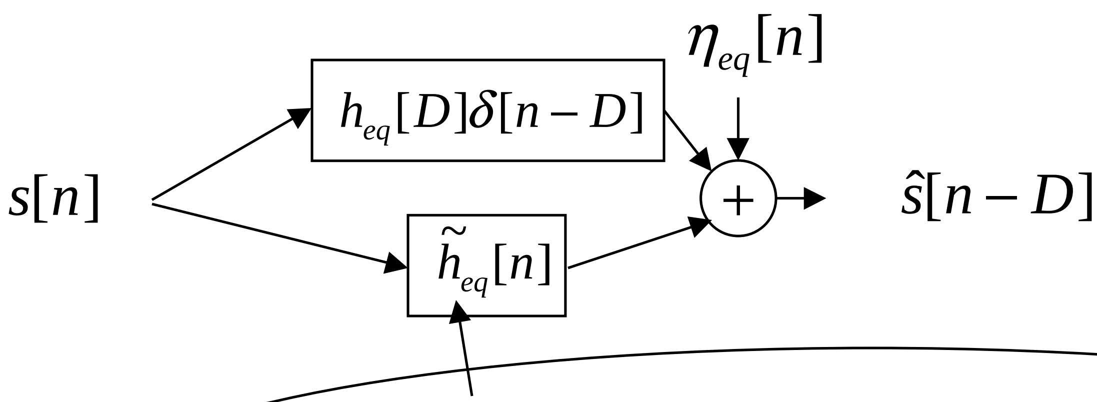


PERFORMANCE ANALYSIS

Combine multiple channels and equalizers into equivalent composite (single input, single output) channel:



Next, break h_{eq} into delay of D plus ISI terms:



Impulse response of ISI portion of channel:

$$\tilde{h}_{eq}[n] = h_{eq}[n] - h_{eq}[D]\delta[n-D]$$

Symbol estimate based on restored chip sequence:

$$\hat{b}_j[m] = \mathbf{c}_j[m]^H \hat{\mathbf{s}}[n]$$

where

$$\hat{\mathbf{s}}[n] = \mathbf{H}_{eq}\mathbf{s}[n] + \boldsymbol{\eta}_{eq}[n]$$

$$= h_{eq}[D]\sum_{i=1}^{N_u} b_i[m]\mathbf{c}_i[m] + \tilde{\mathbf{H}}_{eq}\mathbf{s}[n] + \boldsymbol{\eta}_{eq}[n]$$

Since $\mathbf{c}_j[m]^H \mathbf{c}_i[m] = N_c\delta[i-j]$ (Walsh codes are orthogonal)

$$\hat{b}_j[m] = \underbrace{N_c h_{eq}[D] b_j[m]}_{\text{SIGNAL}} + \underbrace{\mathbf{c}_j[m]^H(\mathbf{H}_{eq}\mathbf{s}[n] + \boldsymbol{\eta}_{eq}[n])}_{\text{INTERFERENCE + NOISE}}$$

$$\text{SINR} = \frac{\text{SIGNAL POWER}}{\text{INTERFERENCE + NOISE POWER}}$$

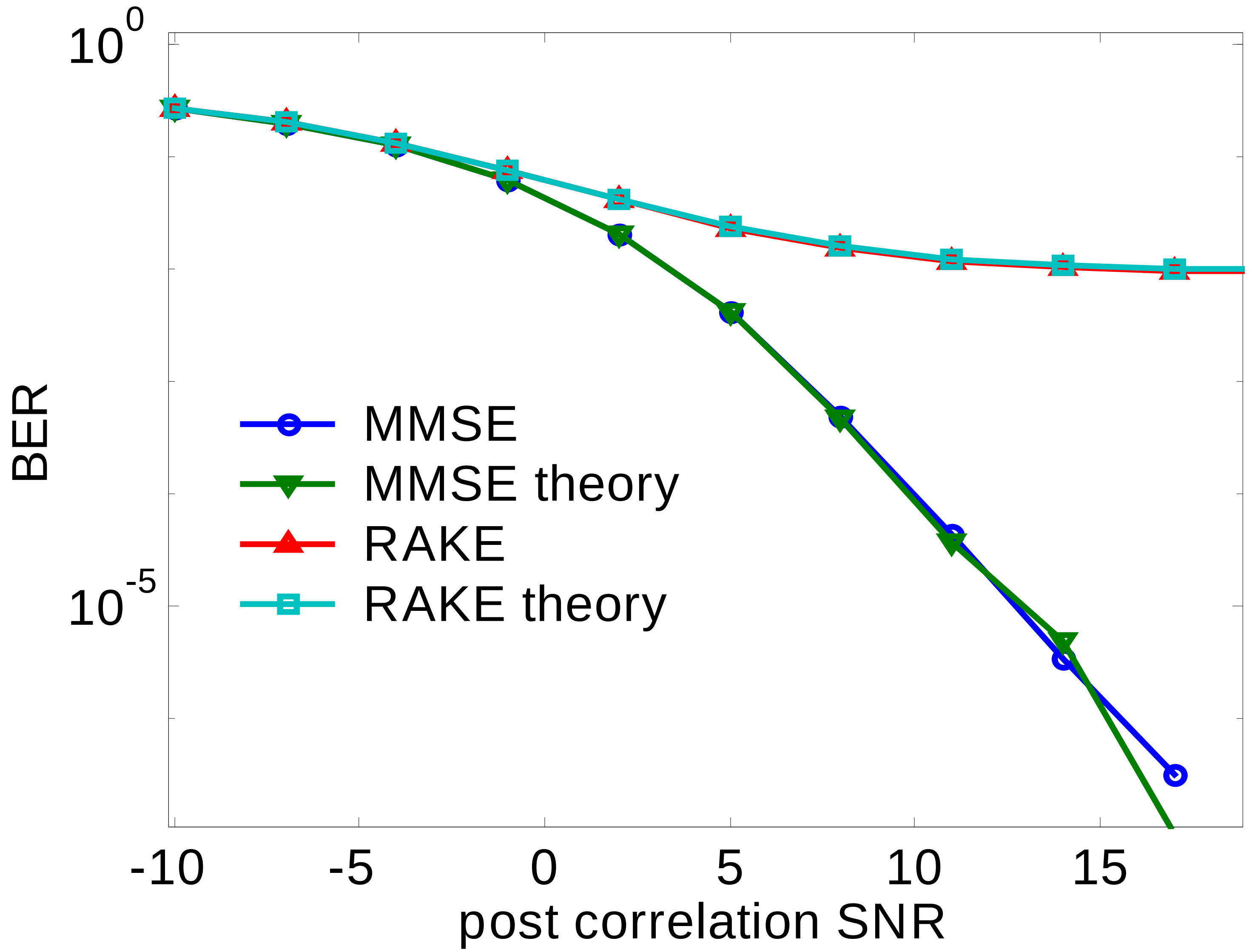
$$= \frac{\left|N_c h_{eq}[D]\right|^2}{tr(\mathbf{R}_{I+N})} \qquad \mathbf{R}_{I+N} = \widetilde{\mathbf{H}}_{eq}\widetilde{\mathbf{H}}_{eq}^H + \mathbf{G}\mathbf{R}_{\eta\eta}\mathbf{G}^H$$

BER: For BPSK symbols;
use Gaussian Approximation -
works well in practice

$$P_e \approx Q(\sqrt{SINR})$$

Note: conditioned on channel, and valid for any chip-level equalizer (RAKE, MMSE, ZF, BLUE)

Comparison of measured and predicted BER



length 57 MMSE chip equalizer, 2 antennas, 1 base station

CDMA Downlink: CONCLUSIONS

- MMSE chip equalizer significantly outperforms RAKE receiver
  - suppress interference
  - result: greater cellular capacity
- Good case for 2 antennas at the mobile
- Future work
  - channel estimation
  - adaptive and computationally efficient implementations

MMSE CHIP-LEVEL EQUALIZER

$$\hat{s}^{(1)}[n-D] = \mathbf{g}^{(1)H}\mathbf{y}[n] \qquad D \text{ is delay in chips}$$

$$\mathbf{g}_i^{(1)}[n] = \begin{bmatrix} g_i^{(1)}[0] \\ g_i^{(1)}[1] \\ \mathrm{M} \\ g_i^{(1)}[N_g - 1] \end{bmatrix} \qquad \mathbf{y}_i[n] = \begin{bmatrix} y_i[n] \\ y_i[n-1] \\ \mathrm{M} \\ y_i[n-(N_g-1)] \end{bmatrix}$$

$$\mathbf{g}^{(1)} = \begin{bmatrix} \mathbf{g}_1^{(1)} \\ \mathbf{g}_2^{(1)} \end{bmatrix} \qquad \mathbf{y}[n] = \begin{bmatrix} \mathbf{y}_1[n] \\ \mathbf{y}_2[n] \end{bmatrix}$$

How do we find the vector $\mathbf{g}^{(1)}$?

Multi-channel Received Data

$$\mathbf{y}[n] = \mathbf{H}^{(1)}\mathbf{s}^{(1)}[n] + \mathbf{H}^{(2)}\mathbf{s}^{(2)}[n] + \boldsymbol{\eta}[n]$$

$$\mathbf{H}_i^{(k)} = \begin{bmatrix} h_i^{(k)}[0] & \cdots & h_i^{(k)}[L-1] & 0 & \cdots & 0 \\ 0 & h_i^{(k)}[0] & \cdots & h_i^{(k)}[L-1] & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_i^{(k)}[0] & \cdots & h_i^{(k)}[L-1] \end{bmatrix}$$

$$\mathbf{H}^{(k)} = \begin{bmatrix} \mathbf{H}_1^{(k)} \\ \mathbf{H}_2^{(k)} \end{bmatrix} \qquad \mathbf{s}^{(k)}[n] = \begin{bmatrix} s^{(k)}[n] \\ s^{(k)}[n-1] \\ \vdots \\ s^{(k)}[n-(N_g+L-2)] \end{bmatrix}$$

Zero-Forcing Criterion

"Best Unbiased Linear Estimator" = BLUE:

$$\text{Minimize } \mathbf{g}^{(1)H}\mathbf{R}_{\eta\eta}\mathbf{g}^{(1)}$$

$$\text{subject to } \mathbf{g}^{(1)H}\tilde{\mathbf{H}} = \boldsymbol{\delta}_D^{(1)}$$

$$\boldsymbol{\delta}_D^{(1)} = \begin{bmatrix} \boldsymbol{\delta}_D \\ \mathbf{0} \end{bmatrix}$$

Solution:

$$\mathbf{g}^{(1)} = \mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}\mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}^{H}\right)^{-1}\boldsymbol{\delta}_D^{(1)}$$

COMPUTATION OF OPTIMAL DELAY

Zero-Forcing Criterion: Minimize Noise gain wrt *D*

$$\mathbf{g}^{(1)H}\mathbf{R}_{\eta\eta}\mathbf{g}^{(1)} = \boldsymbol{\delta}_D^{(1)T}\left(\tilde{\mathbf{H}}\mathbf{R}_{\eta\eta}{}^{-1}\tilde{\mathbf{H}}^H\right)^{-1}\boldsymbol{\delta}_D^{(1)}$$

Compute inverse, find smallest diagonal element

MMSE Criterion: Minimize MSE wrt *D*

$$\mathrm{MSE} = 1 - \boldsymbol{\delta}_D^{(1)T}\tilde{\mathbf{H}}^H\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \mathbf{R}_{\eta\eta}\right)^{-1}\tilde{\mathbf{H}}\boldsymbol{\delta}_D^{(1)}$$

Compute $\tilde{\mathbf{H}}^H\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \mathbf{R}_{\eta\eta}\right)^{-1}\tilde{\mathbf{H}}$

find *largest* diagonal element

ASYMPTOTIC RESULTS

$$\mathbf{g}_{\mathrm{MMSE}} = \left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{H} + \mathbf{R}_{\eta\eta}\right)^{-1}\tilde{\mathbf{H}}\boldsymbol{\delta}_{D}^{(1)}$$

$$= \mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}^{H}\mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}^{H} + \mathbf{I}\right)^{-1}\boldsymbol{\delta}_{D}^{(1)}$$

Alternate form obtained via inversion formula for $(A+BCD)^{-1}$

Lots of noise (Low SNR):

$$\mathbf{g}_{\mathrm{MMSE}} \longrightarrow \mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}\boldsymbol{\delta}_{D}^{(1)}$$

"pre-whitened RAKE"

Little noise (High SNR):

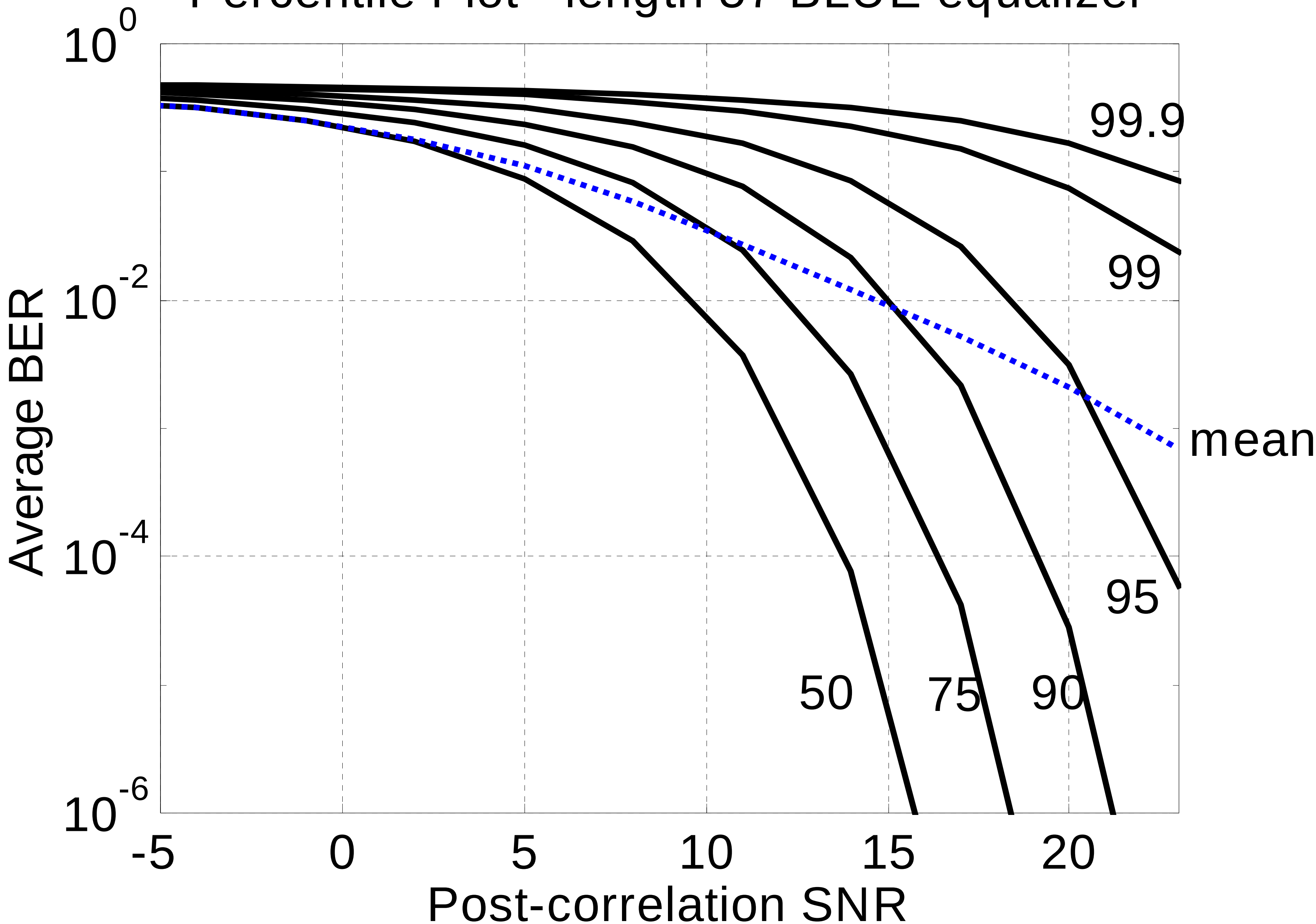
$$\mathbf{g}_{\mathrm{MMSE}} \longrightarrow \mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}\mathbf{R}_{\eta\eta}^{-1}\tilde{\mathbf{H}}^{H}\right)^{-1}\boldsymbol{\delta}_{D}^{(1)}$$

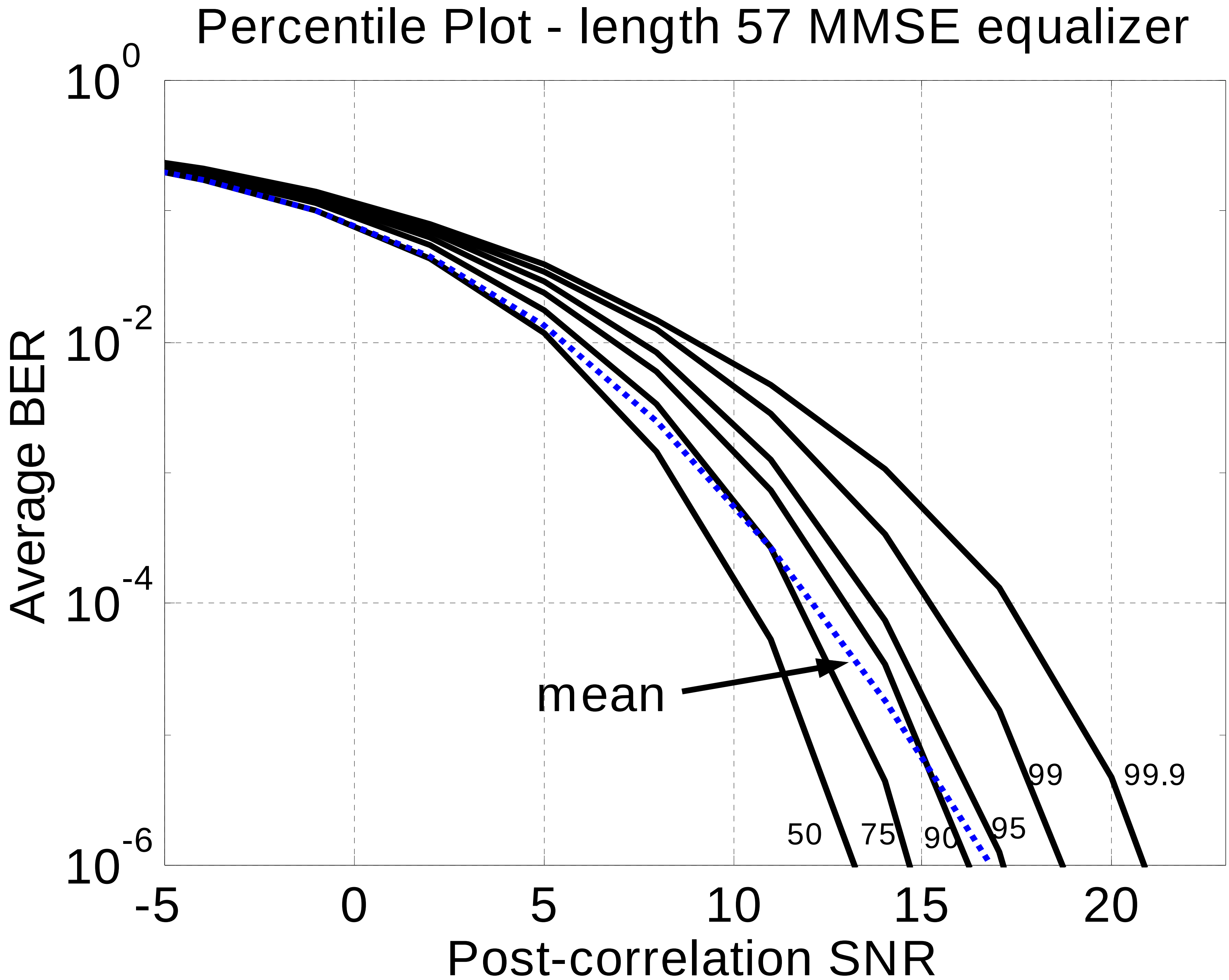
"BLUE"

References

- Klein, VTC, May 97.
  - Initial presentation of idea - Downlink specific processing
- Frank and Visotsky, Allerton, Sept. 98.
  - Closed form SINR expressions
- Ghauri and Slock, Asilomar, Nov. 98.
  - Extension to Multiple channels
- Other related:
  - Werner and Lilleberg, VTC, May 99.
  - Hooli, Latva-aho, and Juntti, Globecom, Dec. 99.

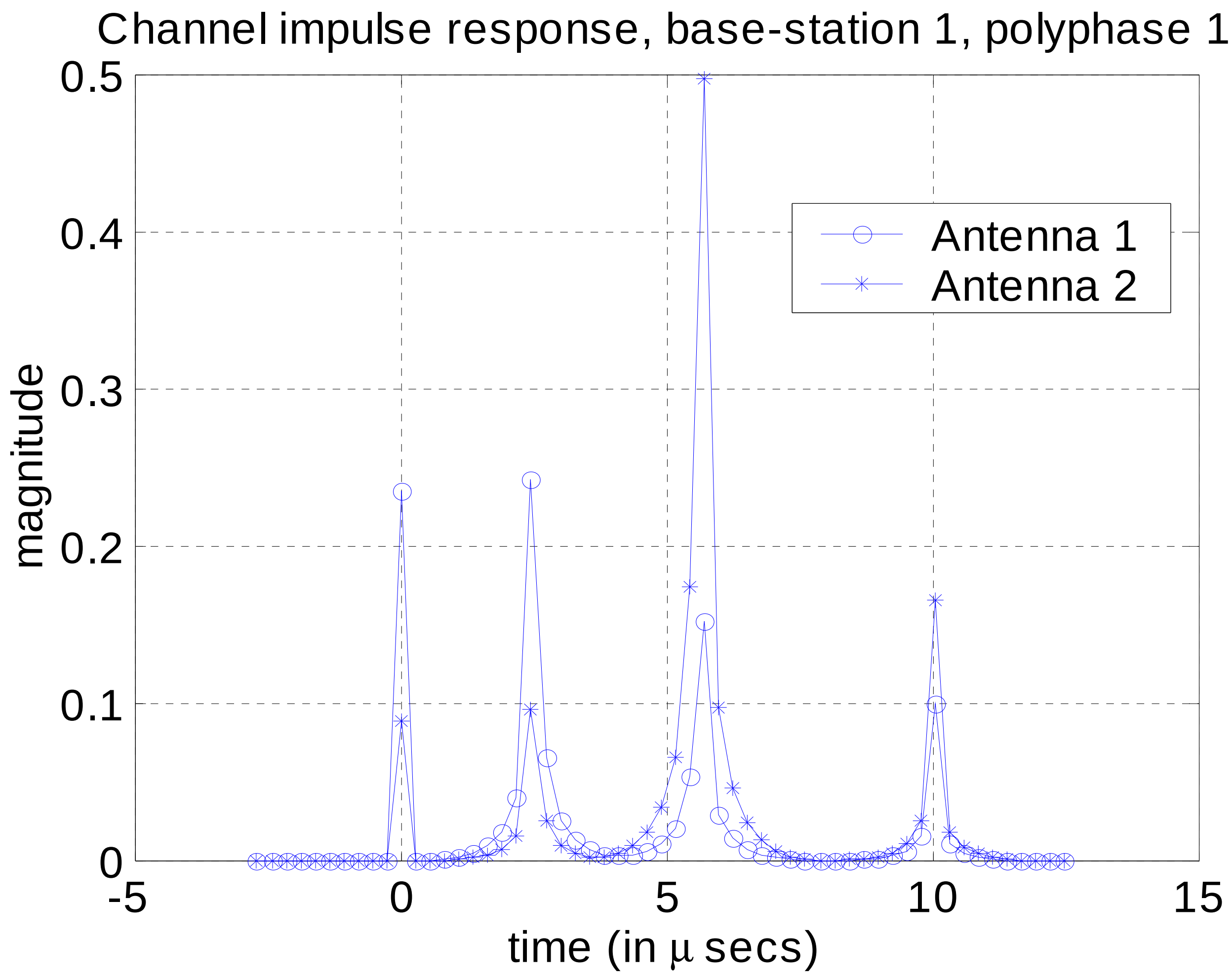
Percentile Plot - length 57 BLUE equalizer




Percentile Plot - length 57 MMSE equalizer
Average BER
10^0
10^-2
10^-4
10^-6
-5
0
5
10
15
20
Post-correlation SNR
mean
50
75
90
95
99
99.9

Channel Model

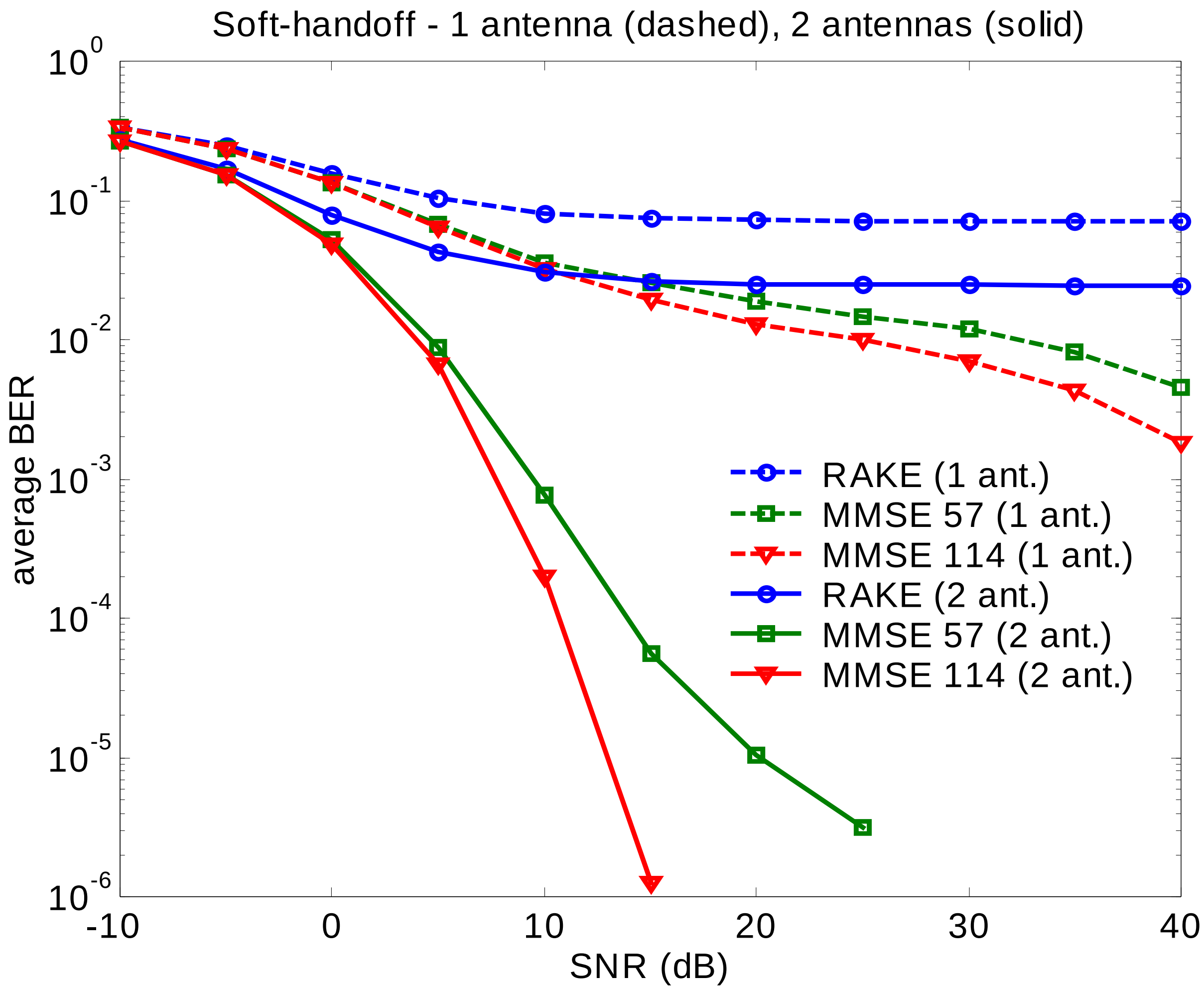
- 4 equal power paths, uniformly arriving between 0 and 10 μsecs
- Antennas experience independent fading



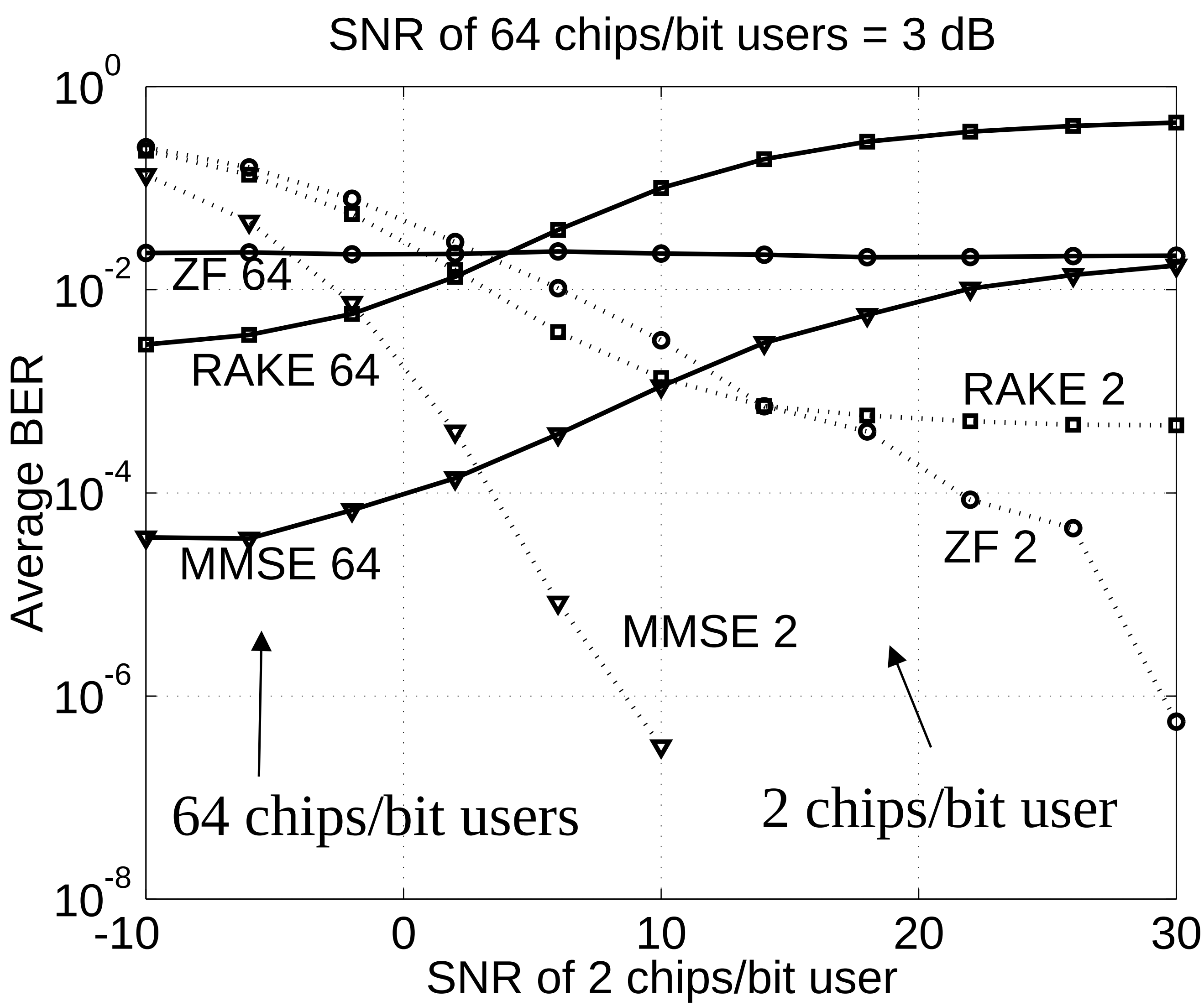
TWO BASE STATIONS

Average BER Performance at Edge of Cell

MMSE Equalizer Length = 57 or 114 Chips



OVSF Results: 32 low rate users, 1 high rate user (32x data rate)



Notes:

- single base-station
- 2 antennas
- no over-sampling
- channel model 1
- ZF = zero-forcing