Problem 1. [30 points]
Let $s_n(m)$ be a windowed speech segment of length $N$ which is nonzero for $0 \leq m \leq N - 1$. The energy of the segment is determined to be unity, i.e.,

$$
\sum_{m=0}^{N-1} s_n^2(m) = 1
$$

In addition, the autocorrelation sequence $R_n(\ell) = \sum_{m=0}^{N+p-1} s_n(m)s_n(m + \ell)$, is computed for lag values $\ell = 1$ and $\ell = 2$ and is determined to be

$$
R_n(1) = \frac{2}{3}, \quad R_n(2) = \frac{7}{12}
$$

(a) Determine the numerical values of the optimum LPC coefficients for a second-order predictor ($p = 2$), $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$, via the Autocorrelation Method. Show all work in how you arrived at your answers.

(b) Determine the numerical value of the minimum mean square prediction error

$$
E_n = \sum_{m=0}^{N+p-1} \{s_n(m) - \hat{s}_n(m)\}^2
$$

where $\hat{s}_n(m) = \alpha_1^{(2)} s_n(m - 1) + \alpha_2^{(2)} s_n(m - 2)$ with $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ determined in (a).

Problem 2. [30 points]

(a) Consider the 2-D convolution

$$
f(x,y) = 0.5 \text{jinc}(x,y) \ast \ast \text{jinc}(x, y/2)
$$

(i) Determine the 2D CSFT of $f(x,y)$, $F(u,v)$, and plot $|F(u,v)|$.

(ii) Determine a simple expression for $f(x,y)$, i.e., the 2D convolution of $0.5\text{jinc}(x,y)$ with jinc$(x, y/2)$.

(b) Consider the 2-D convolution

$$
g(x,y) = \{\text{sinc}(x)\text{rect}(y)\} \ast \ast \{\text{sinc}(2x)\text{rect}(y)\}
$$

(i) Determine the 2D CSFT of $g(x,y)$, $G(u,v)$. Simplify your answer as much as possible. You do not have to plot $G(u,v)$.

(ii) Determine a simple expression for $g(x,y)$, i.e., the 2D convolution of sinc$(x)\text{rect}(y)$ with sinc$(2x)\text{rect}(y)$.

Problem 3. [40 points] IS ON THE NEXT PAGE.
Problem 3. [40 points]
Consider the 2-D sinc function

\[ g_a(x, y) = \text{sinc} \left( \frac{1}{\sqrt{2}} (x - y), \frac{1}{\sqrt{2}} (x + y) \right) = \text{sinc} \left( \frac{1}{\sqrt{2}} (x - y) \right) \text{sinc} \left( \frac{1}{\sqrt{2}} (x + y) \right) \]

(a) Determine the 2D CSFT of \( g_a(x, y) \), denoted \( G_a(u, v) \). Plot \( G_a(u, v) \). \textit{Hint}: For any value of \( u \) and \( v \), \( G_a(u, v) \) is either 1 or 0. With this in mind, your sketch should simply indicate by cross-hatching those regions of the u-v plane for which \( G_a(u, v) = 1 \).

(b) The sampled image \( g_s(x, y) \) is obtained by rectangularly sampling via ideal combing of \( g_a(x, y) \) at the points \((m X, nY)\) where \( X = Y = 0.5 \) inches. Determine the 2D CSFT of \( g_s(x, y) \), denoted \( G_s(u, v) \). Simplify as much as possible. Plot \( G_s(u, v) \). Similar to (a), your sketch should simply indicate those regions of the u-v plane for which \( G_s(u, v) \) is constant by cross-hatching.

(c) The original image \( g_a(x, y) \) is to be reconstructed from the sampled image \( g_s(x, y) \) by passing \( g_s(x, y) \) through a 2D Linear Shift Invariant filter with impulse response \( h(x, y) \) and frequency response \( H(u, v) \). Specify an appropriate \( h(x, y) \) and \( H(u, v) \).