EE438 DSP With Applications Exam 3 18 April 2001

Problem 1. [30 points]
Let $s_n(m)$ be a windowed speech segment of length N which is nonzero for $0 \leq m \leq N-1$. The energy of the segment is determined to be unity, *i.e.,*

$$\sum_{m=0}^{N-1} s_n^2(m) = 1$$

In addition, the autocorrelation seqeunce $R_n(\ell) = \sum_{m=0}^{N+p-1} s_n(m)s_n(m+\ell)$, is computed for lag values $\ell = 1$ and $\ell = 2$ and is determined to be

$$R_n(1) = \frac{2}{3} \qquad\qquad R_n(2) = \frac{7}{12}$$

(a) Determine the numerical values of the optimum LPC coefficients for a second-order predictor ($p = 2$), $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$, via the Autocorrelation Method. Show all work in how you arrived at your answers.

(b) Determine the numerical value of the minimum mean square prediction error

$$E_n = \sum_{m=0}^{N+p-1} \{s_n(m) - \hat{s}_n(m)\}^2$$

where $\hat{s}_n(m) = \alpha_1^{(2)} s_n(m-1) + \alpha_2^{(2)} s_n(m-2)$ with $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ determined in (a).

Problem 2. [30 points]

(a) Consider the 2-D convolution

$$f(x,y) \;=\; 0.5\ \mathrm{jinc}(x,y) ** \mathrm{jinc}(x,y/2)$$

(i) Determine the 2D CSFT of $f(x,y)$, $F(u,v)$, and plot $|F(u,v)|$.

(ii) Determine a simple expression for $f(x,y)$, i. e., the 2D convolution of $0.5\mathrm{jinc}(x,y)$ with $\mathrm{jinc}(x,y/2)$.

(b) Consider the 2-D convolution

$$g(x,y) = \{\mathrm{sinc}(x)\mathrm{rect}(y)\} ** \{\mathrm{sinc}(2x)\mathrm{rect}(y)\}$$

(i) Determine the 2D CSFT of g(x,y), G(u,v). Simplify your answer as much as possible. *You do not have to plot G(u,v).*

(ii) Determine a simple expression for g(x,y), i. e., the 2D convolution of $\mathrm{sinc}(x)\mathrm{rect}(y)$ with $\mathrm{sinc}(2x)\mathrm{rect}(y)$.

Problem 3. [40 points] IS ON THE NEXT PAGE.

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Problem 3. [40 points]

Consider the 2-D sinc function

$$g_a(x,y) = \text{sinc}\left(\frac{1}{\sqrt{2}}(x-y), \frac{1}{\sqrt{2}}(x+y)\right) = \text{sinc}\left(\frac{1}{\sqrt{2}}(x-y)\right)\text{sinc}\left(\frac{1}{\sqrt{2}}(x+y)\right)$$

(a) Determine the 2D CSFT of $g_a(x,y)$, denoted $G_a(u,v)$. Plot $G_a(u,v)$. *Hint* : For any value of u and v, $G_a(u,v)$ is either 1 or 0. With this in mind, your sketch should simply indicate by cross-hatching those regions of the u-v plane for which $G_a(u,v) = 1$.

(b) The sampled image $g_s(x,y)$ is obtained by rectangularly sampling via ideal combing of $g_a(x,y)$ at the points (m X, nY) where X = Y = 0.5 inches. Determine the 2D CSFT of $g_s(x,y)$, denoted $G_s(u,v)$. Simplify as much as possible. Plot $G_s(u,v)$. Similar to (a), your sketch should simply indicate those regions of the u-v plane for which $G_s(u,v)$ is constant by cross-hatching.

(c) The original image $g_a(x,y)$ is to be reconstructed from the sampled image $g_s(x,y)$ by passing $g_s(x,y)$ through a 2D Linear Shift Invariant filter with impulse response $h(x,y)$ and frequency response $H(u,v)$. Specify an appropriate $h(x,y)$ and $H(u,v)$.