EE438 DSP With Applications Exam 2 9 March 2001

Problem 1. [30 points]

Consider the following two $N = 16$ point sequences:

$$x(n) = \cos\left(\frac{\pi}{2}n\right)\{u(n) - u(n-16)\}$$

and

$$h(n) = \left(\frac{1}{2}\right)^n \{u(n) - u(n-16)\},$$

where $u(n)$ is the DT unit step function. Let $X_{16}(k)$ and $H_{16}(k)$ denote the respective $N = 16$ point DFT's of $x(n)$ and $h(n)$.

$$x(n) \overset{DFT}{\underset{16}{\longleftrightarrow}} X_{16}(k) \qquad\qquad h(n) \overset{DFT}{\underset{16}{\longleftrightarrow}} H_{16}(k)$$

(a) Write a closed-form expression (NO summation in the final answer) for both $X_{16}(k)$ and $H_{16}(k)$, respectively (two answers), as a function of k.

(b) Let $y_{16}(n)$ denote the $N = 16$ point inverse DFT of the product $Y_{16}(k) = X_{16}(k)H_{16}(k)$. Determine a closed-form expression (NO summation in final answer) for $y_{16}(n)$. Simplify as much as possible (the final answer is real-valued for all n.) You can make the approximation $1 - \frac{1}{2^{16}} \approx 1$.

(c) Is there any value of n for which $y_{16}(n) = y(n)$, where $y(n)$ is the linear convolution of $x(n)$ and $h(n)$ ($y(n) = x(n) * h(n)$)?? If yes, state which values of n does $y_{16}(n) = y(n)$. Explain your answer.

Problem 2. [30 points]

Let $x(n), n = 0, 1, ..., 11$, be a sequence of length 12 and $X_0(k)$, $X_1(k)$, and $X_2(k)$ be 4 pt. DFT's defined as

$$\{x(0), x(3), x(6), x(9)\} \overset{DFT}{\underset{4}{\longleftrightarrow}} X_0(k)$$

$$\{x(1), x(4), x(7), x(10)\} \overset{DFT}{\underset{4}{\longleftrightarrow}} X_1(k)$$

$$\{x(2), x(5), x(8), x(11)\} \overset{DFT}{\underset{4}{\longleftrightarrow}} X_2(k)$$

In homework we showed that the 12 pt. DFT of $x(n)$, denoted $X(k), k = 0, 1, ..., 11$, may be computed in terms of these three 4 pt. DFT's as

$$\begin{aligned}
X(k) &= X_0(k) + e^{-j\frac{2\pi}{12}k}X_1(k) + e^{-j\frac{2\pi}{12}2k}X_2(k) \quad k = 0,1,2,3 \\
X(k+4) &= X_0(k) + e^{-j\frac{2\pi}{3}}e^{-j\frac{2\pi}{12}k}X_1(k) + e^{j\frac{2\pi}{3}}e^{-j\frac{2\pi}{12}2k}X_2(k) \quad k = 0,1,2,3 \\
X(k+8) &= X_0(k) + e^{j\frac{2\pi}{3}}e^{-j\frac{2\pi}{12}k}X_1(k) + e^{-j\frac{2\pi}{3}}e^{-j\frac{2\pi}{12}2k}X_2(k) \quad k = 0,1,2,3
\end{aligned}$$

(a) Assume each 4 pt. DFT is computed via a radix 2 FFT. How many complex multiplications is needed to compute each one? That is, how many complex mults. is needed to compute $X_0(k)$, k=0,1,2,3, if a radix 2 decimation in time FFT is employed?

(b) How many total complex mults. are needed to compute all 12 values of X(k) according to the procedure above, including the no. of complex mults. needed to compute the three 4 pt. FFT's as described in part (a)?? Note, for example, that for a *particular* value of k between 0 and 3, the multiplication $e^{-j\frac{2\pi}{12}k}X_1(k)$ need only be done once in computing $X(k)$, $X(k+4)$, and $X(k+8)$.

(c) How many total complex mults. would be required to compute all 12 values of X(k) directly via the sum

$$X(k) \;=\; \sum_{n=0}^{11} x(n)\; e^{-j2\pi\frac{k}{12}n}, \quad k = 0, 1, ..., 11$$

What is the savings in computation obtained by decomposing the 12 pt. DFT into three 4 pt. DFT's as described above??

Problem 3. [40 points]
A speech signal, $s(t)$, is sampled at a rate 10 KHz during a voiced interval to obtain the D-T signal,

$$s(n) = \sum_{n=-\infty}^{\infty} h(n - 40k)$$

where $h(n)$ is the impulse response of the vocal tract for this particular voiced sound given by

$$h(n) = \{(.95)^n \cos(2\pi(.066)n) + (.75)^n \cos(2\pi(.172)n) + (.6)^n \cos(2\pi(.241)n)\}\, u(n)$$

(a) What is the pitch period in milliseconds?

(b) What is the most likely sex of the speaker, male or female??

(c) What are the three formant frequencies in Hz??

(d) What is the most likely vowel sound that gave rise to this sampled signal?