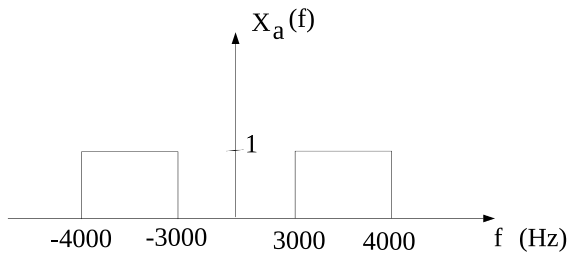
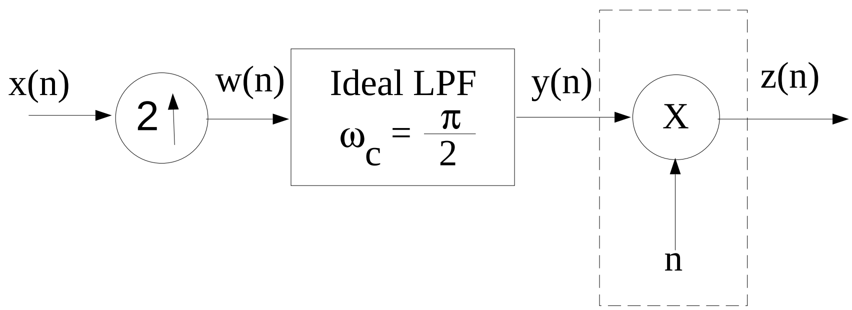
DSP With Applications   Exam 1   9 Feb. 2001

Problem 1. [35 points]

The continuous time signal $x_a(t)$ with CTFT plotted below



is sampled at a rate $f_s = 8$ KHz to produce the discrete-time signal $x(n)$, i.e., $x(n) = x_a(n/f_s)$. The sampled signal, $x(n)$, is then input to the discrete-time system below.



(a) Plot the DTFT, $X(e^{j\omega})$, of $x(n)$ over $-\pi < \omega < \pi$.

(b) Plot the DTFT, $Y(e^{j\omega})$, of $y(n)$ over $-\pi < \omega < \pi$. Note that

$$w(n) = \begin{cases} x(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \qquad \text{and} \qquad H_{ILPF}(e^{j\omega}) = \text{rect}\left\{\frac{\omega}{\pi}\right\} \quad \text{for} \quad -\pi < \omega < \pi$$

where $H_{\text{ILPF}}(e^{j\omega})$ is the frequency response of the ideal lowpass filter.

(c) Consider the sub-system $z(n) = ny(n)$ highlighted in the overall system.

(i) Is this system linear? Substantiate your answer.

(ii) Is this system time-invariant? Substantiate your answer.

(ii) Is this system stable? Substantiate your answer.

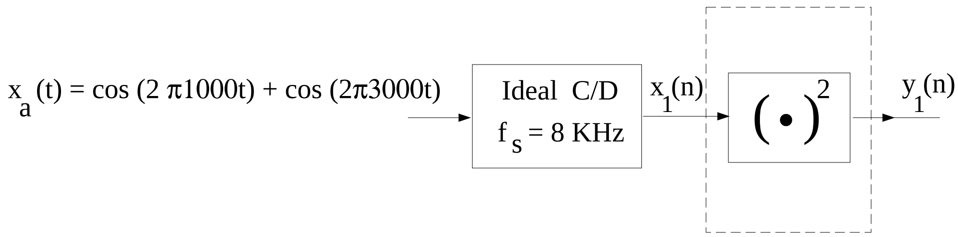
(d) Plot the DTFT, $Z(e^{j\omega})$, of $z(n)$ over $-\pi < \omega < \pi$.

(e) Determine a simple closed-form (no summation in final answer) expression for $z(n)$, the inverse DTFT of $Z(e^{j\omega})$.

Problem 2. [40 points]

As shown in the figure below, the discrete-time signal, $x_1(n)$, is obtained by sampling the continuous-time signal, $x_a(t)$, below at a rate $f_s = 8$ KHz.

$$x_a(t) = \cos(2\pi 1000 t) + \cos(2\pi 3000 t) \tag{1}$$



(a) Plot the DTFT, $X_1(e^{j\omega})$, of $x_1(n)$ over $-\pi < \omega < \pi$.

(b) Consider the sub-system $y_1(n) = x_1^2(n)$ highlighted in the overall system.

(i) Is this system linear? Substantiate your answer.

(ii) Is this system time-invariant? Substantiate your answer.

(ii) Is this system stable? Substantiate your answer.

(c) Plot the DTFT, $Y_1(e^{j\omega})$, of $y_1(n)$ over $-\pi < \omega < \pi$. *Hint:* the following trigonometric identity may be helpful: $\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$.

(d) If $x(n) = x_a(n/f_s)$, where f_s is greater than twice the highest frequency contained in $x_a(t)$, then we know there is no aliasing and the DTFT of $x(n)$, $X(e^{j\omega})$, is related to the CTFT of $x_a(t)$, $X_a(f)$, as $X(e^{j\omega}) = f_s X_a(\frac{f_s\omega}{2\pi})$ for $-\pi < \omega < \pi$.

Let $y_a(t) = x_a^2(t)$ and let the CTFT of $y_a(t)$ be $Y(f)$. **Plot** $Y_2(e^{j\omega}) = f_s Y_a(\frac{f_s\omega}{2\pi})$ for $-\pi < \omega < \pi$, where $f_s = 8$ KHz. Is $Y_2(e^{j\omega}) = Y_1(e^{j\omega})$? Explain your answer.

Problem 3. [25 points]

Consider the causal LTI system where the output, $y(n)$, and the input, $x(n)$, are related as

$$y(n) = \frac{1}{2}y(n-2) + x(n) + x(n-2)$$

(a) Determine the numerical value of $|H(e^{j\omega})|$ for $\omega = 0$, $\omega = \pi/2$, and $\omega = \pi$. Plot $|H(e^{j\omega})|$ over $0 < \omega < \pi$.

(b) Determine the output, $y(n)$, of the system when the input is the D-T signal

$$x(n) = 2 + \cos(\frac{\pi}{2}n) + (-1)^n$$