EE438 | **Digital Signal Processing with Applications** | **Spring 2001**

ASSIGNMENT #6

Due: Wednesday, March 28, 2001 in class

1. The technique used in class to derive the optimum coefficients for LPC is just one example of the least squares approach to approximation/prediction. The following problem illustrates other uses of the least squares approach.
   Suppose we wish to approximate the waveform $x(t) = e^{-1.5t}$ over the interval $0 \le t \le 3$ by the function $\hat{x}(t) = a_0 + a_1 t + a_2 t^2$.

   a) Find the values for the coefficients that minimize

   $$E = \int_0^3 [x(t) - \hat{x}(t)]^2 dt$$

   b) Compare your coefficients with the first 3 terms of a Taylor series for $e^{-1.5t}$ expanded about the point t = 0.

   c) Using matlab, plot on the same axes $x(t), \hat{x}(t),$ and the Taylor series approximation.

   d) Compare the mean-squared error E for $\hat{x}(t)$ and the 3 term Taylor series.

2. Show that if the predictor coefficients satisfy

   $$R_n(l) = \sum_{k=1}^{p} \alpha_k R_n(k-l), \qquad 1 \le l \le p,$$

   then the prediction error simplifies to

   $$E_n = R_n(0) - \sum_{k=1}^{p} \alpha_k R_n(k).$$

3. Consider the signal

   $$x(n) = \{2^{-n} + 4^{-n}\}u(n)$$

   a) Calculate the autocorrelation function

$$R(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

for this signal.

b) Use the Levinson-Durbin recursion to find the coeffiecient $\alpha_1^{(1)}$ of a first order predictor:

$$\hat{x}(n) = \alpha_1^{(1)} x(n-1).$$

c) Calculate the error $E^{(1)}$ using the formula

$$E^{(1)} = (1 - K_1^2)E^{(0)}$$

d) Continue the Levinson-Durbin recursion to find the coefficients $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ for a second order predictor:

$$\hat{x}(n) = \alpha_1^{(2)} x(n-1) + \alpha_2^{(2)} x(n-2).$$

e) Calculate $E^{(2)}$ using the formula

$$E^{(2)} = E^{(1)}(1 - K_2^2).$$

f) Calculate the prediction sequence, $\hat{x}(n)$, $-\infty < n < \infty$ for the second order predictor.

g) Calculate the error sequence

$$e(n) = x(n) - \hat{x}(n), \qquad -\infty < n < \infty$$

for the second order predictor.

h) Using your answer to part g), calculate the mean-squared error

$$E^{(2)} = \sum_{n=-\infty}^{\infty} [x(n) - \hat{x}(n)]^2$$

i) Compare your answers to parts e) and h).

4. Given the discrete-time signal

$$x(n) = \{2^{-n} + 4^{-n}\}u(n)$$

we consider predicting x(n) with a second-order predictor (p = 2)

$$\hat{x}(n) = \alpha_1^{(2)} x(n-1) + \alpha_2^{(2)} x(n-2).$$

$\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ are to be determined according to the Covariance Method as those coefficients that minimize the mean square error defined below with p = 2.

$$E^{(p)} = \sum_{n=p}^{\infty} \{x(n) - \hat{x}(n)\}^2$$

a) Compute the covariance below for arbitrary k and l , note p = 2.

$$\Phi(k, l) = \sum_{n=p}^{\infty} x(n-k)x(n-l)$$

b) Set up the appropriate equations for determining the optimum $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$. Solve for $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$.

c) What is the numerical value of $E^{(2)}$ given the $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ determined in (b)?

d) Repeats parts (a) thru (c) for a third order predictor where p=3 and

$$\hat{x}(n) = \alpha_1^{(3)} x(n-1) + \alpha_2^{(3)} x(n-2) + \alpha_2^{(3)} x(n-3)$$

   i. Compare the optimum coefficients $\alpha_1^{(3)}$, $\alpha_2^{(3)}$, and $\alpha_3^{(3)}$ with $\alpha_1^{(2)}$ and $\alpha_2^{(2)}$ computed in part (b).

   ii. Compare the minimum $E^{(3)}$ with the minimum $E^{(2)}$ computed in part (c).

e) Discuss the results for this problem and the previous problem involving the Autocorrelation Method.