EE 438 Assignment #8 Solutions Spring 2001

1. a) Assume $x(m+iM, n+jN) = x(m,n)$ for any $i, j$

and since $e^{-j2\pi[\frac{(m+iM)k}{M} + \frac{(n+jN)\ell}{N}]}$

$$= e^{-j2\pi(\frac{mk}{M} + \frac{n\ell}{N})} \quad \text{for any } i, j$$

$$X_{10,12}(k,\ell) = \sum_{m=0}^{9} \sum_{n=0}^{11} x(m,n) e^{-j2\pi(\frac{mk}{M} + \frac{n\ell}{N})}$$

$$= \sum_{m=-5}^{4} \sum_{n=-6}^{5} x(m,n) e^{-j2\pi(\frac{mk}{M} + \frac{n\ell}{N})}$$

$$= e^{-j2\pi(3k/10 + 4\ell/12)} + e^{j2\pi(3k/10 + 4\ell/12)}$$

$$= 2\cos[2\pi(3k/10 + \ell/3)]$$

b) $$X_{10,12}(k,\ell) = \sum_{m=0}^{4} \sum_{\ell=0}^{11} e^{-j2\pi[\frac{mk}{10} + \frac{n\ell}{12}]}$$

$$= \frac{1-e^{-j2\pi k\frac{5}{10}}}{1-e^{-j2\pi k/10}} \cdot \frac{1-e^{-j2\pi\ell\frac{12}{12}}}{1-e^{-j2\pi\ell/12}}$$

$$= \frac{1-e^{-j\pi k}}{1-e^{-j2\pi k/10}} \cdot 12\delta(\ell \bmod 12)$$

$$= \frac{e^{-j\pi k/2} \cancel{2j}\sin(\pi k/2)}{e^{-j2\pi k/20} \cancel{2j}\sin(2\pi k/20)} \cdot 12\delta(\ell \bmod 12)$$

$$= \begin{cases} \dfrac{12 e^{j2\pi k/2}\delta(\ell \bmod 12)}{j\sin(2\pi k/20)} & \ell = \ldots, -12, 0, 12, \ldots \quad k \text{ odd} \\ 60, & \ell = \ldots, -12, 0, 12, \ldots, \; k = \ldots, -10, 0, 10, \ldots \\ 0, & \text{otherwise} \end{cases}$$

c) $$X_{10,12}(k,\ell) = \sum_{m=0}^{9} \sum_{\ell=0}^{11} \frac{1}{2}\left[e^{j2\pi(m/10 - n/6)} + e^{-j2\pi(m/10-n/6)}\right]$$

$$\cdot e^{-j2\pi(\frac{mk}{10} + \frac{n\ell}{12})}$$

$$= \frac{1}{2} \sum_{m=0}^{9} \sum_{\ell=0}^{11} e^{-j2\pi\, m(k-1)/10} e^{-j2\pi\, n(\ell+2)/12}$$

$$+ \frac{1}{2} \sum_{m=0}^{9} \sum_{\ell=0}^{11} e^{-j2\pi m(k+1)/10} e^{-j2\pi n(\ell-2)/12}$$

$$= 60 \{ \delta [(k-1) \bmod 10, (\ell+2) \bmod 12]$$

$$+ \delta [(k+1) \bmod 10, (\ell-2) \bmod 12] \}$$

2. a) $$X_{MN}(k,\ell) = \sum_{m=0}^{M-1} \left\{ \sum_{n=0}^{N-1} x(m,n) e^{-j2\pi n\ell/N} \right\} e^{-j2\pi mk/M}$$

Let $X_N^m(\ell) = \sum_{n=0}^{N-1} x(m,n) e^{-j\frac{2\pi n\ell}{N}}$,

1D - N pt DFT for each $m = 0, \cdots, M-1$

Let $X_{MN}(k,\ell) = \sum_{m=0}^{M-1} X_N^m(\ell) e^{-j\frac{2\pi mk}{M}}$.

1D - M pt DFT for each $\ell = 0, \cdots, N-1$

b) Perform M N pt DFT's & N M pt DFT's

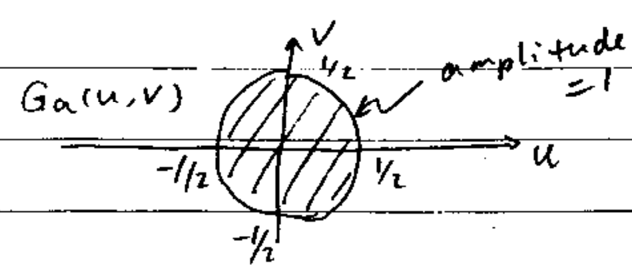
$\Rightarrow$ CO's $= M(N \log_2 N) + N(M \log_2 M)$

$= MN \log_2 (MN)$

So operation count is equivalent to an MN pt. 1-D DFT of all the points in the image

3. a) $G_a(u,v) = e^{-j2\pi(2.5)(u+v)} \text{circ}(u,v)$

$= e^{-j5\pi(u+v)} \text{circ}(u,v)$

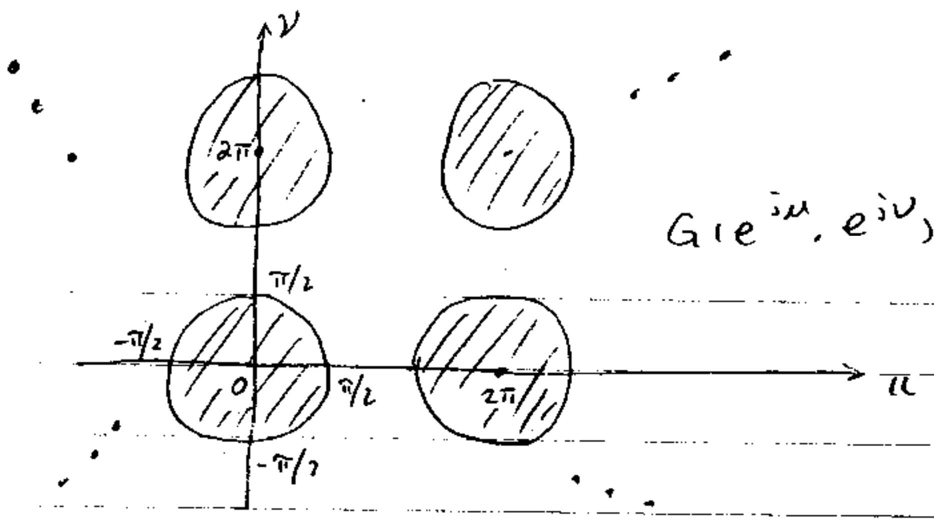



(neglecting linear phase factor)

b) By extending what we did for 1-D case,

$$G(e^{j\mu}, e^{j\nu}) = \frac{1}{0.5^2}\left\{\mathrm{rep}_{\frac{1}{0.5},\frac{1}{0.5}}\left[G_a(u,v)\right]\right\}_{\substack{\mu = \frac{2\pi u}{1/0.5} \\ \nu = \frac{2\pi v}{1/0.5}}}$$

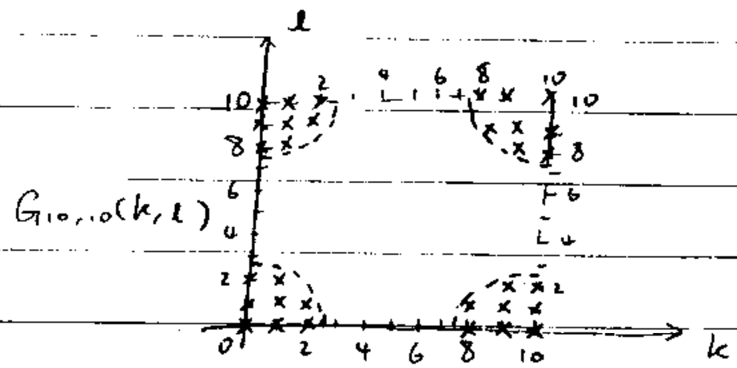
$$= 4\,\mathrm{rep}_{2\pi,2\pi}\, G_a(\mu/\pi, \nu/\pi)$$




Amplitudes = 4

c) Subject to the stated assumption,

$$G_{10,10}(k,\ell) = G(e^{j\frac{2\pi k}{10}}, e^{j\frac{2\pi \ell}{10}})$$




At x's the amplitude is 4, otherwise 0. Periodic in both k and $\ell$ with period 10

4. a) If M and N are both even

$$F_{MN}(k,\ell) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}(-1)^{m+n}g(m,n)e^{-j2\pi\left[\frac{mk}{M}+\frac{n\ell}{N}\right]}$$

$$= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{j\pi(m+n)} g(m,n) e^{-j2\pi\left(\frac{mk}{M} + \frac{nl}{N}\right)}$$

$$= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g(m,n) e^{-j2\pi\left[\frac{m(k-M/2)}{M} + \frac{n(l-N/2)}{N}\right]}$$

$$= G_{MN}(k - M/2,\ l - N/2)$$

If M and N are not both even

$$g(m,n) = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} G_{MN}(i,j) e^{j2\pi\left[\frac{mi}{M} + \frac{nj}{N}\right]}$$

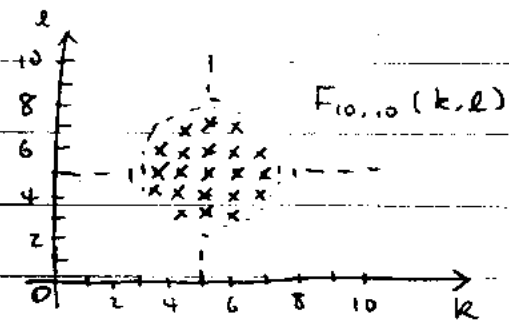
$$F_{MN}(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{j\pi(m+n)} \left\{\frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} G_{MN}(i,j) e^{j2\pi\left[\frac{mi}{M} + \frac{nj}{N}\right]}\right\} \cdot e^{-j2\pi\left(\frac{mk}{M} + \frac{nl}{N}\right)}$$

$$= \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} G_{MN}(i,j) \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{-j\pi m(2k-2i-M)/M} e^{-j\pi n(2l-2j-N)/N}$$

$$= \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} G_{MN}(i,j) \frac{1 - e^{-j\pi(2k-2i-M)}}{1 - e^{-j\pi(2k-2i-M)/M}} \cdot \frac{1 - e^{-j\pi(2l-2j-N)}}{1 - e^{-j\pi(2l-2j-N)/N}}$$

$$= \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} G_{MN}(i,j) \frac{1 - e^{j\pi M}}{1 - e^{-j\pi(2k-2i-M)/M}} \cdot \frac{1 - e^{j\pi N}}{1 - e^{-j\pi(2l-2j-N)/N}}$$

b)




At x's amplitude is 4 otherwise 0. periodic with period 10 in both k and l directions

5. a) $g(m,n) = \sum_k \sum_l h(k,l) f(m-k, n-l)$

$$= f(m,n) + \tfrac{1}{2}[f(m-1,n) + f(m+1,n)$$

$$-f(m,n-1) - f(m,n+1)]$$

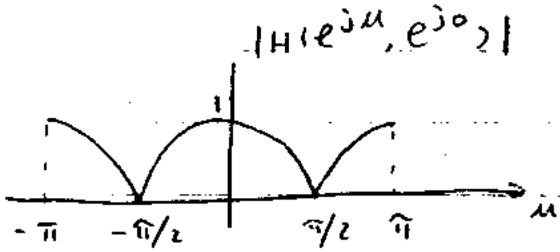
b) Filter kernel

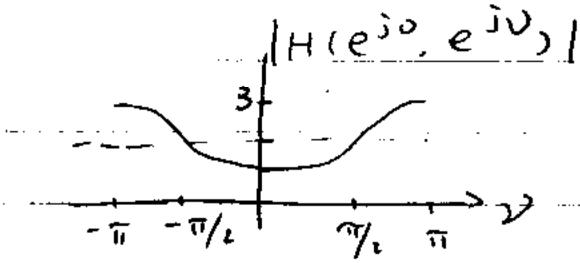
$$\begin{array}{ccc} 0 & -\frac{1}{2} & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{array}$$

$$\begin{array}{ccccccccccc}
 & & & \vdots & & \vdots & & & & & \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 & 0 & 0 & -1/2 & -1/2 & -1/2 & -1/2 & -1/2 & 0 & 0 & \\
 & 0 & 1/2 & 1 & 3/2 & 3/2 & 3/2 & 1 & 1/2 & 0 & \\
\cdots & 0 & 1/2 & 1/2 & 1 & 1 & 1 & 1/2 & 1/2 & 0 & \cdots \\
\cdots & 0 & 1/2 & 1/2 & 1 & 1 & 1 & 1/2 & 1/2 & 0 & \\
 & 0 & 1/2 & 1/2 & 1 & 1 & 1 & 1/2 & 1/2 & 0 & \cdots \\
 & 0 & 1/2 & 1 & 3/2 & 3/2 & 3/2 & 1 & 1/2 & 0 & \\
 & 0 & 0 & -1/2 & -1/2 & -1/2 & -1/2 & -1/2 & 0 & 0 & \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 & & & \vdots & & \vdots & & & & &
\end{array}$$

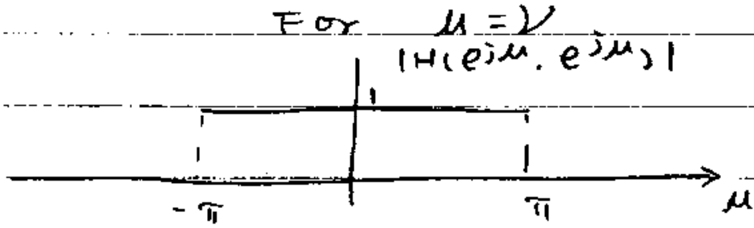
c) $H(e^{j\mu}, e^{j\nu}) = 1 + \frac{1}{2}[e^{-j\mu} + e^{j\mu} - e^{-j\nu} - e^{j\nu}]$

$\qquad = 1 + \cos\mu - \cos\nu$




For $\mu = \nu$




d) Filter preserves DC since $H(e^{j0}, e^{j0}) = 1$, it attenuates freq. along $\mu$ axis $\Rightarrow$ smoothes vertical edges, amplifies freq. along $\nu$ axis $\Rightarrow$ sharpens horizontal edges.