# EE438 Signals and Systems – Spring 2001

**Homework 7**

**Due Date:** Monday, 9 April 2001

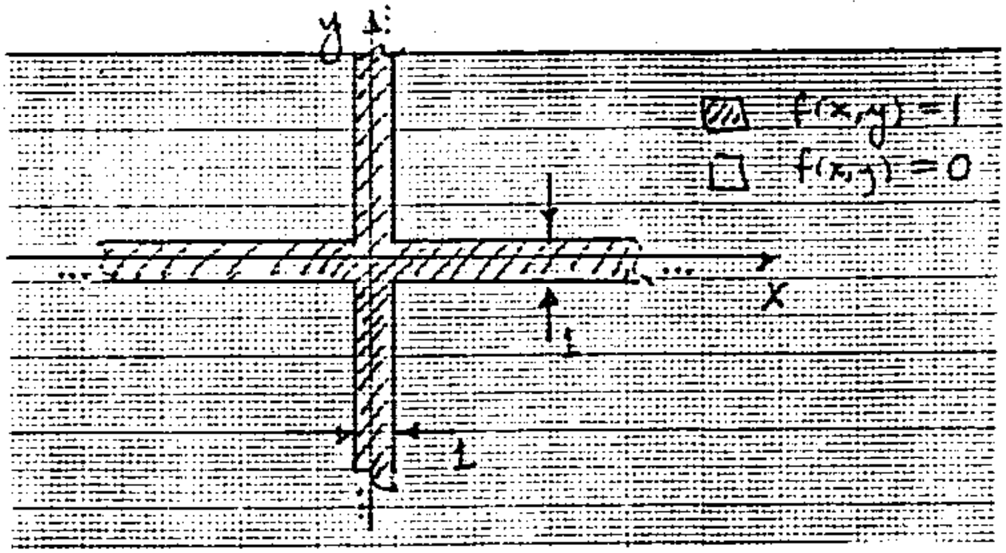
1. **For each function given below, do the following:**

   i) **Express f(x,y) in terms of special functions given in class.**

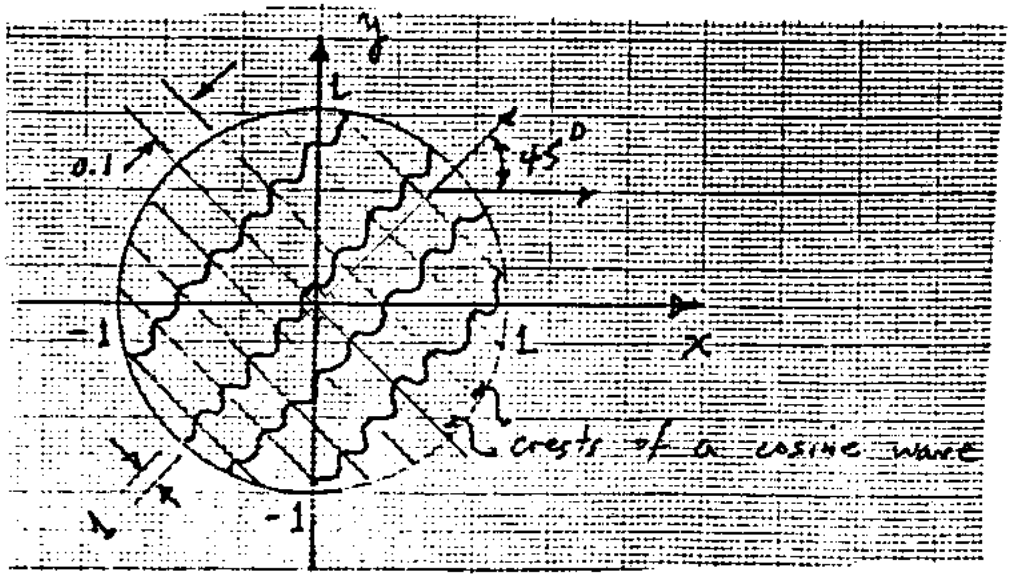
   ii) **Find its CFT using transform pairs and properties.**

   iii) **Sketch the CFT in enough detail to show that you know what it looks like.**

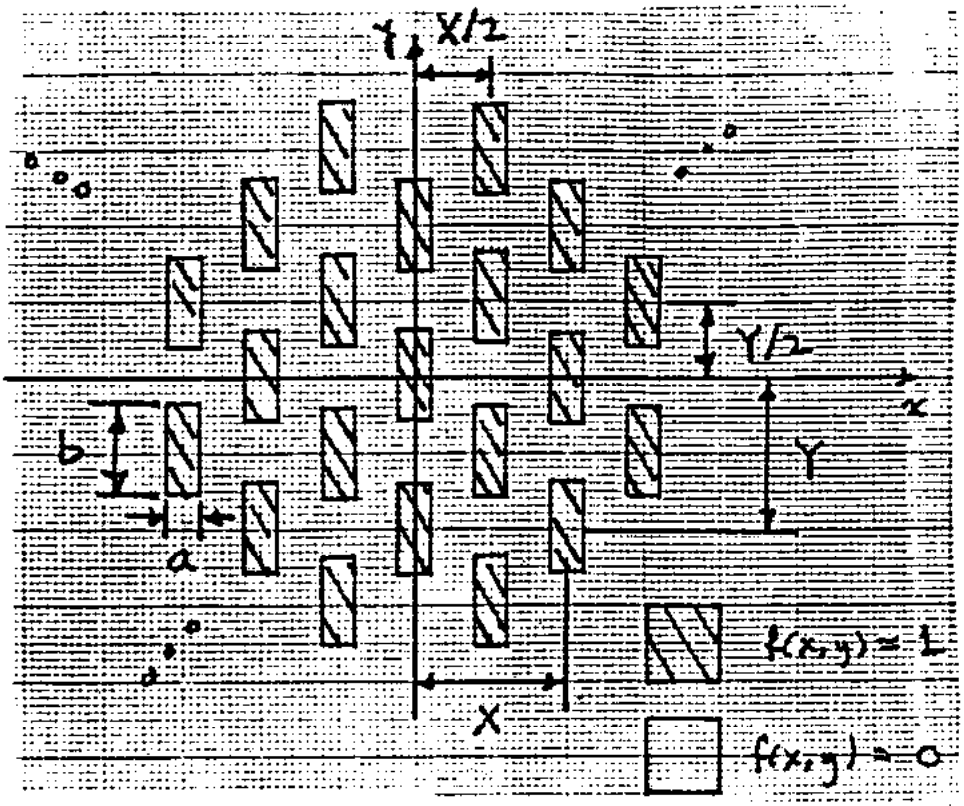
**a)**




**b)**



c)




2. Consider the spatial frequency component $\cos[2\pi(u_0 x + v_0 y)]$. Let $\lambda$ be the distance between crests in this wave; and let $\theta$ be the angle between the normal to the crests and the x-axis. Show that

   a) $\lambda = [u_0^2 + v_0^2]^{-1/2}$

   b) $\theta = \arctan(v_0/u_0)$

3. For a real image f(x,y), show that the 2-D CSFT $F(u,v) = |F(u,v)|e^{j\angle F(u,v)}$ has the following symmetry:

   a) $|F(u,v)| = |F(-u,-v)|$

   b) $\angle F(u,v) = -\angle F(-u,-v)$

4. Suppose that a 2-D function f(x,y) has 2-D CSFT

$$F(u,v) = \begin{cases} j, & 0 \le u \le 1 \text{ and } 0 \le v \le 1 \\ -j, & -1 \le u \le 0 \text{ and } -1 \le v \le 0 \\ 1+j, & 0 \le u \le 1 \text{ and } -1 \le v \le 0 \\ 1-j, & -1 \le u \le 0 \text{ and } 0 \le v \le 1 \end{cases}$$

   a) Sketch $|F(u,v)|$ and $\angle F(u,v)$.

b) Find a *simple expression* for f(x,y).

c) Discuss relation between this problem and Problem 3.

5. Consider an imaging system that obeys superposition. The image of an ideal point object at $(x_0, y_0) = (1,2)$ is measured and found to be $\text{rect}(x+2)\delta(y+4)$, i.e.,

$$\delta(x-1, y-2) \rightarrow \begin{matrix}\text{Imaging}\\ \text{System}\end{matrix} \rightarrow \text{rect}(x+2)\delta(y+4)$$

a) Calculate and sketch the image of the object

$$g(x,y) = \text{rect}(x, y/4)$$

b) What is the magnification of this imaging system?

c) What is the point spread function h(x,y)?

d) What is the frequency response H(u,v)?

6. Suppose we represent an object g(x,y) and its CFT G(u,v) in polar coordinates and use a tilde to denote the polar coordinate versions, i.e.,

$$\tilde{g}(r,\theta) = g(x,y)$$

$$\tilde{G}(\rho,\phi) = G(u,v)$$

where

$$r = \sqrt{x^2 + y^2} \qquad \rho = \sqrt{u^2 + v^2}$$

$$\theta = \arctan\left[\frac{y}{x}\right] \qquad \phi = \arctan\left[\frac{v}{u}\right]$$

a) Show that the CFT in polar coordinates is given by

$$\tilde{G}(\rho,\phi) = \int_0^{2\pi} \int_0^{\infty} \tilde{g}(r,\theta) e^{-i2\pi\rho r\cos(\phi-\theta)} r \, dr \, d\theta$$

b) Show that rotation of the object by $\theta_0$ degrees causes the spectrum to also rotate by $\theta_0$, i.e.,

$$\text{If} \quad \tilde{g}(r,\theta) \overset{CFT}{\leftrightarrow} \tilde{G}(\rho,\phi), \quad \text{then}$$

$$\tilde{g}(r,\theta+\theta_0) \overset{CFT}{\leftrightarrow} \tilde{G}(\rho,\phi+\theta_0)$$

c) Show that if the object is circularly symmetric, then its transform will also be circularly symmetric, i.e.,

$$\tilde{g}(r,\theta) = \tilde{g}_0(r) \rightarrow \tilde{G}(\rho,\phi) = \tilde{G}_0(\rho)$$

7. The object

$$g(x,y) = \cos[2\pi(100x + 200y)] \quad (x,y) \text{ inches}$$

is sampled at 300 samples/inch in the x and y directions. The sampled signal $g_s(x,y)$ is then filtered with an ideal rectangular low pass filter having cut-off at 150 cycles/inch along the horizontal and vertical frequency axes. Let $g_r(x,y)$ denote the reconstructed object that results.

Calculate and sketch the following functions:

a) $g(x,y)$
b) $G(u,v)$
c) $g_s(x,y)$
d) $G_s(u,v)$
e) $G_r(u,v)$
f) $g_r(x,y)$