2-D Transform Relations

1. linearity

$$a_1 f_1(x,y) + a_2 f_2(x,y) \overset{\text{CSFT}}{\leftrightarrow} a_1 F_1(u,v) + a_2 F_2(u,v)$$

2. scaling and shifting

$$f\left(\frac{x-x_0}{a}, \frac{y-y_0}{b}\right) \overset{\text{CSFT}}{\leftrightarrow} |ab|\, F(au, bv)\, e^{-j2\pi(ux_0+vy_0)}$$

3. modulation

$$f(x,y)\, e^{j2\pi(u_0 x+v_0 y)} \overset{\text{CSFT}}{\leftrightarrow} F(u-u_0,\, v-v_0)$$

Transform Relation for Separable Functions

7. Let

$$g(x) \overset{\text{1–D CSFT}}{\leftrightarrow} G(u)$$

$$h(y) \overset{\text{1–D CSFT}}{\leftrightarrow} H(v)$$

then

$$g(x)\, h(y) \overset{\text{2–D CSFT}}{\leftrightarrow} G(u)\, H(v)$$

Transform Relations

8. rotation

$$\tilde{f}(r,\ \theta + \theta_0) \overset{\text{CSFT}}{\leftrightarrow} \tilde{F}(\rho,\ \phi + \theta_0)$$

9. circular symmetry

$$\tilde{f}(r,\theta) = \tilde{f}_0(r) \Leftrightarrow \tilde{F}(\rho,\phi) = \tilde{F}_0(\rho)$$

Important Transform Pairs

1. $\text{rect}(x,y) \overset{\text{CSFT}}{\leftrightarrow} \text{sinc}(u,v)$

2. $\text{circ}(x,y) \overset{\text{CSFT}}{\leftrightarrow} \text{jinc}(u,v)$

3. $\delta(x,y) \overset{\text{CSFT}}{\leftrightarrow} 1$ (by sifting property)

4. $1 \overset{\text{CSFT}}{\leftrightarrow} \delta(u,v)$ (by reciprocity)

5. $e^{j2\pi[u_0x+v_0y]} \overset{\text{CSFT}}{\leftrightarrow} \delta(u-u_0,\ v-v_0)$
(by modulation property)

Polar Coordinate CSFT

Polar coordinate transformation

spatial domain	frequency domain
$x = r\cos\theta$	$u = \rho\cos\phi$
$y = r\sin\theta$	$v = \rho\sin\phi$
$\tilde{f}(r,\theta) = f(x,y)$	$\tilde{F}(\rho,\phi) = F(u,v)$

2-D Comb and Replication Operators

$$\mathrm{comb}_{XY}[f(x,y)] = \sum_m \sum_n f(mX,\ nY)\delta(x-mX,\ y-nY)$$

$$\mathrm{rep}_{XY}[f(x,\ y)] = \sum_m \sum_n f(x-mX,\ y-nY)$$

Transform relation

$$\mathrm{rep}_{XY}[f(x,y)] \overset{\text{CSFT}}{\leftrightarrow} \frac{1}{XY}\,\mathrm{comb}\,\frac{1}{X}\,\frac{1}{Y}\,[F(u,v)]$$

$$\mathrm{comb}_{X,Y}[f(x,y)] \overset{\text{CSFT}}{\leftrightarrow} \frac{1}{XY}\,\mathrm{rep}_{\frac{1}{X},\frac{1}{Y}}\,[F(u,v)]$$