EE 438 Exam 3 Solution Spring 2001

Solution to Prob. 1

(a) $R_n(0)$ is determined immediately as

$$R_n(0) = \sum_{m=0}^{N+p-1} s_n^2(m) = \sum_{m=0}^{N-1} s_n^2(m) = 1$$

- to determine $R_n(1)$ and $R_n(2)$, recall for $p=2$, the LPC coefficients satisfy

$$\begin{bmatrix} R_n(0) & R_n(1) \\ R_n(1) & R_n(0) \end{bmatrix} \begin{bmatrix} \alpha_1^{(2)} \\ \alpha_2^{(2)} \end{bmatrix} = \begin{bmatrix} R_n(1) \\ R_n(2) \end{bmatrix}$$

- substituting: $$\begin{bmatrix} 1 & \frac{2}{3} \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} \alpha_1^{(2)} \\ \alpha_2^{(2)} \end{bmatrix} = \begin{bmatrix} 2/3 \\ \frac{7}{12} \end{bmatrix}$$

- sol'n: $\alpha_1^{(2)} = \frac{1}{2}$ $\quad\alpha_2^{(2)} = \frac{1}{4}$

- check: $(1)\left(\frac{1}{2}\right) + \frac{2}{3}\left(\frac{1}{4}\right) = \frac{3}{6} + \frac{1}{6} = \frac{2}{3}$

$\frac{2}{3}\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right) = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

(b) Solution A: From prob. 2 on hmwk. 6:

$$E_n^{(p)} = R_n(0) - \sum_{k=1}^{p} \alpha_k^{(p)} R_n(k)$$

$$E_n^{(2)} = R_n(0) - \alpha_1^{(2)} R_n(1) - \alpha_2^{(2)} R_n(2)$$

$$= 1 - \frac{1}{2}\left(\frac{2}{3}\right) - \frac{1}{4}\,\frac{7}{12}$$

$$= 1 - \frac{1}{3} - \frac{7}{48} = \frac{32-7}{48} = \frac{25}{48}$$

Solution B: From Levinson-Durbin algorithm,

$$E^{(k+1)} = \left(1 - \alpha_k^{(k)\,2}\right) E^{(k)}$$

with $E^{(0)} = R_n(0)$

EE438 Exam 3 Solution Spring 2001

2

Sol'n. to Prob. 1 (cont.)

(b) $E^{(2)} = \left(1 - \alpha_1^{(1)2}\right)\left(1 - \alpha_2^{(2)2}\right) R_n(0)$

$$= \left(1 - \frac{4}{9}\right)\left(1 - \frac{1}{16}\right)(1) = \frac{25}{48}$$

where: $\alpha_1^{(1)} = \frac{R_n(1)}{R_n(0)} = \frac{2}{3}$

EE438 Exam 3 Solution Spring '01

Solution to Problem 2 (3)

(a) $f(x,y) = .5 \operatorname{jinc}(x,y) ** \operatorname{jinc}(x, \frac{y}{2})$

i.) $F(u,v) = \frac{1}{2} \operatorname{circ}(u,v) \; 2 \operatorname{circ}(u, 2v)$

• invoked properties

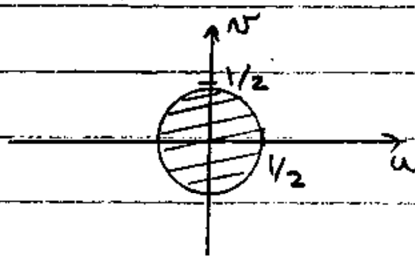
of 2-D CSFT properties and pairs

$\cancel{F(u,v) = \operatorname{circ}(u,v)} \operatorname{circ}(u, 2v)$

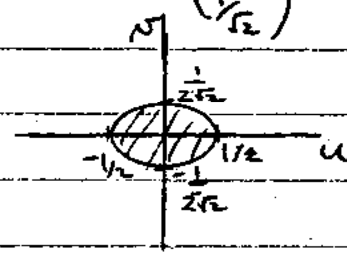
$= 1$ $\qquad\qquad$ $=1$

$u^2 + v^2 < 1/4$ $\qquad\qquad$ $u^2 + 2v^2 < 1/4$

$u^2 + \left(\frac{v}{1/\sqrt{2}}\right)^2 < 1/4$



multiply



$\Rightarrow \cancel{F(u,v) = \operatorname{circ}(u, 2v)}$

• answer: $f(x,y) = .5 \operatorname{jinc}(x, y/2)$

(b) $g(x,y) = \{\operatorname{sinc}(x) \operatorname{rect}(y)\} ** \{\operatorname{sinc}(2x) \operatorname{rect}(y)\}$

$G(u,v) = G_1(u,v) \; G_2(u,v)$

• invoking separability:

$G_1(u,v) = \operatorname{rect}(u) \operatorname{sinc}(v)$

$G_2(u,v) = \frac{1}{2} \operatorname{rect}(\frac{u}{2}) \operatorname{sinc}(v)$

$G(u,v) = \frac{1}{2} \operatorname{rect}(u) \operatorname{rect}(\frac{u}{2}) \operatorname{sinc}(v) \operatorname{sinc}(v)$

i) $\cancel{= \frac{1}{2} \operatorname{rect}(u) \operatorname{sinc}^2(v)}$

$\Lambda(y) =$ triang functi

ii) $g(x,y) = \frac{1}{2} \operatorname{sinc}(x) \; \Lambda(y)$

Solution to Prob. 3

- in polar coordinates, $g_a(x,y)$ may be expressed as

$$\tilde{g}_a(r,\theta) = \text{sinc}\left(r\cos(\theta+45^\circ),\ r\sin(\theta+45^\circ)\right)$$

- note: $r\cos(\theta+45^\circ) = r\cos\theta\cos 45^\circ - r\sin\theta\sin 45^\circ$

$$= \tfrac{1}{\sqrt{2}} r\cos\theta - \tfrac{1}{\sqrt{2}} r\sin\theta = \tfrac{1}{\sqrt{2}}(x-y)$$

$$r\sin(\theta+45^\circ) = r\sin\theta\cos 45^\circ + r\cos\theta\sin 45^\circ$$

$$= \tfrac{1}{\sqrt{2}}(x+y)$$

- thus, we see that $g_a(x,y)$ is $\text{sinc}(x,y)$ rotated by 45° (counter-clockwise) in the x-y plane
- recall the property of 2D CSFT's:

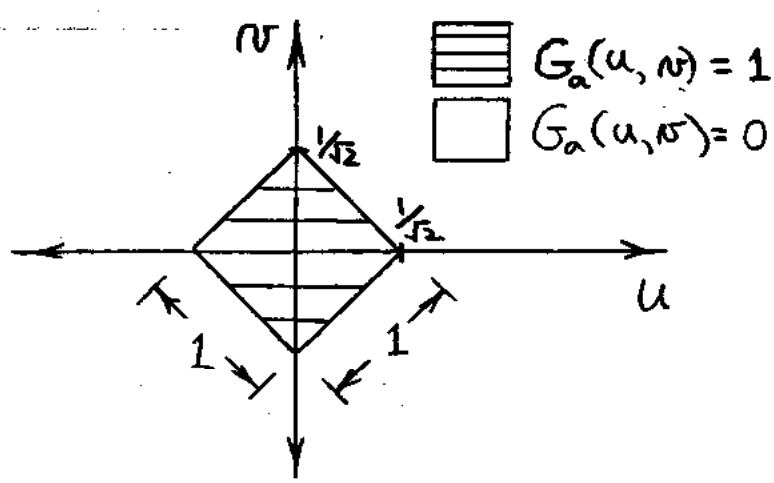
$$\tilde{g}(r,\theta+\theta_0) \xleftrightarrow{\text{CSFT}} \tilde{G}(\rho,\phi+\theta_0)$$

- since the 2D CSFT of $g(x,y) = \text{sinc}(x,y)$ is $G(u,v) = \text{rect}(u,v)$, it follows that the 2D CSFT of $\tilde{g}_a(r,\theta) = \text{sinc}\left(r\cos(\theta+45^\circ), r\sin(\theta+45^\circ)\right)$ is

$$\tilde{G}_a(\rho,\phi) = \text{rect}\left(\rho\cos(\phi+45^\circ),\ \rho\sin(\phi+45^\circ)\right)$$

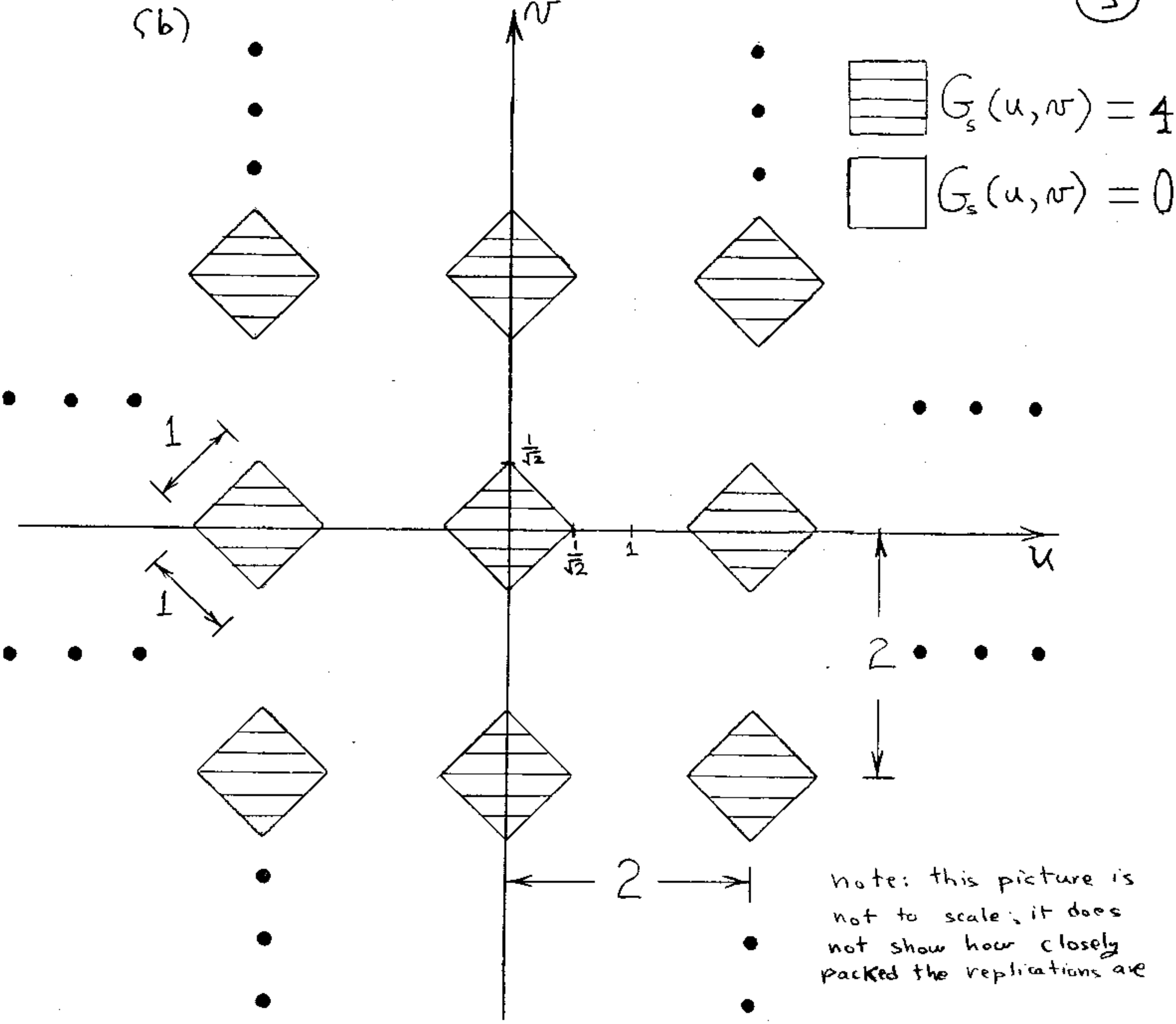
or in rectangular coordinates:

$$G_a(u,v) = \text{rect}\left(\tfrac{1}{\sqrt{2}}(x-y),\ \tfrac{1}{\sqrt{2}}(x+y)\right)$$



Sol'n. to Prob. 3 (cont.)

(b)



note: this picture is not to scale; it does not show how closely packed the replications are

$$G_s(u,v) = \frac{1}{.5}\cdot\frac{1}{.5}\ \mathrm{rep}_{\frac{1}{.5},\frac{1}{.5}}\left\{G_a(u,v)\right\}$$

(c) since $2 - \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{2}}$

can use a filter with a rectangular region of support in u-v domain $H_r(u,v) = \mathrm{rect}\left(\frac{u}{a}, \frac{v}{a}\right)$

Answer: $h_r(x,y) = a^2 \mathrm{sinc}(ax, ay)$

where: $2/\sqrt{2} < a < 2\left(2 - \frac{1}{\sqrt{2}}\right) = 1.293$