Solution to Prob. 1

(a) \[ x(n) = \cos\left(\frac{\pi}{2} n\right) \{ u(n) - u(n-8) \} \]

\[ \frac{2\pi(4)}{16} \]

\[ \cos\left(\frac{2\pi(4)}{16} n\right) \xrightarrow{\text{DFT}} \begin{array}{l}
8 \delta(k-4) + 8 \delta(k-12) = X_16(k) \\
\end{array} \]

\[ k = 0, 1, \ldots, 15 \]

\[ N = 16 \text{ pt. DFT of } h(n) : \]

\[ H_{16}(k) = \sum_{n=0}^{15} \left(\frac{1}{2}\right)^n e^{-j\frac{2\pi kn}{16}} = \sum_{n=0}^{15} \left(\frac{1}{2} e^{-j\frac{2\pi}{16}}\right)^n \]

\[ = \frac{1 - \frac{1}{2^4} e^{-j\frac{2\pi}{16}}}{1 - \frac{1}{2} e^{-j\frac{2\pi}{16}}} = \frac{1}{1 - \frac{1}{2} e^{-j\frac{2\pi}{8}}} \]

\[ k = 0, 1, \ldots, 15 \]

where: \( 1 - \frac{1}{2^{16}} \approx 1 \)

(b) \[ y_{16}(k) = H_{16}(k) X_{16}(k) \]

\[ = 8 \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi}{2}}} \delta(k-4) + 8 \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi}{2}}} \delta(k-12) \]

\[ = \frac{16}{2 + j} \delta(k-4) + \frac{16}{2 - j} \delta(k-12) \]

\[ = \frac{16}{\sqrt{5}} e^{j26.6^\circ} \delta(k-4) + \frac{16}{\sqrt{5}} e^{j26.6^\circ} \delta(k-12) \]

\[ y_8(n) = \frac{2}{\sqrt{5}} \cos\left(\frac{\pi}{2} n - 26.6^\circ\right) \{ u(n) - u(n-15) \} \]
Solin. to Prob. 1 (cont.)

(c) $x(n)$ is of length $L=16$,
$h(n)$ is of length $M=16$,

\[ y(n) = x(n) * h(n) \]

is of length $M+L-1 = 31$.

(time-domain aliasing)

\[ y_{16}(n) = \sum_{l=-\infty}^{\infty} y(n-l16)[x(n-l16)-u(n-l16)] \]

\[ = y(n) + y(n+16) \]

\( n = 0, 1, \ldots, 15 \)

Since $y(n)$ is only nonzero for $n = 0, 1, \ldots, 15$.

Thus: $y_{16}(15) = y(15) + y(31) = y(15)$ \( \text{only unaliased point} \)

all other 15 points are aliased: $y_{16}(n) \neq y(n)$ for $n = 0, 1, \ldots, 14$. 


Solution to Problem 2

the computation involved in computing $X(k)$, $X(k+4)$, and $X(k+8)$ from $X_0(k)$, $X_1(k)$, and $X_2(k)$ may be represented in terms of a 3 pt. DFT as

\[ X_0(k) \xrightarrow{W_{12}^{k}} X(k) \]

\[ X_1(k) \xrightarrow{W_{12}^{k}} X(k+4) \quad k = 0, 1, 2, 3 \]

\[ X_2(k) \xrightarrow{W_{12}^{k}} X(k+8) \]

where: $W_{12} = e^{-j\frac{2\pi}{12}}$ and $\delta = e^{-j\frac{2\pi}{3}}$

note: the 3 pt. butterfly involves 6 complex mults. for each $k$.

(a) from class we know that to compute an N pt. DFT via a decimation-in-time radix 2 FFT requires a total of $\frac{N}{2} \log_2 N$ complex mults.

answer: for $N = 4$ $\Rightarrow$ $\frac{4}{2} \log_2 4 = 4$ complex mults

(b) When an N pt. DFT is factored as $N = ML$ and the Divide and Conquer Approach is applied, we know from class (and the Demo Divide Conquer.m) that the no. of complex multiplies is $ML^2 + N + LM^2$. Here $M = 3$ and $L = 4$. However, we will compute the 4 pt. DFT in terms of a radix 2 FFT. The formula is thus modified as $N \left( \frac{L}{2} \log_2(L) \right) + N + LM^2 = 3(4) + 12 + 4(3)^2 = 60$

(c) direct computation of a 12 pt. DFT requires

$12^2 = 144$ complex mults.

$\frac{60}{144} = 41.7\% \Rightarrow 58.3\%$ reduction in complex mults.
Sol'n to Problem 3

(a) Pitch period: \( \frac{40}{F_5} = \frac{40}{10^9/\text{sec}} = 4 \text{ ms} \)

(b) Pitch frequency: \( \frac{1}{4 \times 10^{-3} s} = 250 \text{ Hz} \) \( \Rightarrow \) female

(c) \( F_1 = (0.066) \times 10^4 = 660 \text{ Hz} \)
\( F_2 = (0.172) \times 10^4 = 1720 \text{ Hz} \)
\( F_3 = (0.241) \times 10^4 = 2410 \text{ Hz} \)

(d) Consulting the table of average formants for the vowels,
the most likely voiced phoneme is AE (or AE) --
the a sound in bat.

Since periodic forever, get spectral lines, i.e.,
Dirac Delta functions at every integer multiple
of the pitch frequency.