BALAKRISHNAN
FEB 9, 1999

EE301 Midterm #1

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This exam has two parts.

   Part I consists of three questions, whose answers you need not justify. Enter the answers to Part I on Page 2, which is provided to you separately for your convenience. When you return your exam, simply place Page 2 between pages 1 and 3. You may staple it in place, if you wish (this is not necessary).

   Make sure you enter your name, student ID number and e-mail address in the space provided on that page, **NOW!**

   Part II consists of two problems. Unless otherwise instructed, justify your answers to these problems completely. Please note that answers provided without justification to those problems requiring a full justification will be given zero credit.
3. You have **one hour**.
4. There are **11** pages in the exam booklet (including Page 2). Use the back of each page for rough work, if necessary.
5. You are **not** allowed the use of crib sheets.
6. You are **not** allowed the use of calculators.
7. Tip: Make sure you read through the exam once before beginning. Work as quickly and efficiently as you can. If you get stuck on a certain problem, move on to others.

IMPORTANT!
Enter the answer to the questions in Part I on Page 2.

In Part II, whenever a certain space is provided for the final answer, be sure to enter your answer there.

Name:	
Student ID #:	KEY
E-mail address:	
Signature:	

Enter your answers to Part I here

Problem 1: (30 points)

For items (a) and (b), enter the numerical values of E_∞ and P_∞ respectively. For items (c) through (j), enter one of the the letters "A" through "H".

Table I

(a) E_∞	(b) P_∞	(c) $x_{\text{even}}(t)$	(d) $x_{\text{odd}}(t)$	(e) $x(t+1)$	(f) $x(t-1)$	(g) $x(2t+1)$	(h) $x(2t+2)$	(i) $x(1-2t)$	(j) $x(2-2t)$
1/3	0	E	A	D	B	C	H	G	F

Problem 2: (20 points)

Enter "Y" in the Table if you can conclude that the property listed on the left in each row holds for the system listed at the top of the column. Enter "N" if it can be concluded that the property does not hold. (For each case, there is sufficient data to make this decision.)

Every entry in the table is worth 1.5 points. You will get two additional points if all the entries are correct.

Table II

	System I	System II
Memoryless	N	N
Invertible	N	N
Causal	N	N
Stable	Y	Y
Time-invariant	N	N
Linear	Y	N

Problem 3: (10 points)

In the second row of the following table, enter "Y" if the signal corresponding to the item label listed at the top of the column is periodic; enter "N" otherwise. In the third row, enter the fundamental period, if appropriate.

Table III

Item label	(a)	(b)	(c)	(d)	(e)
Periodic?	Y	N	N	Y	Y
Fundamental period (if periodic)	$2\pi/5$	–	–	16	16

Name:

Student ID #:

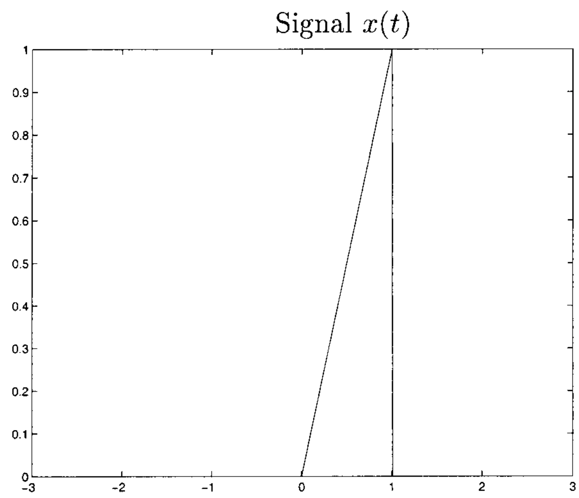
E-mail address:

Signature:

Questions for Part I

- Enter your answers on Page 2.
- Do not justify your answers.
- No partial credit will be given for any questions in Part I. Therefore work as carefully as you can.

1. (30 points) A continuous-time signal $x(t)$ is shown below.



(a) (4 points) What is the energy in $x(t)$ over the infinite interval, that is, what is E_∞?

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt \quad \& \quad x(t) = t\{u(t) - u(t-1)\}$$

So $$E_\infty = \int_0^1 t^2 dt = \frac{t^3}{3}\Big|_0^1 = 1/3 \;//$$

(b) (2 points) What is the power in $x(t)$ over the infinite interval, that is, what is P_∞?

$$P_\infty = \lim_{T\to\infty} \frac{\int_{-T}^{T} |x(t)|^2 dt}{2T}$$

We see that if $E_\infty < \infty$ then $P_\infty = 0$ //

Eight signals labeled "Signal A" through "Signal H" are shown on the next page. Match these signals with the following eight signals, and enter your answers in Table I. Each entry must be a letter from "A" through "H". Each answer is worth three points.

(c) The even part of $x(t)$, that is, $x_{\text{even}}(t)$.

$x_{even}(t) = \frac{x(t)+x(-t)}{2}$ should be symmetric wrt the y axis $\Rightarrow$ E

(d) The odd part of $x(t)$, that is, $x_{\text{odd}}(t)$.

$x_{odd}(t) = \frac{x(t)-x(-t)}{2}$ should be symmetric wrt the origin $\Rightarrow$ A

(e) $x(t+1)$.

Shift $x(t)$ to the left by 1 $\Rightarrow$ D

(f) $x(t-1)$.

Shift $x(t)$ to the right by 1 $\Rightarrow$ B

(g) $x(2t+1)$.

Shift $x(t)$ to the left by 1 & contract t by 2 $\Rightarrow$ C

(h) $x(2t+2)$.

Shift $x(t)$ to the left by 2 & contract t by 2 $\Rightarrow$ H

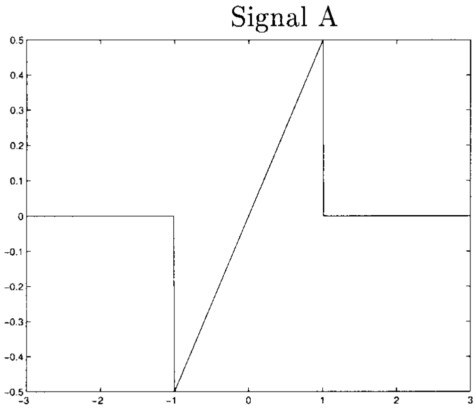
(i) $x(1-2t)$.

Shift $x(t)$ to the left by 1, flip, & contract t by 2 $\Rightarrow$ G

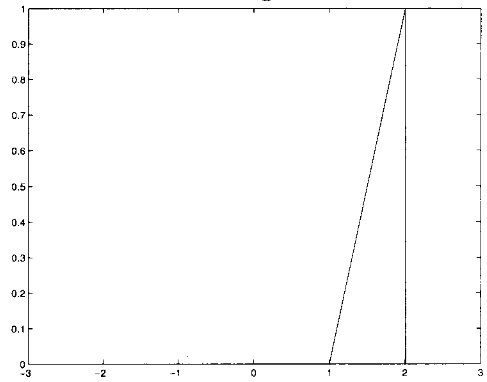
(j) $x(2-2t)$.

Shift $x(t)$ to the left by 2, flip, & contract t by 2 $\Rightarrow$ F

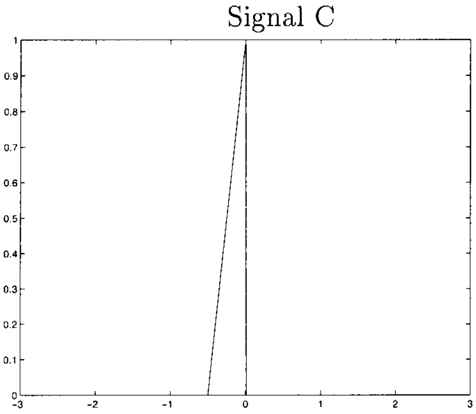
Signal A



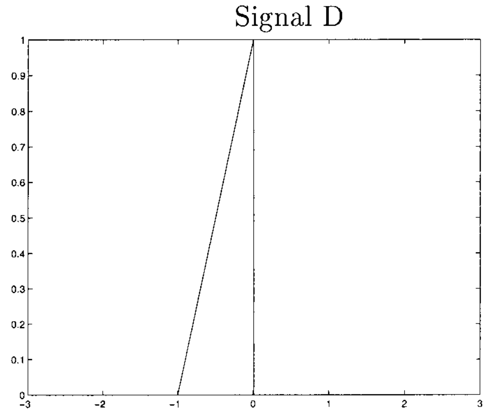
Signal B



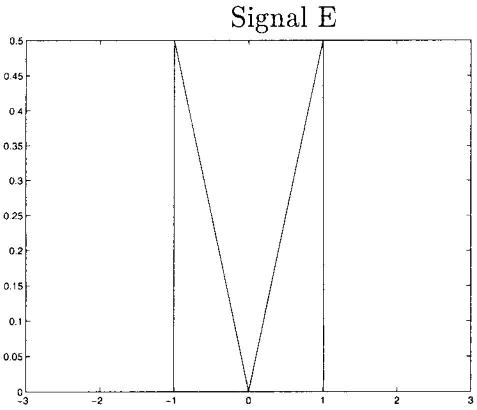
Signal C



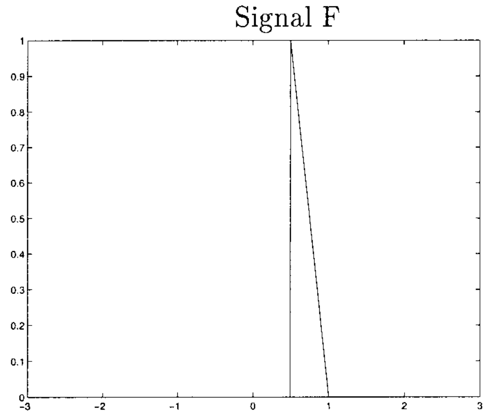
Signal D



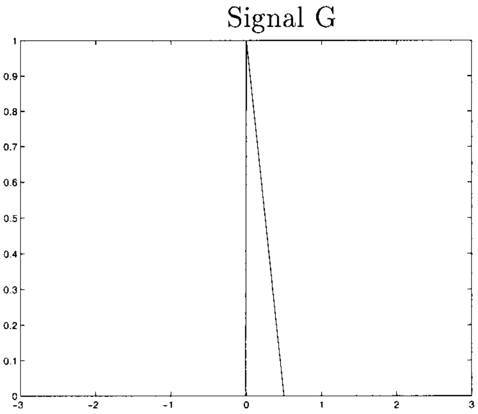
Signal E



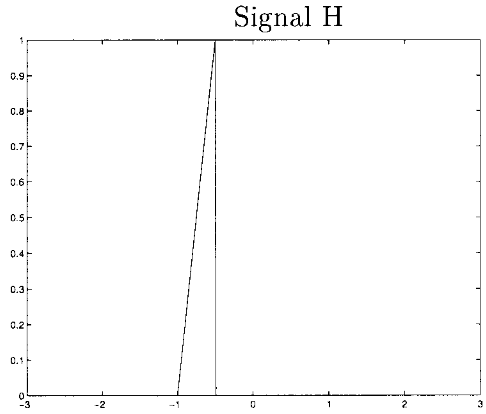
Signal F



Signal G



Signal H



2. (20 points) Determine if each of the following systems (with input x and output y) is memoryless or with-memory; invertible or non-invertible; causal or non-causal; stable or unstable; time-invariant or time-varying; linear or nonlinear.

Enter your answers in Table II on Page 2. In each entry in the table, write "**Y**" if you can conclude that the property listed on the left holds for the system listed at the top of the column. Enter "**N**" if it can be concluded that the property does not hold. (For each case, there is sufficient data to make this decision.)

Every entry in the table is worth 1.5 points. You will get two additional points if all the entries are correct.

(a) **System I:**

$$y[n] = x[n] + x[-n].$$

- Linearity : $x_1 - \boxed{} - y_1 = x_1[n] + x_1[-n]$

  $x_2 - \boxed{} - y_2 = x_2[n] + x_2[-n]$

  $ax_1 + bx_2 - \boxed{} - y_3 = \left(ax_1[n] + bx_2[n]\right) + \left(ax_1[-n] + bx_2[-n]\right)$

  Is $y_3[n] = ay_1 + by_2$ yes $\Rightarrow$ Linear

- Causality : Let $n = -1$. Then, $y[-1] = x[-1] + \underset{\uparrow \text{future}}{x[1]}$

  we see that the system is noncausal

$\Longrightarrow$

(b) **System II:**

$$y(t) = x(t)x(-t).$$

- Linearity : $x_1 - \boxed{} - y_1 = x_1(t)\, x_1(-t)$

  $x_2 - \boxed{} - y_2 = x_2(t)\, x_2(-t)$

  $ax_1 + bx_2 - \boxed{} - y_3 = [ax_1(t) + bx_2(t)][ax_1(-t) + bx_2(-t)]$

  Is $y_3(t) = ay_1(t) + by_2(t)$ No $\Rightarrow$ Nonlinear

- Causality : Let $t = -1$ $\Rightarrow$ noncausal
- Time invariance : Similar reasoning to (a) $\Rightarrow$ TV

$\Longrightarrow$

Q2 | (a) cont'd :

- Time invariance : $x_1 \rightarrow \boxed{} \rightarrow y_1(n) = x(n) + x(-n)$

$x_1(n-n_0) \rightarrow \boxed{} \rightarrow y_1'(n) = x(n-n_0) + x(-n-n_0)$

Is $y_1'(n) = y_1(n-n_0) \Rightarrow x(n-n_0) + x(-[n-n_0]) \stackrel{?}{=} y_1'(n)$ NO

$\Rightarrow$ It's time varying.

- Memory: Let $n=1 \Rightarrow y(1) = x(1) + x(-1)$

Input at $n=-1$ is used to compute output at $n=1$

$\Rightarrow$ with memory.

- Stability: $|y(n)| = |x(n) + x(-n)|$

$\leq |x(n)| + |x(-n)|$

$< B + B = 2B$

So if $|x(n)| < B$ then $|y(n)| < 2B \Rightarrow$ Stable

- Invertibility : we observe that $y(n) = y(-n)$. Hence, the system is not invertible

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(b) cont'd :

Proceeding similar to part (c) we find that the system is with memory & not invertible.

- Stability : $|y(t)| = |x(t)\,x(-t)|$

$= |x(t)|\,|x(-t)|$

$< B \cdot B$

If $|x(t)| < B$, then $|y(t)| < B^2 \Rightarrow$ Stable

3. (10 points) Classify the following signals as periodic or non-periodic; for periodic signals, calculate the fundamental period. Enter your answers in Table III on Page 2.

(a) (2 points) $x(t) = e^{j5t}$.

Periodic $\Rightarrow T = 2\pi/5$ //

(b) (2 points) $x(t) = te^{j5t}$.

Product of a periodic & nonperiodic signal

$\Rightarrow$ not periodic //

(c) (2 points) $x[n] = e^{jn}$.

$2\pi/\omega_0 = 2\pi/1 = 2\pi$: irrational

$\Rightarrow$ not periodic //

(d) (2 points) $x[n] = e^{j\frac{\pi}{8}n}$.

$2\pi/\omega_0 = 2\pi/\pi/8 = 16 \Rightarrow T = 16$ //

(e) (2 points) $x[n] = e^{j\frac{\pi}{4}n}e^{j\frac{\pi}{8}n}$.

$2\pi/\omega_1 = 2\pi/\pi/4 \Rightarrow T_1 = 8$

$2\pi/\omega_2 = 2\pi/\pi/8 \Rightarrow T_2 = 16$

$T = \text{l.c.m}(8, 16) = 16$ //

Questions for Part II

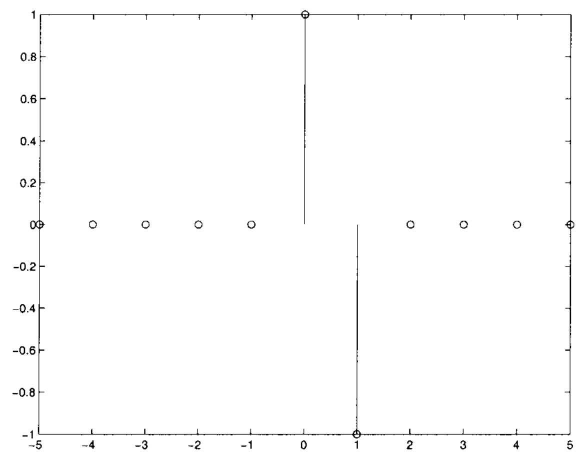
Justify your answers completely. Answers provided without a complete justification may be given zero credit!

4. (20 points)

The impulse response of a discrete-time linear time-invariant system is given by

$$h[n] = \begin{cases} 1 & \text{if } n = 0, \\ -1 & \text{if } n = 1, \\ 0 & \text{otherwise} \end{cases}$$

A plot of $h[n]$ is shown below.



(a) (5 points)
Let $x[n]$ be an input to this LTI system. Let $y[n]$ be the corresponding output. Find real numbers α, β and γ such that

$$y[n] = \alpha x[n+1] + \beta x[n] + \gamma x[n-1].$$

$h(n) = \delta(n) - \delta(n-1)$

$y(n) = x(n) * h(n) = x(n) * [\delta(n) - \delta(n-1)]$

$= x(n) - x(n-1)$

$\alpha = 0$

$\beta = 1$

$\gamma = -1$ //

(b) (5 points)
Is the LTI system causal? Completely justify your answer.

① $y(n)$ only depends on past & present values $\Rightarrow$ Causal

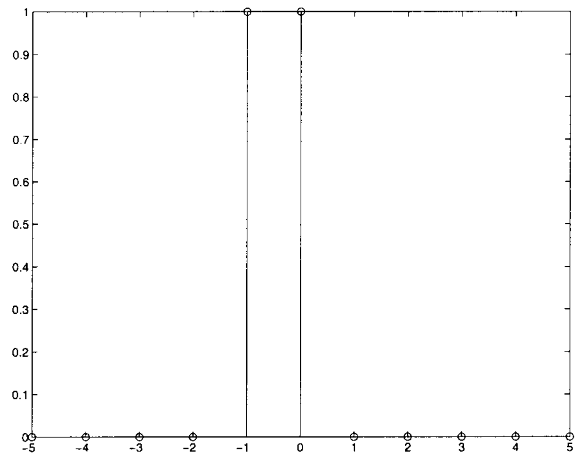
OR

② $h(n) = 0$ for $n < 0$ $\Rightarrow$ Causal

(c) (10 points) The input to the LTI system is

$$x[n] = \begin{cases} 1 & \text{if } n = -1, \\ 1 & \text{if } n = 0, \\ 0 & \text{otherwise} \end{cases}$$

A plot of $x[n]$ is shown below, for your convenience.



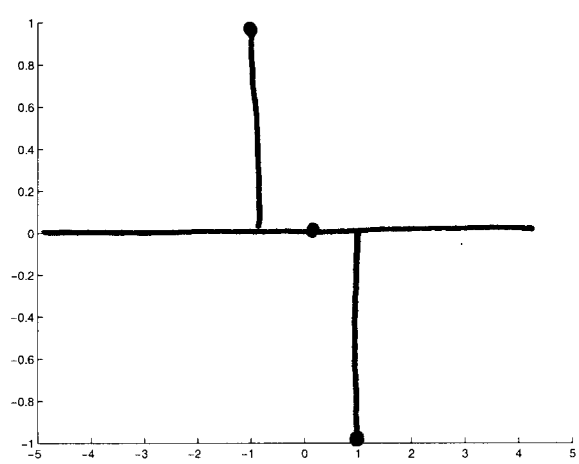
Find the output $y[n]$, and plot it in the space shown below. Show all work.

we know that $y(n) = x(n) - x(n-1)$

$= [\delta(n+1) + \delta(n)] - [\delta(n) + \delta(n-1)]$

$\downarrow$

$y(n) = \delta(n+1) - \delta(n-1)$



5. (20 points)

For a discrete-time LTI system, you are given that when the input signal is

$$x[n] = \begin{cases} 1 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ 0 & \text{otherwise} , \end{cases}$$

the output signal is

$$y[n] = \begin{cases} 0 & \text{if } n < 0, \\ \left(\frac{1}{2}\right)^n & \text{if } n \geq 0. \end{cases}$$

(a) (10 points) Find the output $y_1[n]$ of the system when the input is

$$x_1[n] = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{if } n = 1, \\ -1 & \text{if } n = 2, \\ 0 & \text{otherwise.} \end{cases}$$

We see that $x_1[n] = x[n] - x[n-1]$

Since our system is LTI, we know that

$$y_1[n] = y[n] - y[n-1]$$
$$= \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^{n-1} u(n-1) \quad //$$

If you play with this expression, then you can get:

$$y_1[n] = \left(\frac{1}{2}\right)^n u(n) - \left\{\left(\frac{1}{2}\right)^{n-1} u(n) - 2\delta(n)\right\}$$
$$= \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^n \cdot 2u(n) + 2\delta(n)$$
$$= 2\delta(n) - \left(\frac{1}{2}\right)^n u(n) \quad //$$

(b) (10 points) You are given that the impulse response satisfies

$$h[n] = 0, \quad \text{for} \quad n < 0.$$

Find the numerical values of

$$h[0], \quad h[1], \quad h[2], \quad h[3], \quad \text{and} \quad h[4].$$

Since $x(n) = \delta(n) + \delta(n-1)$, we know that

$$y(n) = x(n) * h(n) = [\delta(n) + \delta(n-1)] * h(n)$$
$$= h(n) + h(n-1).$$

Let's plug in a few values for n

$n = 0$: $y(0) = h(0) + \overbrace{h(-1)}^{=0} = 1 \Rightarrow h(0) = 1$ //

$n = 1$: $y(1) = h(1) + h(0) = 1/2 \Rightarrow h(1) = -1/2$ //

$n = 2$: $y(2) = h(2) + h(1) = 1/4 \Rightarrow h(2) = 3/4$ //

$n = 3$: $y(3) = h(3) + h(2) = 1/8 \Rightarrow h(3) = -5/8$ //

$n = 4$: $y(4) = h(4) + h(3) = 1/16 \Rightarrow h(4) = 11/16$ //

(b) (10 points) You are given that the impulse response satisfies

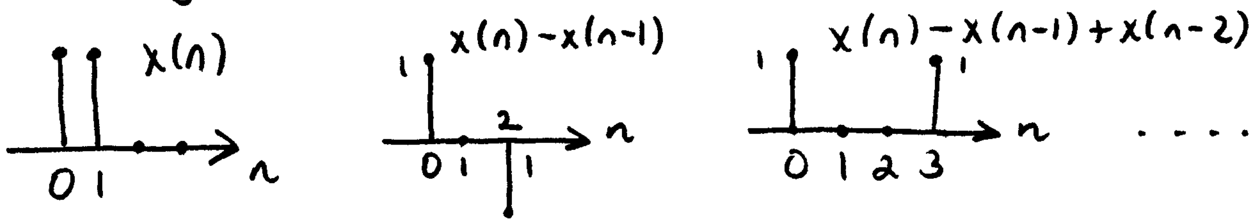
$$h[n] = 0, \quad \text{for} \quad n < 0.$$

Find the numerical values of

$$h[0], \quad h[1], \quad h[2], \quad h[3], \quad \text{and} \quad h[4].$$

$\mapsto$ Another way to solve this problem:

In order to find $h(n)$ we need to have $\delta(n)$ as the input. Is it possible to obtain $\delta(n)$ from the $x(n)$ signal of part (a)?



In the limit where we take ∞'ly many terms:

$$\delta(n) = \sum_{k=0}^{\infty} (-1)^k x(n-k)$$

So $h(n) = \sum_{k=0}^{\infty} (-1)^k y(n-k)$ since the system is LTI.

$$h(0) = \sum_{k=0}^{\infty} y(-k) = y(0) = 1 \quad \leftarrow \text{since } y(n) = 0 \text{ for } n < 0$$

$$h(1) = \sum_{k=0}^{\infty} y(1-k) \cdot (-1)^k = y(1) - y(0) = 1/2 - 1 = 1/2$$

$$h(2) = \sum_{k=0}^{\infty} y(2-k)(-1)^k = y(2) - y(1) + y(0)$$
$$= 1/4 - 1/2 + 1 = 3/4 \quad \text{etc..}$$