EE301 Midterm #1

1. Enter your name, student ID number, e-mail address and your full signature in the space provided on this page.

2. You have fifty minutes.

3. There are 10 pages in the exam booklet. Use the back of each page for rough work, if necessary.

4. You are not allowed the use of crib sheets.

5. You are not allowed the use of calculators.

6. Tip: Make sure you read through the exam once before beginning. Work as quickly and efficiently as you can. If you get stuck on a certain problem, move on to others.

7. Unless otherwise instructed, no justification is necessary.

8. Unless otherwise stated, no partial credit will be given, therefore work as carefully as you can.

9. Enter your answers in the spaces provided.

Name:

Student ID #:

E-mail address:

Signature:

Solutions
(a) (3 pts) Calculate $E_\infty$ for $x(t)$, i.e., the energy of $x(t)$ over the interval $(-\infty, \infty)$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = 3$$

$E_\infty = 3$

(b) (3 pts) Calculate $P_\infty$ for $x(t)$, i.e., the power of $x(t)$ over the interval $(-\infty, \infty)$.

$$P_\infty = 0, \text{ since } E_\infty = 3 < \infty$$

$P_\infty = 0$

(c) (2 pts) Sketch $x(t - 1)$ below.

(Shift right by 1)

(d) (2 pts) Sketch $x(t + 1)$ below.

(Shift left by 1)

(e) (5 pts) Sketch $x(0.5t)$ below.

(Expand time by 2)

(f) (5 pts) Sketch $x(1 - 0.5t)$ below.

(Shift left by 1; Expand time by 2; Reverse time)

(g) (5 pts) Sketch the even part of $x(t)$ below.

$$\frac{x(t) + x(-t)}{2}$$

(h) (5 pts) Sketch the odd part of $x(t)$ below.

$$\frac{x(t) - x(-t)}{2}$$
2. (20 pts) Classify the following signals as periodic or Non-periodic; for periodic signals, calculate the fundamental period. Enter your answers in the spaces provided.

- No partial credit will be given for any part of this problem, therefore work as carefully as you can.
- No justification is necessary.

(a) (4 pts) \( x(t) = \sin(4t + \pi/8) \).

\( \sin(\omega_0 t + \phi) \) is always periodic for any \( \omega_0 \neq 0 \).

So \( \omega_0 = \text{fundamental frequency} = 4 \)
\[ T = \text{fund. period} = \frac{2\pi}{\omega_0} = \frac{\pi}{2} \]

(b) (4 pts) \( x(t) = \sin t + \sin(2t + \pi/8) \).

Periodic, with fund. period = \( 2\pi \) (by inspection)

\[ \square \text{Non-periodic} \]
\[ \checkmark \text{Periodic; Period} = \frac{\pi}{2} \]

(c) (4 pts) \( x[n] = e^{j\pi n/8} \).

\[ x[n] = e^{j\omega_0 n} \] with \( \omega_0 = \pi/8 \)
\[ \omega_0 = \frac{\pi}{8}, \quad \frac{1}{2\pi} = \frac{1}{16} = \frac{m}{N}, \quad \text{so N} = 16 \]

\[ \square \text{Non-periodic} \]
\[ \checkmark \text{Periodic; Period} = 16 \]

(d) (4 pts) \( x[n] = 2^{-n} e^{j\pi n/8} \).

Not periodic, because \( 2^n \) term

\[ \square \text{Non-periodic} \]
\[ \square \text{Periodic; Period} = \_\_\_ \]

(e) (4 pts) \( x[n] = e^{j\pi n/8} \).

\[ x[n] = e^{j\omega_0 n}, \quad \text{with} \quad \omega_0 = \frac{\pi}{8} \]
\[ \omega_0 = \frac{1}{16\pi}, \quad \text{not rational, so not} \]

\[ \square \text{Non-periodic} \]
\[ \square \text{Periodic; Period} = \_\_\_ \]
3. (10 pts) Determine if the following systems (with input $x$ and output $y$) are linear or nonlinear; time-invariant or time-varying; memoryless or with-memory; causal or non-causal; stable or unstable.

- No partial credit will be given for any part of this problem, therefore work as carefully as you can.
- No justification is necessary.

(a) (5 points) $y(t) = \frac{d}{dt}(x(t))$.

While the first four properties are fairly straightforward to verify, stability is a bit tricky: To see why $y = \frac{dx}{dt}$ represents an unstable system, consider a family of bounded inputs:

![Graph showing bounded input and unbounded output](image)

(b) (5 points) $y[n] = \begin{cases} 0, & n < 0, \\ \sum_{k=0}^{n} x[k], & n \geq 0. \end{cases}$

- Time varying because $\delta[n] \mapsto \delta[n]$ but $\delta[n+1] \mapsto 0$

- Unstable: Consider $y[n]$ for the bounded input $x[n] = u[n]$ (step input)

<table>
<thead>
<tr>
<th>Linear</th>
<th>Nonlinear</th>
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<tbody>
<tr>
<td>✔️</td>
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<tr>
<td>Time-invariant</td>
<td>Time-varying</td>
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<td>✔️</td>
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<tr>
<td>Memoryless</td>
<td>With-memory</td>
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<td>✔️</td>
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<tr>
<td>Causal</td>
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<td>Stable</td>
<td>Unstable</td>
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<td>✔️</td>
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</tbody>
</table>
4. (20 points) Let the impulse response $h(t)$ of a continuous-time LTI system be

$$h(t) = \begin{cases} 0 & t < 0, \\ 1 & t \geq 0. \end{cases}$$

Thus, the impulse response is simply the unit step function.

For this system, suppose the input signal is

$$x(t) = \begin{cases} 0 & -\infty < t \leq 0, \\ t & 0 < t \leq 1, \\ 1 & 1 < t \leq \infty. \end{cases}$$

A sketch of $x(t)$ is shown below:

Find the output $y(t)$, and sketch it on the graph provided on the next page.

- **You must show all the work you did in determining** $y(t)$. Merely writing down your answer with no justification will likely earn you zero credit.
- Work as neatly as you can.
- Use the back of this page, if you need extra space.

**When** $t < 0$

\[ x(\tau) \uparrow \]

\[ h(t-\tau) \]

\[ \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau = 0 \]

\[ y(t) = 0 \]
When $t \in [0,1]$

\[
\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \text{Shaded area} = \frac{1}{2} t^2 = y(t)
\]

When $t > 1$

\[
y(t) = \text{Shaded area} = \frac{1}{2} + t - 1
\]

Sketch the output $y(t)$ of the system here.
5. (20 points)

For all parts of this problem:

- **You must show all the work you did in obtaining your answers.** Merely writing down your answers with no justification will likely earn you zero credit.
- Work as neatly as you can.

The output $y(t)$ of a causal LTI system corresponding to the input

$$x(t) = \begin{cases} 0 & -\infty < t \leq 0, \\ 1 & 0 < t \leq 1, \\ 0 & 1 < t < \infty. \end{cases}$$

is

$$y(t) = \begin{cases} 0 & -\infty < t \leq 0.5, \\ 2(t - 0.5) & 0.5 < t \leq 1.5, \\ 2 & 1.5 < t < \infty. \end{cases}$$

A sketch of the input $x(t)$ and the output $y(t)$ are shown below:

(a) (5 pts) Find the output of the system corresponding to the input $x(0.5t)$, and sketch it on the graph provided on the next page.

By inspection  \[ x(0.5t) = x(t) + x(t-1) \]
So \( y(0.5t) = y(t) + y(t-1) \) (from linearity)

Another method is to solve part (b) first, i.e., find the impulse response. This turns out to be \( 2u(t-0.5) \), so \( x(0.5t) * 2u(t-0.5) = y(0.5t) \), which you can verify is the same as before.

By the way, it is NOT TRUE that for any linear system

\[ x(t) \rightarrow y(t) \]

implies \( x(0.5t) \rightarrow y(0.5t) \)

(Can you think of conditions under which this is true though?)

Sketch the output of the system corresponding to the input \( x(0.5t) \) here.
(b) (15 pts) Find the impulse response of the system, and sketch it on the graph below.

**Method I.** \[ x(t) = u(t) - u(t-1), \] for which the output is \[ y(t) = 2 \left[ (t-0.5)u(t-0.5) - (t-1.5)u(t-1.5) \right] \]

But with \( s(t) \) denoting the step response, we must have \( s(t) - s(t-1) = y(t) \), so that

\[ s(t) - s(t-1) = 2 \left[ (t-0.5)u(t-0.5) - (t-1.5)u(t-1.5) \right] \]

So differentiating both sides w.r.t. \( t \),

\[ h(t) - h(t-1) = 2 \left[ u(t-0.5) - u(t-1.5) \right], \]

from which we get \( h(t) = 2u(t-0.5) \)

**Method II.** \( x(t) \rightarrow y(t) \) means \( \frac{dx}{dt} \rightarrow \frac{dy}{dt} \)

Now, \( \frac{dx}{dt} = \delta(t) - \delta(t-1) \), so that \( \frac{dy}{dt} = h(t) - h(t-1) \)

(-rest is similar to Method I)