BALAKRISHNAN
HANDOUT #13
~~SEP 30~~, 1998
Oct 5

EE301 Midterm #1

1. Enter your name, student ID number, e-mail address and your full signature in the space provided on this page.
2. You have **fifty minutes**.
3. There are **10** pages in the exam booklet. Use the back of each page for rough work, if necessary.
4. You are **not** allowed the use of crib sheets.
5. You are **not** allowed the use of calculators.
6. Tip: Make sure you read through the exam once before beginning. Work as quickly and efficiently as you can. If you get stuck on a certain problem, move on to others.
7. **Unless otherwise instructed, no justification is necessary.**
8. **Unless otherwise stated, no partial credit will be given, therefore work as carefully as you can.**
9. **Enter your answers in the spaces provided.**

Name:	
Student ID #:	Solutions
E-mail address:	
Signature:	

(a) (3 pts) Calculate E_∞ for $x(t)$, i.e., the energy of $x(t)$ over the interval $(-\infty, \infty)$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = 3$$

$\boxed{E_\infty = 3}$

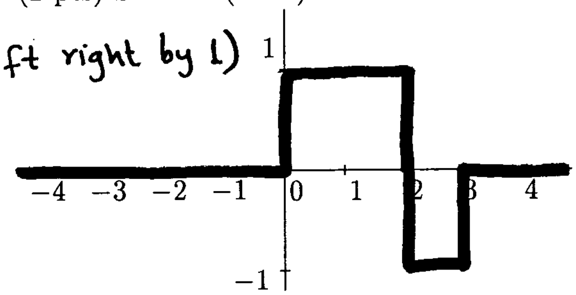
(b) (3 pts) Calculate P_∞ for $x(t)$, i.e., the power of $x(t)$ over the interval $(-\infty, \infty)$.

$P_\infty = 0$, since $E_\infty = 3 < \infty$

$\boxed{P_\infty = 0}$

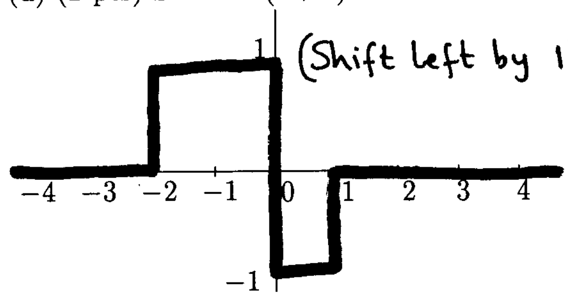
(c) (2 pts) Sketch $x(t-1)$ below.

(Shift right by 1)



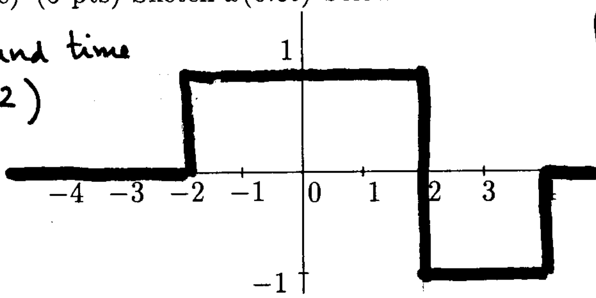
(d) (2 pts) Sketch $x(t+1)$ below.

(Shift left by 1)



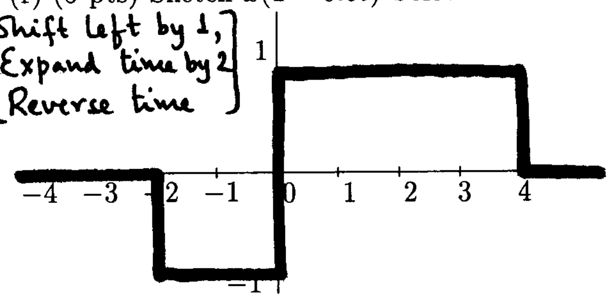
(e) (5 pts) Sketch $x(0.5t)$ below.

(Expand time by 2)



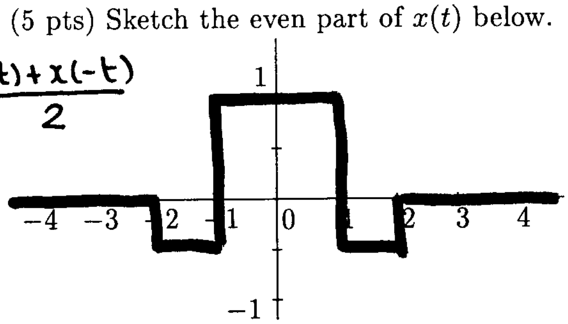
(f) (5 pts) Sketch $x(1-0.5t)$ below.

[Shift left by 1, Expand time by 2, Reverse time]



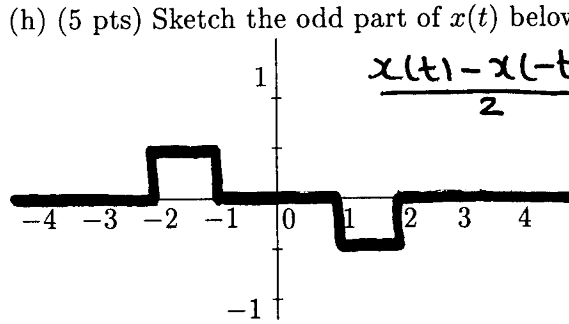
(g) (5 pts) Sketch the even part of $x(t)$ below.

$\frac{x(t)+x(-t)}{2}$



(h) (5 pts) Sketch the odd part of $x(t)$ below.

$\frac{x(t)-x(-t)}{2}$



2. (20 pts) Classify the following signals as periodic or Non-periodic; for periodic signals, calculate the fundamental period. Enter your answers in the spaces provided.

- **No partial credit will be given for any part of this problem, therefore work as carefully as you can.**
- **No justification is necessary.**

(a) (4 pts) $x(t) = \sin(4t + \pi/8)$.

$\sin(\omega_0 t + \phi)$ is always periodic for any $\omega_0 \neq 0$.

So ω_0 = fundamental frequency = 4

T = fund. period = $\frac{2\pi}{\omega_0} = \frac{\pi}{2}$

☐ Non-periodic

☑ Periodic; Period = $\pi/2$

(b) (4 pts) $x(t) = \sin t + \sin(2t + \pi/8)$.

Periodic, with fund. period = 2π (by inspection)

☐ Non-periodic

☑ Periodic; Period = 2π

(c) (4 pts) $x[n] = e^{j\pi n/8}$.

$x[n] = e^{j\omega_0 n}$ with $\omega_0 = \pi/8$

$\frac{\omega_0}{2\pi} = \frac{\pi}{8} \cdot \frac{1}{2\pi} = \frac{1}{16} = \frac{m}{N}$, so $N = 16$

☐ Non-periodic

☑ Periodic; Period = 16

(d) (4 pts) $x[n] = 2^{-n} e^{j\pi n/8}$.

Not periodic, because of 2^{-n} term

☑ Non-periodic

☐ Periodic; Period = ____

(e) (4 pts) $x[n] = e^{jn/8}$.

$x[n] = e^{j\omega_0 n}$, with $\omega_0 = 1/8$

$\frac{\omega_0}{2\pi} = \frac{1}{16\pi}$, not rational, so not

☐ Non-periodic

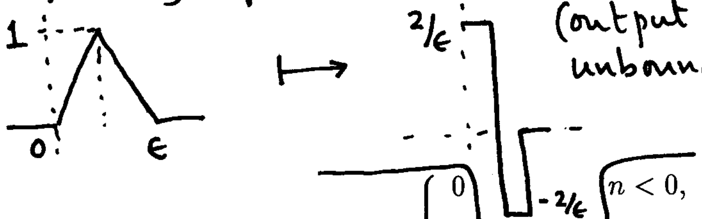
☐ Periodic; Period = ____

3. (10 pts) Determine if the following systems (with input x and output y) are linear or nonlinear; time-invariant or time-varying; memoryless or with-memory; causal or non-causal; stable or unstable.

- **No partial credit will be given for any part of this problem, therefore work as carefully as you can.**
- **No justification is necessary.**

(a) (5 points) $y(t) = \frac{d}{dt}(x(t))$.

While the first four properties are fairly straightforward to verify, stability is a bit tricky: To see why $y = \frac{dx}{dt}$ represents an unstable system, consider a family of bounded inputs:



(output is unbounded as $\epsilon \to 0$)

☑ Linear	☐ Nonlinear
☑ Time-invariant	☐ Time-varying
☑ Memoryless	☐ With-memory
☑ Causal	☐ Noncausal
☐ Stable	☑ Unstable

(b) (5 points) $y[n] = \begin{cases} 0 & n < 0, \\ \sum_{k=0}^{n} x(k) & n \geq 0. \end{cases}$

Time varying because $\delta[n] \mapsto \delta[n]$ but $\delta[n+1] \mapsto 0$

Unstable : Consider $y[n]$ for the bounded input $x[n] = u[n]$ (step input)

☑ Linear	☐ Nonlinear
☐ Time-invariant	☑ Time-varying
☐ Memoryless	☑ With-memory
☑ Causal	☐ Noncausal
☐ Stable	☑ Unstable

4. (20 points) Let the impulse response $h(t)$ of a continuous-time LTI system be

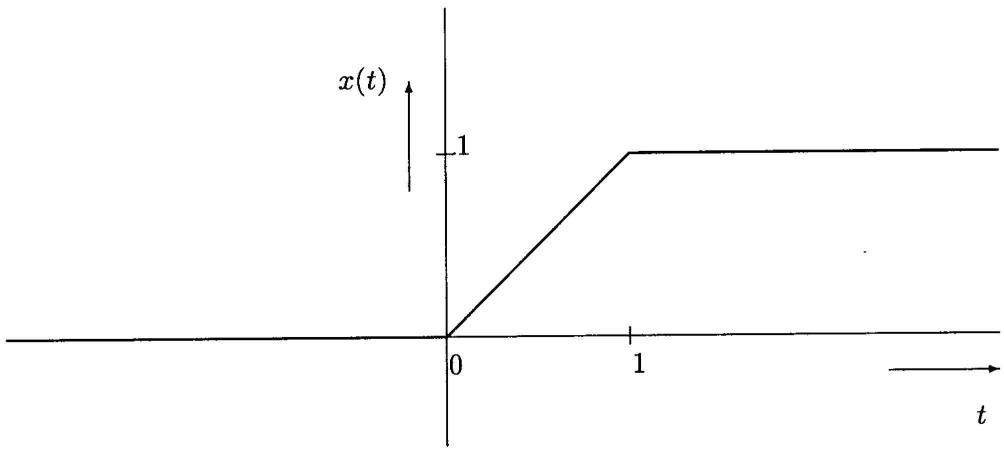
$$h(t) = \begin{cases} 0 & t < 0, \\ 1 & t \geq 0. \end{cases}$$

Thus, the impulse response is simply the unit step function.

For this system, suppose the input signal is

$$x(t) = \begin{cases} 0 & -\infty < t \leq 0, \\ t & 0 < t \leq 1, \\ 1 & 1 < t \leq \infty. \end{cases}$$

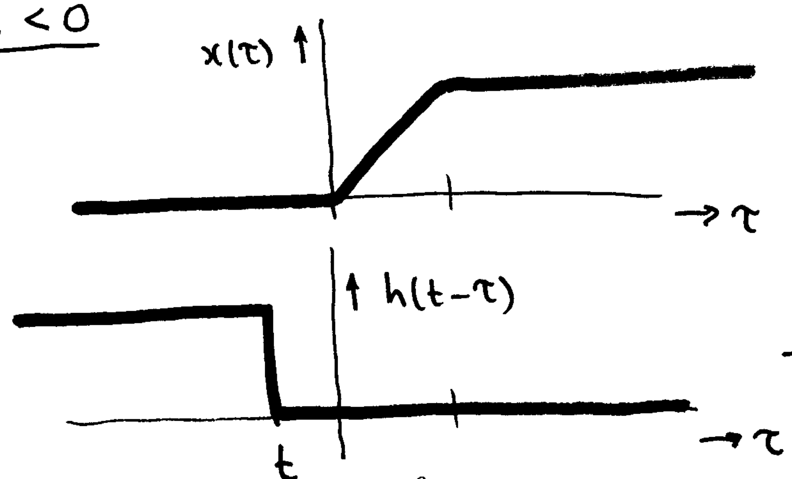
A sketch of $x(t)$ is shown below:



Find the output $y(t)$, and sketch it on the graph provided on the next page.

- **You must show all the work you did in determining** $y(t)$**.** Merely writing down your answer with no justification will likely earn you zero credit.
- Work as neatly as you can.
- Use the back of this page, if you need extra space.

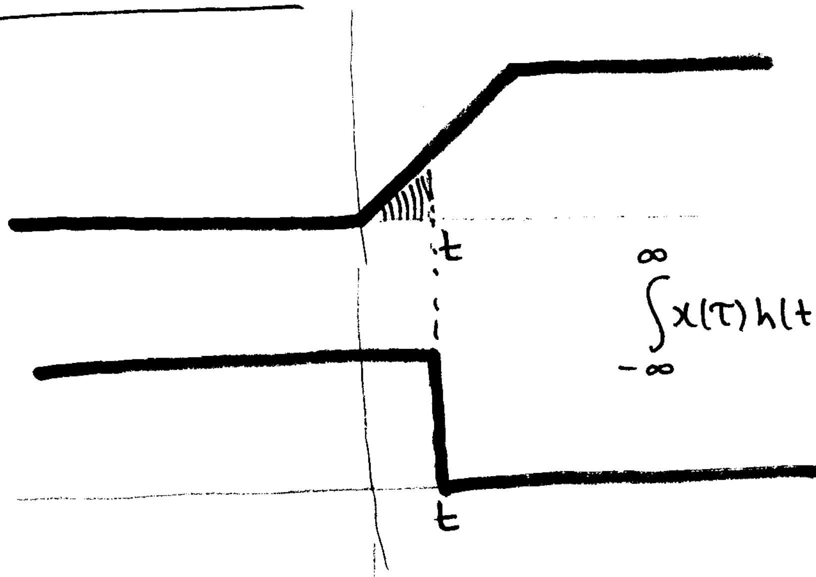
When $t < 0$



$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = 0$$

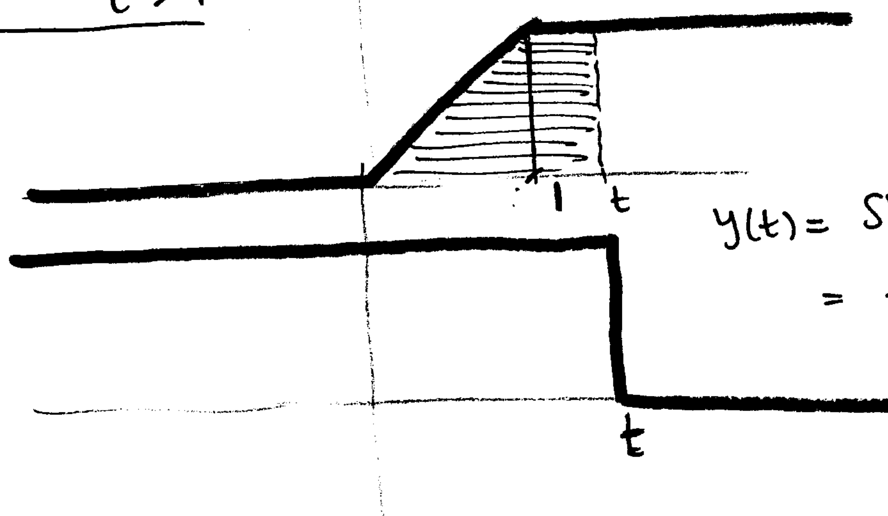
$$\boxed{y(t) = 0}$$

When $t \in [0,1]$



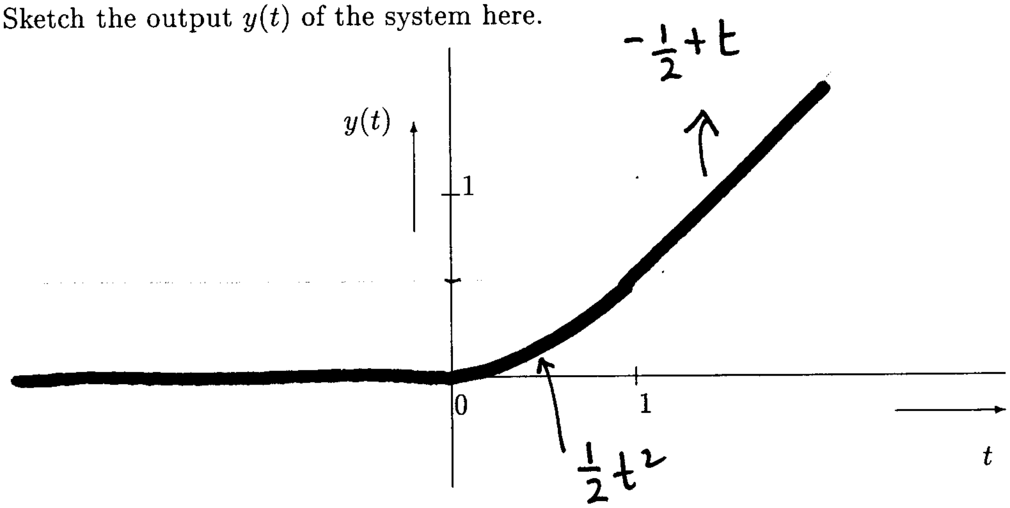
$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \text{Shaded area}$$
$$= \frac{1}{2}t^2$$
$$= y(t)$$

When $t > 1$



$$y(t) = \text{Shaded area}$$
$$= \frac{1}{2} + t - 1$$

Sketch the output $y(t)$ of the system here.



5. (20 points)

For all parts of this problem:

- **You must show all the work you did in obtaining your answers.** Merely writing down your answers with no justification will likely earn you zero credit.
- Work as neatly as you can.

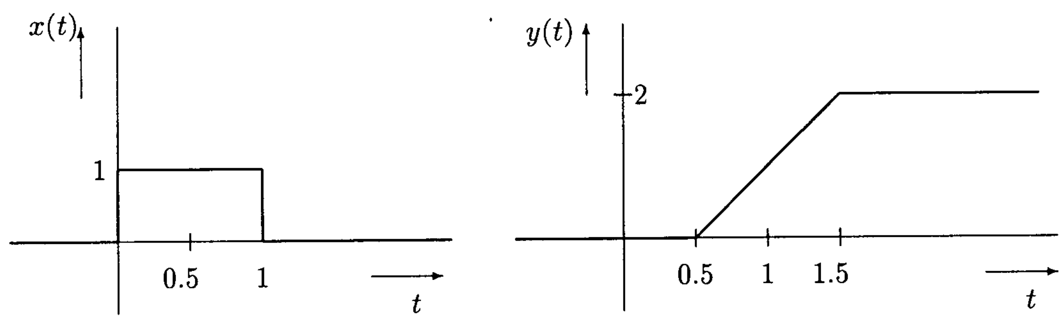
The output $y(t)$ of a causal LTI system corresponding to the input

$$x(t) = \begin{cases} 0 & -\infty < t \le 0, \\ 1 & 0 < t \le 1, \\ 0 & 1 < t < \infty. \end{cases}$$

is

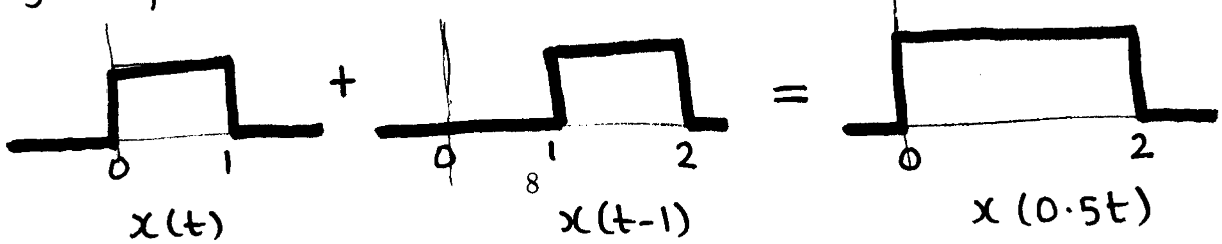
$$y(t) = \begin{cases} 0 & -\infty < t \le 0.5, \\ 2(t-0.5) & 0.5 < t \le 1.5, \\ 2 & 1.5 < t < \infty. \end{cases}$$

A sketch of the input $x(t)$ and the output $y(t)$ are shown below:



(a) (5 pts) Find the output of the system corresponding to the input $x(0.5t)$, and sketch it on the graph provided on the next page.

By inspection $x(0.5t) = x(t) + x(t-1)$



So $y(0.5t) = y(t) + y(t-1)$ (from linearity)

Another method is to solve part (b) first, i.e., find the impulse response. This turns out to be $2u(t-0.5)$, so $x(0.5t) * 2u(t-0.5) = y(0.5t)$, which you can verify is the same as before.

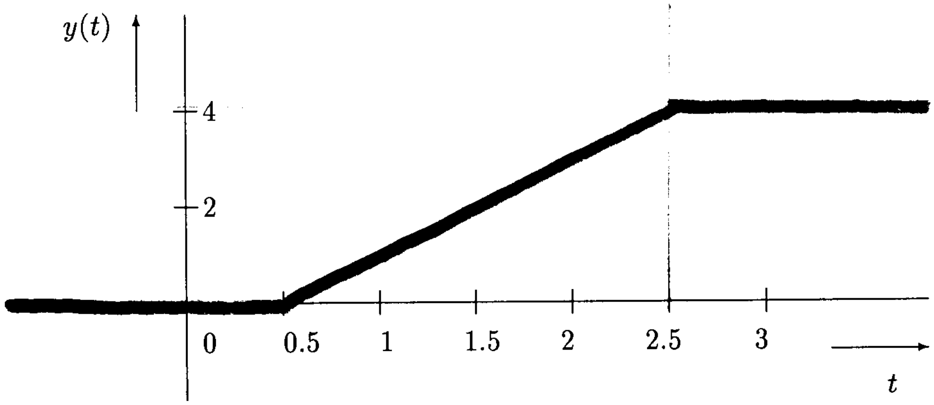
By the way, it is NOT TRUE that for any linear system

$$x(t) \mapsto y(t)$$

implies $x(0.5t) \mapsto y(0.5t)$

(Can you think of conditions under which this is true though?)

Sketch the output of the system corresponding to the input $x(0.5t)$ here.



(b) (15 pts) Find the impulse response of the system, and sketch it on the graph below.

<u>Method I</u>. $x(t) = u(t) - u(t-1)$, for which the output is

$$y(t) = 2\Big[(t-0.5)u(t-0.5) - (t-1.5)u(t-1.5)\Big]$$

But with $s(t)$ denoting the step response, we must have $s(t) - s(t-1) = y(t)$, so that

$$s(t) - s(t-1) = 2\big[(t-0.5)u(t-0.5) - (t-1.5)u(t-1.5)\big]$$

So differentiating both sides w.r.t. t,

$$h(t) - h(t-1) = 2\big[u(t-0.5) - u(t-1.5)\big],$$

from which we get $h(t) = 2u(t-0.5)$

<u>Method II</u>. $x(t) \mapsto y(t)$ means $\frac{dx}{dt} \mapsto \frac{dy}{dt}$

Now, $\frac{dx}{dt} = \delta(t) - \delta(t-1)$, so that $\frac{dy}{dt} = h(t) - h(t-1)$ (rest is similar to Method I)