EE 301 Midterm Exam #1

October 1, Fall 2003

Name: Key

Instructions:

- Follow all instructions carefully!
- This is a 60 minute exam containing **4** problems totaling 120 points.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.

Good Luck.

Name: ______________________________

Problem 1.(30pt) *LTI Systems*

a) Consider an LTI system with impulse response $h(t) = e^{-at}u(t)$ and input $x(t) = u(t)$. Compute the output $y(t)$.

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} u(t-\tau)\, e^{-a\tau} u(\tau)\, d\tau$$

$$= \left(\int_{0}^{t} e^{-a\tau} d\tau\right) u(t)$$

$$= \frac{1}{-a} e^{-a\tau}\Big|_0^t u(t)$$

$$= \frac{1}{-a}\left(e^{-at} - 1\right) u(t)$$

$$= \left(1 - e^{-at}\right) u(t)$$

b) Negate the following statement

$$\forall x,\ Px \Rightarrow Qx$$

$\exists x$ s.t. P_x & $(\neg Q_x)$

b) Consider the following definition: "A signal $x(t)$ is **bounded** if there exists an $M > 0$ such that for all time t, $|x(t)| < M$."

Give the definition of an **unbounded** signal.

A signal $x(t)$ is unbounded if for all $M > 0$ there exists a t such that $|x(t)| \geq M$.

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Problem 2.(30pt) *LTI System Properties*

Consider the following LTI system

$$y(t) = \int_0^\infty g(\tau)x(t-\tau)d\tau$$

a) Prove that the system is linear.

b) Compute the system's impulse response.

c) Is the system, memoryless? Why?

d) Is the system, causal? Why?

a) Let $y_1(t) \triangleq S[x_1(t)]$

$y_2(t) \triangleq S[x_2(t)]$

Then $\forall \alpha, \beta, x_1(t)$ and $x_2(t)$

$S[\alpha x_1(t) + \beta x_2(t)]$

$$= \int_0^\infty g(\tau)(\alpha x_1(t-\tau) + \beta x_2(t-\tau))d\tau$$

$$= \alpha \int_0^\infty g(\tau)x_1(t-\tau)d\tau + \beta \int_0^\infty g(\tau)x_2(t-\tau)d\tau$$

$$= \alpha S[x_1(t)] + \beta S[x_2(t)]$$

Q.E.D.

b) $$h(t) = S[\delta(t)] = \int_0^\infty g(\tau)\,\delta(t-\tau)d\tau$$

$$= \begin{cases} g(t) & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$$= g(t)\,u(t)$$

c) No, the system has memory
because $g(t) > 0$ for $t > 0$

d) Yes, because $h(t) = 0$ for all $t < 0$

Problem 3.(30pt) *Sinusoidal Inputs to LTI Systems*
Consider the discrete-time LTI system

$$y_n = h_n * x_n$$

where h_n is the real-valued impulse response of the system.
a) Show that if $x_n = e^{j\omega n}$ then the output has the form $y_n = Ce^{j\omega n}$ were C is a complex number.
b) Calculate an expression for C.

a)
$$
\begin{aligned}
y_n &= h_n * x_n \\
&= x_n * h_n \\
&= \sum_{k=-\infty}^{\infty} x_{n-k} h_k \\
&= \sum_{k=-\infty}^{\infty} e^{j\omega(n-k)} h_k \\
&= \sum_{k=-\infty}^{\infty} e^{j\omega n} e^{-j\omega k} h_k \\
&= e^{j\omega n} \sum_{k=-\infty}^{\infty} h_k e^{-j\omega k} \\
&= C e^{j\omega n}
\end{aligned}
$$

b)
$$C = \sum_{k=-\infty}^{\infty} h_k e^{j\omega k}$$

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Problem 4.(30pt) *Digital to Analog Conversion*

Consider a linear system

$$y(t) = \mathcal{S}[x_n]$$

with discrete-time input x_n and continuous time output $y(t)$.

Assume there exists a period T such that if $y(t) = \mathcal{S}[x_n]$ then $y(t - kT) = \mathcal{S}[x_{n-k}]$ for all integers k. Notice that this property is similar to time invariance.

Further define the function

$$h(t) \triangleq \mathcal{S}[\delta_n]$$

as the response of the system to a discrete-time impulse.

a) Determine the response of the system to $x_n = \delta_{n-k}$, a DT delta function at time k. Justify (prove) your answer.

b) Determine the response of the system to any DT input x_n. Justify (prove) your answer.

c) Explain in words why this system might be useful.

a) $S[\delta_{n-k}] = h(t-kT)$

by assumed property

b) $x_n = \sum_{k=-\infty}^{\infty} x_k \, \delta(n-k)$

$S[x_n] = S\left[\sum_{k=-\infty}^{\infty} x_k \delta(n-k)\right]$

by linearity $= \sum_{k=-\infty}^{\infty} x_k \, S[\delta(n-k)]$

$y(t) = \sum_{k=-\infty}^{\infty} x_k \, h(t-kT)$

c) This system can be used to convert DT signals to CT signals. It is a D to A converter.