EE 301 Midterm Exam #1

October 1, Fall 2003

**Name:** _____

**Instructions:**

- Follow all instructions carefully!
- This is a 60 minute exam containing **4** problems totaling 120 points.
- You may **only** use your brain and a pencil (or pen) to complete this exam. You **may not** use your book, notes or a calculator.

# Good Luck.

**Name:** _____

**Problem 1.**(30pt)

**a) *LTI Systems***

Consider an LTI system with impulse response $h(t) = e^{-at}u(t)$ and input $x(t) = u(t)$. Compute the output $y(t)$.

**b) *Formal Logic***

Negate the logical statement $\forall x,\ Px \Rightarrow Qx$

**c) *Logical Statements***

Consider the following definition: "A signal $x(t)$ is **bounded** if there exists an $M > 0$ such that for all $t$, $|x(t)| < M$."

Give the definition of an **unbounded** signal.

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**Problem 2.**(30pt) *LTI System Properties*

Consider the following LTI system

$$y(t) = \int_0^{\infty} g(\tau)x(t-\tau)d\tau$$

where $g(t) > 0$ for all $t$.

**a)** Prove that the system is linear.

**b)** Compute the system's impulse response.

**c)** Is the system, memoryless? Why?

**d)** Is the system, causal? Why?

**Problem 3.**(30pt) *Sinusoidal Inputs to LTI Systems*
Consider the discrete-time LTI system

$$y_n = h_n * x_n$$

where $h_n$ is the real-valued impulse response of the system.
**a)** Show that if $x_n = e^{j\omega n}$ then the output has the form $y_n = Ce^{j\omega n}$ were $C$ is a complex number.
**b)** Calculate an expression for $C$ in terms of the function $h_n$.

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**Problem 4.**(30pt) *Digital to Analog Conversion*

Consider a linear system

$$y(t) = \mathcal{S}[x_n]$$

with discrete-time input $x_n$ and continuous time output $y(t)$.

Assume there exists a period $T$ such that if $y(t) = \mathcal{S}[x_n]$ then $y(t - kT) = \mathcal{S}[x_{n-k}]$ for all integers $k$. Notice that this property is similar to time invariance.

Further define the function

$$h(t) \stackrel{\triangle}{=} \mathcal{S}[\delta_n]$$

as the response of the system to a discrete-time impulse.

**a)** Determine the response of the system to $x_n = \delta_{n-k}$, a DT delta function at time $k$. Justify (prove) your answer.

**b)** Determine the response of the system to any DT input $x_n$. Justify (prove) your answer.

**c)** Explain in words why this system might be useful.