Name: _________________________________

Instructions:

• Follow all instructions carefully!

• This is a 60 minute exam containing 4 problems totaling 120 points.

• You may only use your brain and a pencil (or pen) to complete this exam. You may not use your book, notes or a calculator.

Good Luck.
Problem 1. (30pt)

a) **LTI Systems**
Consider an LTI system with impulse response $h(t) = e^{-at}u(t)$ and input $x(t) = u(t)$. Compute the output $y(t)$.

b) **Formal Logic**
Negate the logical statement $\forall x, P x \Rightarrow Q x$

c) **Logical Statements**
Consider the following definition: “A signal $x(t)$ is **bounded** if there exists an $M > 0$ such that for all $t$, $|x(t)| < M$.”
Give the definition of an **unbounded** signal.
Problem 2. (30pt) LTI System Properties

Consider the following LTI system

\[ y(t) = \int_{0}^{\infty} g(\tau)x(t - \tau)d\tau \]

where \( g(t) > 0 \) for all \( t \).

a) Prove that the system is linear.
b) Compute the system’s impulse response.
c) Is the system, memoryless? Why?
d) Is the system, causal? Why?
Problem 3. (30pt) Sinusoidal Inputs to LTI Systems
Consider the discrete-time LTI system

\[ y_n = h_n * x_n \]

where \( h_n \) is the real-valued impulse response of the system.

a) Show that if \( x_n = e^{j\omega n} \) then the output has the form \( y_n = Ce^{j\omega n} \) were \( C \) is a complex number.

b) Calculate an expression for \( C \) in terms of the function \( h_n \).
Problem 4. (30pt) Digital to Analog Conversion

Consider a linear system

\[ y(t) = S[x_n] \]

with discrete-time input \( x_n \) and continuous time output \( y(t) \).

Assume there exists a period \( T \) such that if \( y(t) = S[x_n] \) then \( y(t - kT) = S[x_{n-k}] \) for all integers \( k \). Notice that this property is similar to time invariance.

Further define the function

\[ h(t) \triangleq S[\delta_n] \]

as the response of the system to a discrete-time impulse.

a) Determine the response of the system to \( x_n = \delta_{n-k} \), a DT delta function at time \( k \). Justify (prove) your answer.

b) Determine the response of the system to any DT input \( x_n \). Justify (prove) your answer.

c) Explain in words why this system might be useful.