System Examples

1. \( y(t) = x(at) \)

\[ x(t) \rightarrow S \rightarrow x(at) \]

Linear? Yes

\[ a_1 x_1(t) + a_2 x_2(t) \rightarrow S' \rightarrow a_1 x_1(at) + a_2 x_2(at) \]

where:

\[ y(t) = a_1 y_1(t) + a_2 y_2(t) \]

\[ x_i(t) \rightarrow S' \rightarrow y_i(t) = x_i(at) \]

\( i = 1, 2 \)
$T$ I? No, not time invariant

\[ x(t) \rightarrow \mathcal{S} \rightarrow y(t) = x(at) \]

\[ x(t-t_0) \rightarrow \mathcal{S} \rightarrow z(t) = x(at-t_0) \]

Is \( z(t) = y(t-t_0) \)? No

\[ x(at-t_0) \neq x(a(t-t_0)) = x(at-at_0) \]

See Example 1.16 in text on pg. 52

System is stable.

System is not causal.

Same for DT system

\[ y[n] = x[n] \]
Example 2. DT System

\[ y[n] = g[n] x[n] \]

Linear? Yes, due to distributive property of multiplication

\[ a_1 x_1[n] + a_2 x_2[n] \rightarrow \rightarrow y[n] = g[n] \{ a_1 x_1[n] + a_2 x_2[n] \} \]
\[ = a_1 g[n] x_1[n] + a_2 g[n] x_2[n] \]
\[ = a_1 y_1[n] + a_2 y_2[n] \]
TI? No, system is not Time-Invariant

\[ x[n-n_0] \rightarrow \int \rightarrow z[n] = g[n] \times [n-n_0] \]

Is \( z[n] \equiv y[n-n_0] \)?

\[ g[n] \times [n-n_0] \neq g[n-n_0] \times [n-n_0] \]

System is not TI

System is BIBO Stable if \( |g[n]| < \infty \) for all n

System is causal (it is memoryless)

Examples: \( g[n] = \cos(\omega_0 n) \)

\( g[n] = n \)  

See HwK 2
Comments:

- Similar observations hold with respect to the CT System \( y(t) = g(t) \times x(t) \).
- See Eg. 1.17 pg. 59 where \( g(t) = t \).
- Even though this system is not both linear and time-invariant (LTI), it is still a very useful system in practice.
- The only ramification of not being LTI is that the output is not related to the input through convolution with the impulse response of the system but still useful.
Example 3. Square-law System

\[ x(t) \rightarrow S \rightarrow y(t) = x^2(t) \]

Linear? No. Superposition does not apply.

\[ a_1 x_1(t) + a_2 x_2(t) \rightarrow S \rightarrow y(t) = \]

\[ \left( a_1 x_1(t) + a_2 x_2(t) \right)^2 = \]

\[ = a_1^2 x_1^2(t) + a_1 a_2 x_1(t)x_2(t) + a_2^2 x_2^2(t) \]

违反了线性

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\[ \neq a_1 x_1^2(t) + a_2 x_2^2(t) \]
- See Example 1.18 in text on pg. 54

- System is TI: \( x(t-t_0) \rightarrow \int \rightarrow x^2(t-t_0) \)

- Stable and causal too

- Similar observations hold wrt DT system

  \[ y[n] = x^2[n] \]

- Again, even though not LTI, this system is used often in practice, as we will see later in this course

- See handout posted at course website for further examples of systems
Example: Hard-limiter

\[ y(t) = \begin{cases} 
V_{\text{max}}, & \text{if } x(t) > V_{\text{max}} \\
 x(t), & \text{if } -V_{\text{max}} < x(t) < V_{\text{max}} \\
-V_{\text{max}}, & \text{if } x(t) < -V_{\text{max}} 
\end{cases} \]

- Used to remove noisy spikes in radios (FM)
- Used, after amplitude scaling, to fit signal into range of quantizer in process of A/D conversion

- System is TI but not linear
- You can show nonlinearity with a single counter-example \( x_1(t) = A_1 u(t) \) and \( x_2(t) = A_2 u(t) \), where \( A_1 < V_{\text{max}} \) and \( A_2 < V_{\text{max}} \) but \( A_1 + A_2 > V_{\text{max}} \)
\( \chi_1(t) \rightarrow S \rightarrow y_1(t) = A_1 u(t) \)

\( \chi_2(t) \rightarrow S \rightarrow y_2(t) = A_2 u(t) \)

\( \chi_1(t) + \chi_2(t) \rightarrow S \rightarrow y(t) = \gamma_{\text{max}} u(t) \)

\( \equiv y_1(t) + y_2(t) \)

\( = (A_1 + A_2) u(t) \)
Example. Integrator

\[ y(t) = \int_{t-T_b}^{t+T_a} x(\tau) \, d\tau = \begin{cases} \text{area under} & x(t) \\ \text{from} & t-T_b \\ \text{to} & t+T_a \end{cases} \]

\[ T_a > 0, \ T_b > 0. \text{ note: } T_b = \infty \text{ is valid} \]

System is LT1.

Linearity:

\[ y(t) = \int_{t-T_b}^{t+T_a} \left( a_1 x_1(\tau) + a_2 x_2(\tau) \right) \, d\tau \]

\[ = a_1 \int_{t-T_b}^{t+T_a} x_1(\tau) \, d\tau + a_2 \int_{t-T_b}^{t+T_a} x_2(\tau) \, d\tau \]

\[ = a_1 y_1(t) + a_2 y_2(t) \]
TI: input $x(t-t_0)$

$z(t) = \int_{t-T_b}^{t+T_a} x(\tau-t_0) \, d\tau$

Change of variables: $\lambda = \tau - t_0$

$d\lambda = d\tau$, $\tau = \lambda + t_0$

New limits:

$\lambda \int_{t+T_a-t_0}^{t+T_a-t_0}$

Thus:

$z(t) = \int_{t-t_0-T_b}^{t-t_0+T_a} x(\lambda) \, d\lambda = y(t-t_0)$

System is LTI