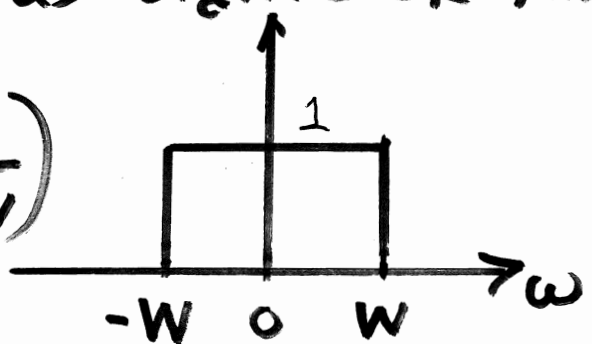


# Sine Functions

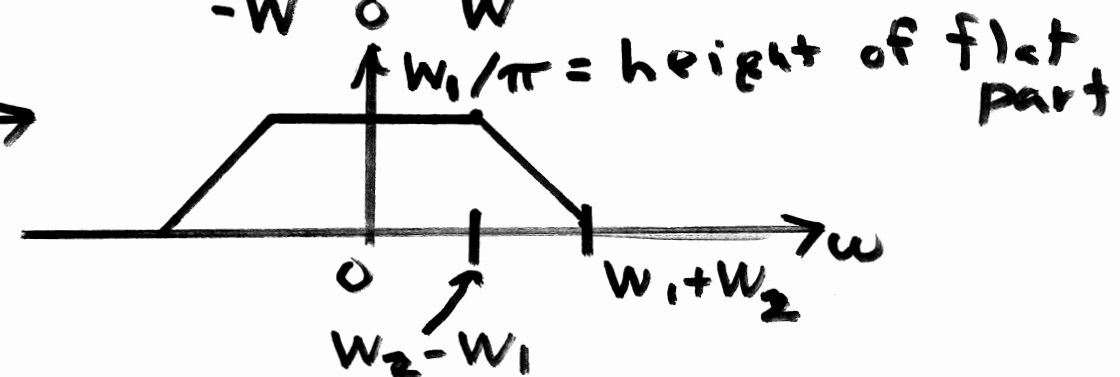
Used as signals OR impulse responders on Exams

$$\frac{\sin(Wt)}{\pi t} \xleftrightarrow{+} \text{rect}\left(\frac{\omega}{2W}\right)$$



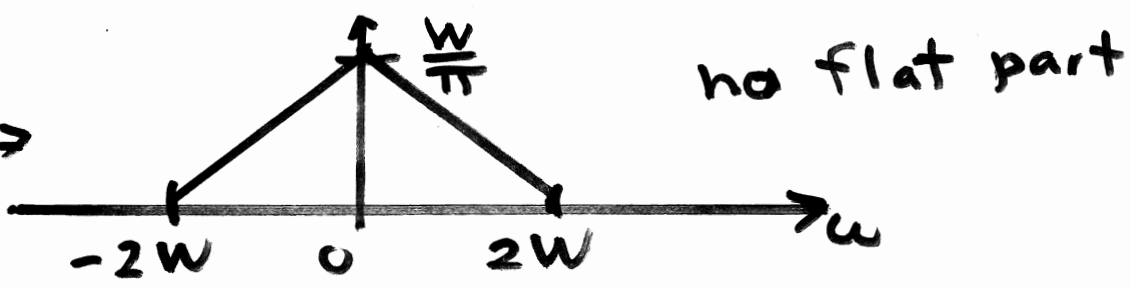
$$\frac{\sin(W_1 t)}{\pi t} \frac{\sin(W_2 t)}{\pi t} \xleftrightarrow{+}$$

$W_2 \geq W_1$



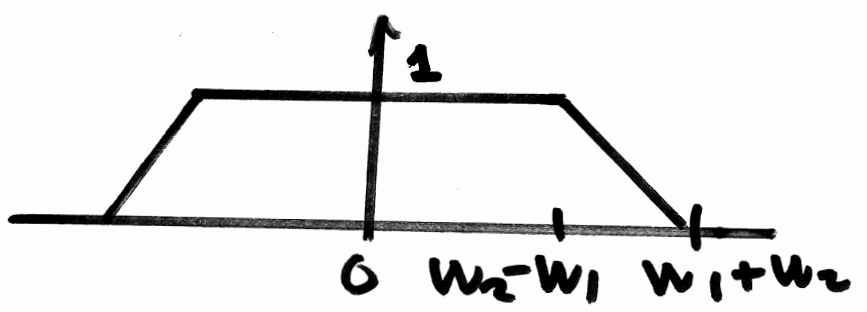
Special Case:  $W_2 = W_1$ :

$$\left\{ \frac{\sin(Wt)}{\pi t} \right\}^2 \xleftrightarrow{+}$$

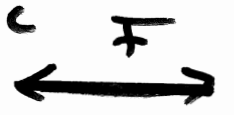


Simple Linearity implies

$$\frac{\pi}{W_1} \frac{\sin(W_1 t)}{\pi t} \frac{\sin(W_2 t)}{\pi t} \xleftrightarrow{+}$$



Real and even-symmetric in time-domain



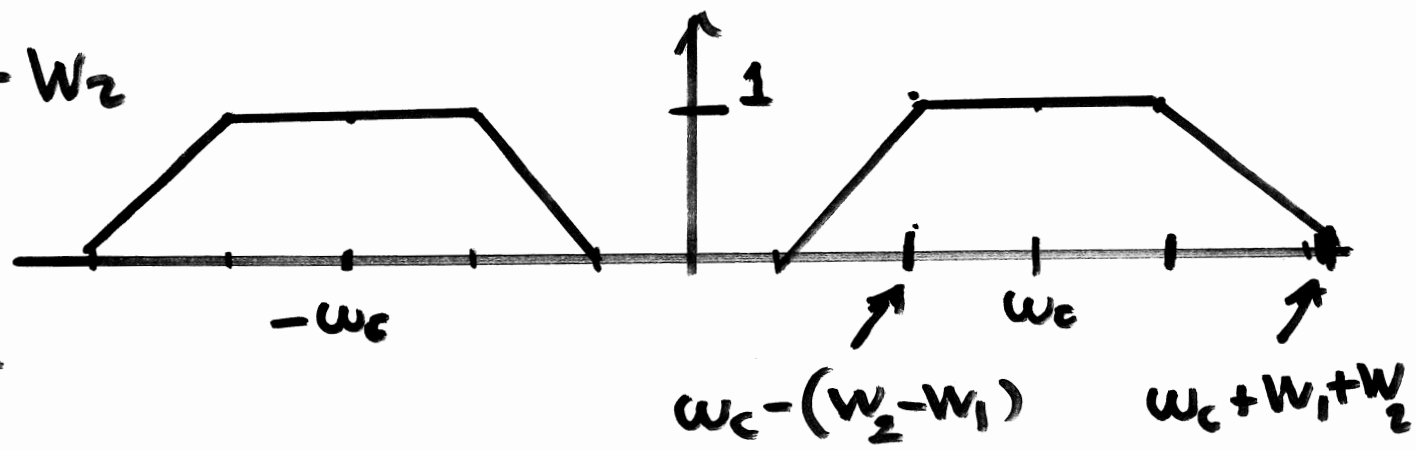
Real and even-symmetric in frequency domain

$$\frac{\pi \sin(\omega_1 t)}{\omega_1 \pi t} \frac{\sin(\omega_2 t)}{\pi t} \stackrel{+}{\longleftrightarrow} 2 \cos(\omega_c t)$$

$$\omega_2 \geq \omega_1$$

$$\omega_c \geq \omega_1 + \omega_2$$

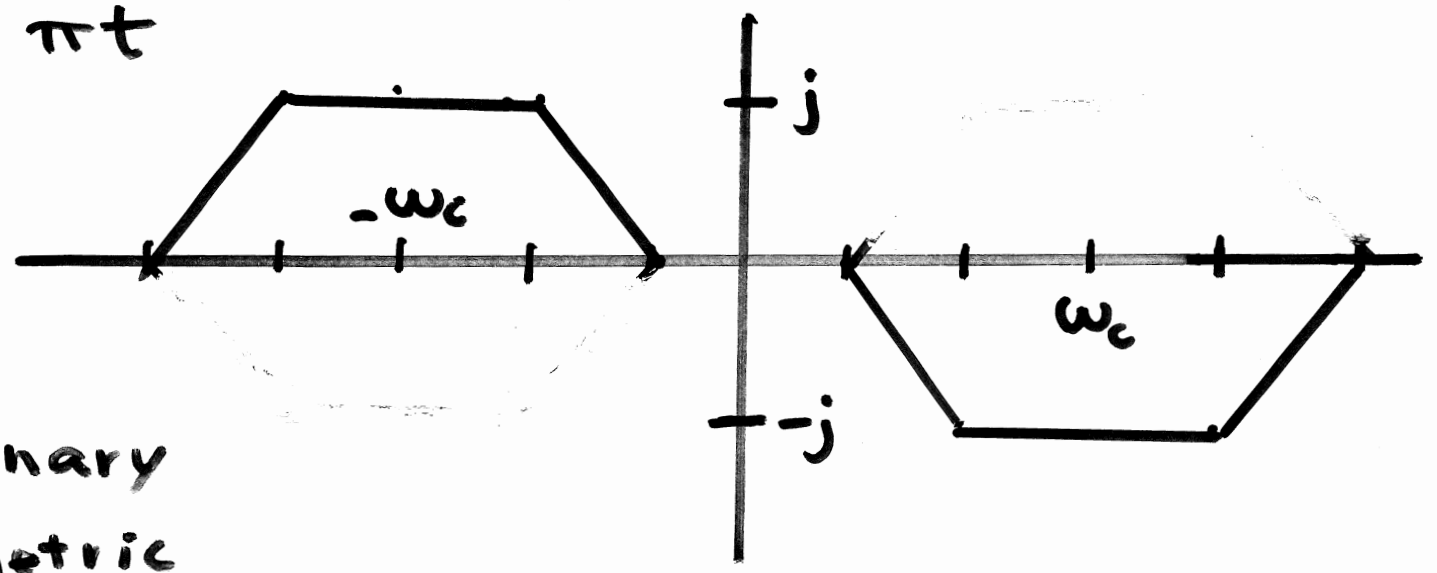
Still real-valued  
and even-symmetric



$$\frac{\pi \sin(\omega_1 t)}{\omega_1 \pi t} \frac{\sin(\omega_2 t)}{\pi t} \stackrel{+}{\longleftrightarrow} 2 \sin(\omega_c t)$$

$$\omega_c \geq \omega_1 + \omega_2$$

Now,  
purely imaginary  
and odd-symmetric



$$\frac{\sin(\omega(t-t_0))}{\pi(t-t_0)} + \frac{\sin(\omega(t+t_0))}{\pi(t+t_0)}$$

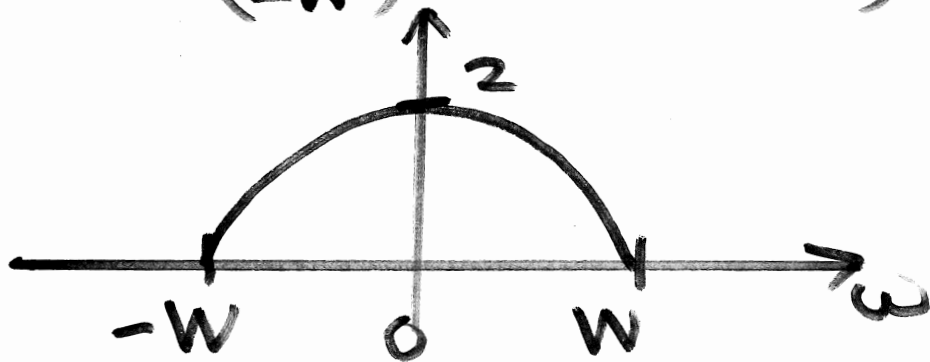
$$\xleftrightarrow{\mathcal{F}} \text{rect}\left(\frac{\omega}{2W}\right) \left\{ e^{-j\omega t_0} + e^{j\omega t_0} \right\}$$

$$= \text{rect}\left(\frac{\omega}{2W}\right) 2 \cos(\omega t_0)$$

$$\underline{\text{If } t_0 = \frac{\pi}{2W} \Rightarrow \text{rect}\left(\frac{\omega}{2W}\right) 2 \cos\left(\frac{\pi \omega}{2W}\right)}$$

real-valued  
and

even-symmetric



$$\frac{\sin(\omega(t-t_0))}{\pi(t-t_0)} - \frac{\sin(\omega(t+t_0))}{\pi(t+t_0)}$$

$$\xleftrightarrow{\mathcal{F}} \text{rect}\left(\frac{\omega}{2W}\right) \left\{ e^{-j\omega t_0} - e^{j\omega t_0} \right\} \frac{2j}{2j}$$

$$= \text{rect}\left(\frac{\omega}{2W}\right) 2j \sin(\omega t_0) (-1)$$

$$\text{If } t_0 = \frac{\pi}{\omega} \Rightarrow -\text{rect}\left(\frac{\omega}{2W}\right) 2j \sin\left(\frac{\pi}{W}\right)$$

purely imaginary  
and  
odd-symmetric

