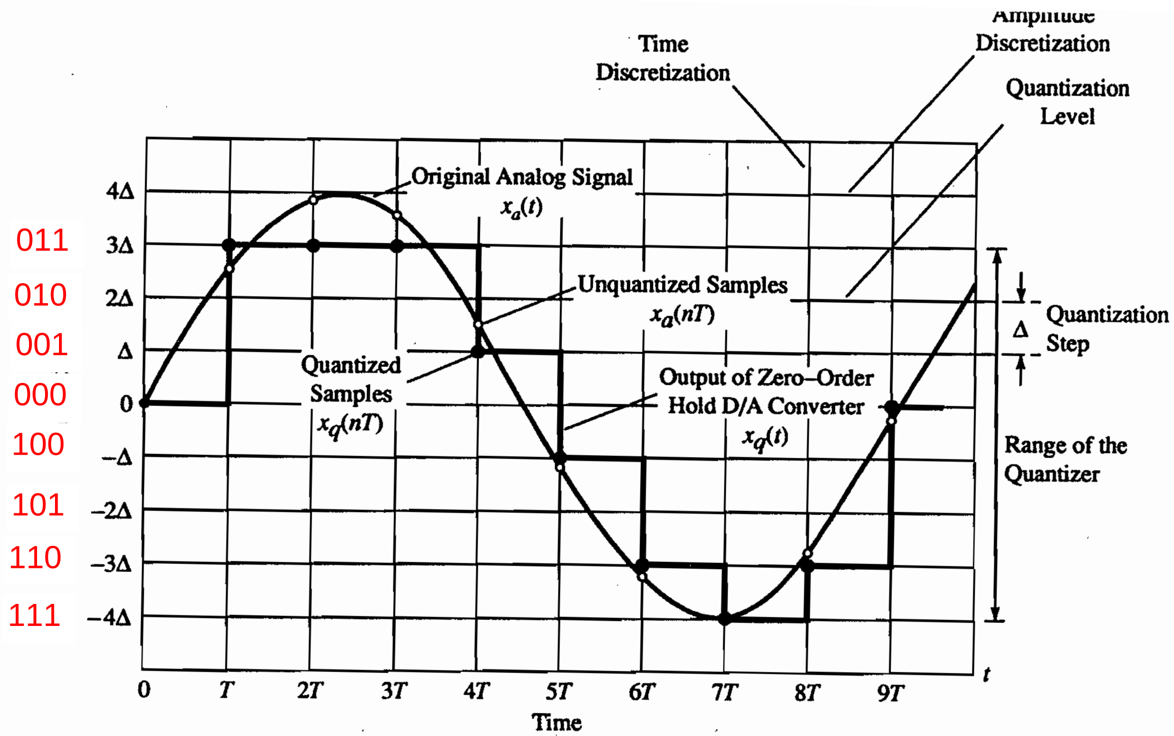

Figure 1.4.8 Sampling and quantization of a sinusoidal signal.

1.4.4 Quantization of Sinusoidal Signals

Figure 1.4.8 illustrates the sampling and quantization of an analog sinusoidal signal $x_a(t) = A\cos\Omega_0 t$ using a rectangular grid. Horizontal lines within the range of the quantizer indicate the allowed levels of quantization. Vertical lines indicate the sampling times. Thus, from the original analog signal $x_a(t)$ we obtain a discrete-time signal $x(n) = x_a(nT)$ by sampling and a discrete-time, discrete-amplitude signal $x_q(nT)$ after quantization. In practice, the staircase signal $x_q(t)$ can be obtained by using a zero-order hold. This analysis is useful because sinusoids are used as test signals in A/D converters.

If the sampling rate F_s satisfies the sampling theorem, quantization is the only error in the A/D conversion process.

Thus we can evaluate the quantization error by quantizing the analog signal $x_a(t)$ instead of the discrete-time signal $x(n) = x_a(nT)$. Inspection of Fig. 1.4.8 indicates that the signal $x_a(t)$ is almost linear between quantization levels (see Fig. 1.4.9). The

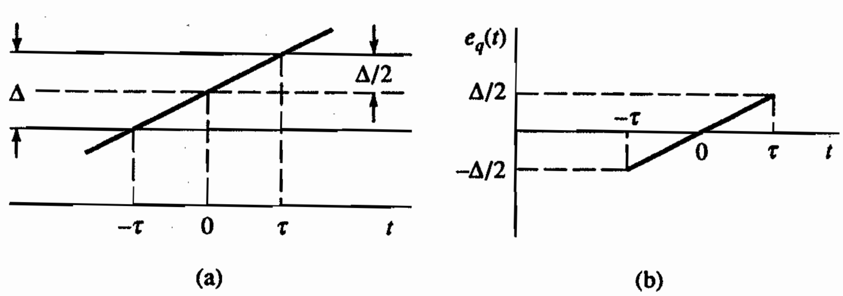

Figure 1.4.9 The quantization error $e_q(t) = x_a(t) - x_q(t)$.

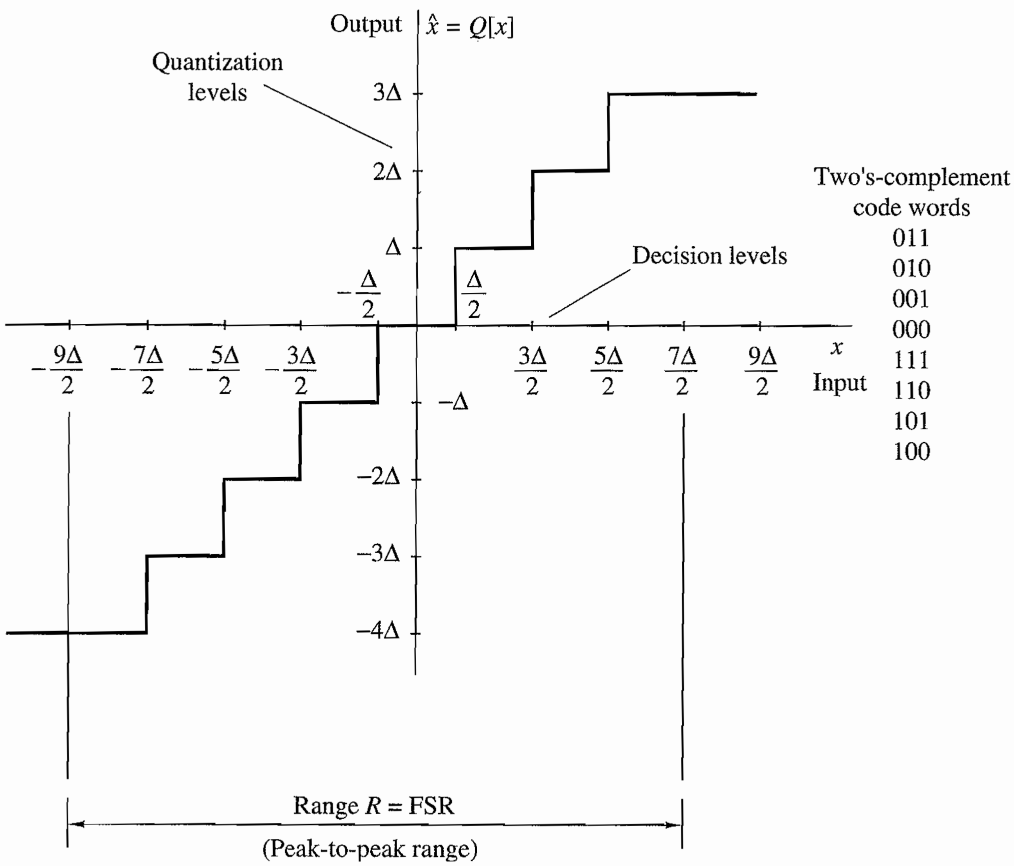

Figure 6.3.3 Example of a midtread quantizer.

There are various binary coding schemes, each with its advantages and disadvantages. Table 6.1 illustrates some existing schemes for 3-bit binary coding. These number representation schemes are described in more detail in Section 9.4.

The two's-complement representation is used in most digital signal processors. Thus it is convenient to use the same system to represent digital signals because we can operate on them directly without any extra format conversion. In general, a $(b+1)$-bit binary fraction of the form $\beta_0\beta_1\beta_2 \cdots \beta_b$ has the value

$$-\beta_0 \cdot 2^0 + \beta_1 \cdot 2^{-1} + \beta_2 \cdot 2^{-2} + \cdots + \beta_b \cdot 2^{-b}$$

if we use the two's-complement representation. Note that β_0 is the most significant bit (MSB) and β_b is the least significant bit (LSB). Although the binary code used to represent the quantization levels is important for the design of the A/D converter and the subsequent numerical computations, it does not have any effect in the performance of the quantization process. Thus in our subsequent discussions we ignore the process of coding when we analyze the performance of A/D converters.

The only degradation introduced by an ideal converter is the quantization error, which can be reduced by increasing the number of bits. This error, which dominates the performance of practical A/D converters, is analyzed in the next section.

Practical A/D converters differ from ideal converters in several ways. Various degradations are usually encountered in practice. Specifically, practical A/D converters may have an *offset* error (the first transition may not occur at exactly $+\frac{1}{2}$LSB),

TABLE 6.1 Commonly Used Bipolar Codes

Number	Decimal Fraction: Positive Reference	Decimal Fraction: Negative Reference	Sign + Magnitude	Two's Complement	Offset Binary	One's Complement
+7	$+\frac{7}{8}$	$-\frac{7}{8}$	0111	0111	1111	0111
+6	$+\frac{6}{8}$	$-\frac{6}{8}$	0110	0110	1110	0110
+5	$+\frac{5}{8}$	$-\frac{5}{8}$	0101	0101	1101	0101
+4	$+\frac{4}{8}$	$-\frac{4}{8}$	0100	0100	1100	0100
+3	$+\frac{3}{8}$	$-\frac{3}{8}$	0011	0011	1011	0011
+2	$+\frac{2}{8}$	$-\frac{2}{8}$	0010	0010	1010	0010
+1	$+\frac{1}{8}$	$-\frac{1}{8}$	0001	0001	1001	0001
0	0+	0−	0000	0000	1000	0000
0	0−	0+	1000	(0000)	(1000)	1111
−1	$-\frac{1}{8}$	$+\frac{1}{8}$	1001	1111	0111	1110
−2	$-\frac{2}{8}$	$+\frac{2}{8}$	1010	1110	0110	1101
−3	$-\frac{3}{8}$	$+\frac{3}{8}$	1011	1101	0101	1100
−4	$-\frac{4}{8}$	$+\frac{4}{8}$	1100	1100	0100	1011
−5	$-\frac{5}{8}$	$+\frac{5}{8}$	1101	1011	0011	1010
−6	$-\frac{6}{8}$	$+\frac{6}{8}$	1110	1010	0010	1001
−7	$-\frac{7}{8}$	$+\frac{7}{8}$	1111	1001	0001	1000
−8	$-\frac{8}{8}$	$+\frac{8}{8}$		(1000)	(0000)	

scale-factor (or gain) error (the difference between the values at which the first tr sition and the last transition occur is not equal to FS − 2LSB), and a *linearity* e (the differences between transition values are not all equal or uniformly changi If the *differential linearity* error is large enough, it is possible for one or more c words to be missed. Performance data on commercially available A/D conver are specified in manufacturers' data sheets.

6.3.3 Analysis of Quantization Errors

To determine the effects of quantization on the performance of an A/D conver we adopt a statistical approach. The dependence of the quantization error on characteristics of the input signal and the nonlinear nature of the quantizer mak deterministic analysis intractable, except in very simple cases.

In the statistical approach, we assume that the quantization error is randon nature. We model this error as noise that is added to the original (unquantized) sig If the input analog signal is within the range of the quantizer, the quantization e