Help for Prob. 2.22 (d)

Convolve:

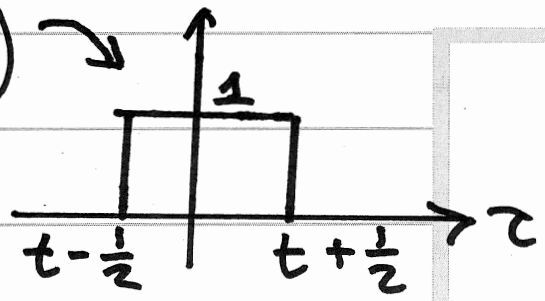
$$x(t) = at + b \qquad h(t) = \frac{4}{3}\,\mathrm{rect}\left(t - \tfrac{1}{2}\right) - \frac{1}{3}\,\delta(t-2)$$

First, consider:

$$\tilde{y}(t) = x(t) * \mathrm{rect}(t)$$

$$\mathrm{rect}(t) = u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$

$$\mathrm{rect}(t-\tau) = \mathrm{rect}\left(-(\tau - t)\right)$$



$$\tilde{y}(t) = \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} (a\tau + b)\, d\tau = \text{(on next page)}$$

$$\tilde{y}(t) = a\frac{\tau^2}{2} + b\tau \Big]_{t-\frac{1}{2}}^{t+\frac{1}{2}}$$

$$= \frac{a}{2}\left(t^2 + t + \frac{1}{4}\right) + b\left(t + \frac{1}{2}\right) - \frac{a}{2}\left(t^2 - t + \frac{1}{4}\right) - b\left(t - \frac{1}{2}\right)$$

$$= at + b$$

Thus: $(at+b) * \text{rect}(t) = at + b$ interesting result

In this problem, we have (invoking Linearity & TI)

$$(at+b) * \left(\frac{4}{3}\text{rect}\left(t-\tfrac{1}{2}\right) - \frac{1}{3}\delta(t-2)\right)$$

$$= \frac{4}{3}\left(a\left(t-\tfrac{1}{2}\right)+b\right) - \frac{1}{3}\left(a(t-2)+b\right)$$

$$= at + b + \left(-\frac{2}{3}a + \frac{2}{3}a\right)$$

$$= at + b \quad \text{(final answer)}$$