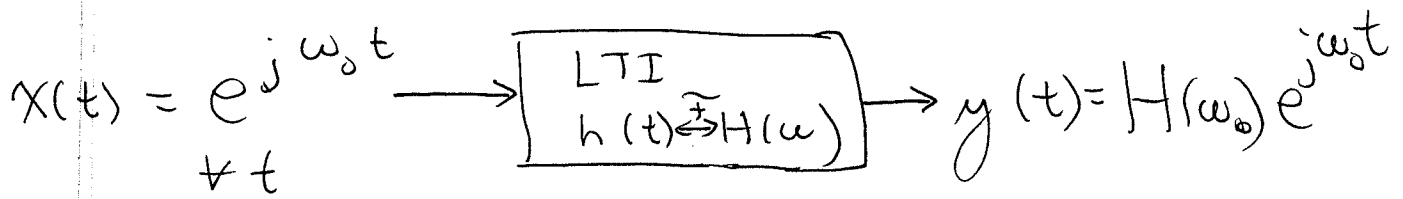


Passing sinewaves thru LTI System (1)



• Recall: this is where the defining ^{integral} equation for the Fourier Transform came from:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) \underbrace{e^{j\omega_0(t-\tau)}}_{x(t-\tau)} d\tau$$
$$= e^{j\omega_0 t} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega_0 \tau} d\tau$$

$$= H(\omega_0) e^{j\omega_0 t}$$

• where: $H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$

• THUS: $e^{j\omega_0 t} * h(t) = H(\omega_0) e^{j\omega_0 t}$

\Rightarrow no reason to do convolution

• In frequency domain \Rightarrow same result:

$$y(t) = e^{j\omega_0 t} * h(t) \xleftrightarrow{\mathcal{F}} Y(\omega) = 2\pi \delta(\omega - \omega_0) H(\omega)$$

$$= H(\omega_0) e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} = H(\omega_0) 2\pi \delta(\omega - \omega_0)$$

\Rightarrow no reason to have to do this either (see next page)
on exam

Using Euler's Identities

(2)

$$\cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$\sin(\omega_0 t) = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

and invoking linearity, can easily show:

$$\cos(\omega_0 t) \rightarrow \boxed{\begin{array}{c} \text{LTI} \\ h(t) \xleftrightarrow{F} H(\omega) \end{array}} \rightarrow |H(\omega_0)| \cos(\omega_0 t + \angle H(\omega_0))$$

$$\sin(\omega_0 t) \rightarrow \boxed{h(t)} \rightarrow |H(\omega_0)| \sin(\omega_0 t + \angle H(\omega_0))$$

$$H(\omega) = |H(\omega_0)| e^{j\angle H(\omega_0)}$$

polar form

Further, invoking linearity:

$$\underbrace{\sum_{k=-\infty}^{\infty} a_k e^{j\omega_k t}}_{\text{sum of sinewaves}} \rightarrow \boxed{h(t)} \rightarrow \sum_{k=-\infty}^{\infty} a_k H(\omega_k) e^{j\omega_k t}$$

• special case: periodic signal where $\omega_k = k \frac{2\pi}{T}$

• FURTHER:

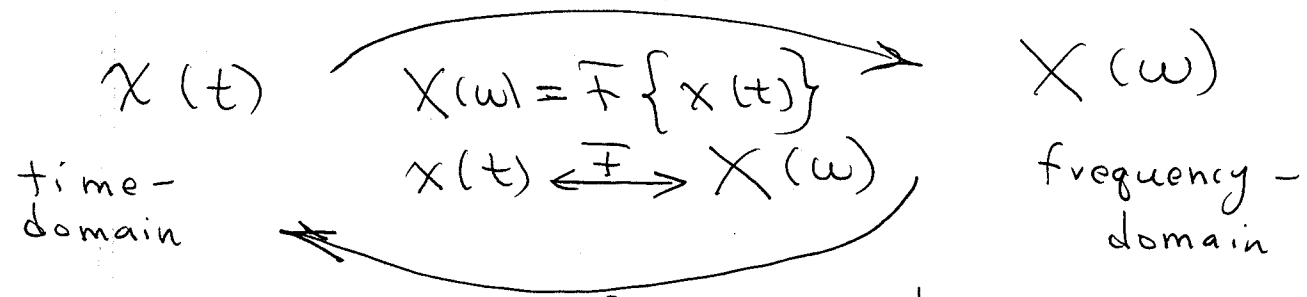
$$\sum_{k=-\infty}^{\infty} A_k \cos(\omega_k t + \phi_k) \rightarrow \boxed{h(t)} \rightarrow \sum_{k=-\infty}^{\infty} A_k |H(\omega_k)| \cos(\omega_k t + \phi_k + \angle H(\omega_k))$$

- Def'n of FT came from passing sinewave thru LTI system => lead to Fourier Transform of Impulse Response

$h(t) \xleftrightarrow{\mathcal{F}} H(\omega)$: characterizes response of LTI system as a function of frequency

- THEN used same definition for signal, but interpret differently:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$



$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \mathcal{F}^{-1}\{X(\omega)\}$$

- $X(\omega)$, or more specifically $|X(\omega)|^2$, characterizes the energy distribution of $x(t)$ as a function of frequency

Recall, Parseval's Theorem:

energy $E_x = \int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

Note: Chap. 5 Discrete Time
Fourier Transform (DTFT) (4)

• $x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] = x[n] * h[n]$
 $= \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

• Similar to CT, the Definition of the DTFT comes from passing DT sine wave thru DT LTI system:

$x[n] = e^{j\omega_0 n} \rightarrow \boxed{h[n]} \rightarrow y[n] = x[n] * h[n]$
 $t = n$
 $= \sum_{k=-\infty}^{\infty} h[k] e^{j\omega_0 (n-k)}$
 $= e^{j\omega_0 n} \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega_0 k}$
 $= H(\omega_0) e^{j\omega_0 n}$

• where:

$h[n] \xleftrightarrow{\text{DTFT}} H(\omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$

• characterizes how DT LTI system responds as a function of frequency BUT remember DT frequencies are only unique over a 2π interval (e.g. $-\pi < \omega < \pi$)

• For DT signals: $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$