

Example. Prob. 2.5

$$y[n] = x[n] * h[n]$$

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{otherwise} \end{cases} = u[n] - u[n-10]$$

$$h[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{otherwise} \end{cases} = u[n] - u[n-(N+1)]$$

Find N given $y[4] = 5$ $y[14] = 0$

From convolution of two DT rectangles result on bottom of pg. 2:

$$\text{length of } y[n] = 10 + (N+1) - 1 = N+10$$

is nonzero from $n=0$ to $n=N+10-1 = N+9$

$$\text{So: } N+9 < 14 \quad \text{so } N < 5$$

So $N_1 = N$ and $N_2 = 9$ at bottom of pg. 2

$$\text{Check } N=4 = N_1, \Rightarrow y[N_1] = N_1 + 1 \Rightarrow y[4] = 5 \quad \checkmark$$

checks

Answer: $N=4$

Prob. 2.21 (a) done in class notes

$$\underbrace{\alpha^n u[n]}_{x[n]} * \underbrace{\beta^n u[n]}_{h[n]} = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{n-k} u[n-k] = \left\{ \sum_{k=0}^{\infty} \left(\frac{\alpha}{\beta} \right)^k u[-(k-n)] \right\} \beta^n$$

$$= \beta^n \sum_{k=0}^n \left(\frac{\alpha}{\beta} \right)^k = \beta^n \left\{ \frac{1 - \left(\frac{\alpha}{\beta} \right)^{n+1}}{1 - \frac{\alpha}{\beta}} \right\}$$

flipped around
k=0 axis and
slid to right
by n

$$= \frac{\beta}{\beta - \alpha} \beta^n u[n] + \frac{\alpha}{\alpha - \beta} \alpha^n u[n] \quad \text{when } \alpha \neq \beta$$

(b) when $\alpha = \beta$

$$y[n] = \alpha^n \sum_{k=0}^n (1)^k = (n+1) \alpha^n u[n]$$

no part (c)
for 11 mult.

(8)

Prob. 2.21 part (d)

$$y[n] = x[n] * h[n]$$

$$x[n] = u[n] - u[n-5] = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

Define:

$$\tilde{h}[n] = u[n] - u[n-6] = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

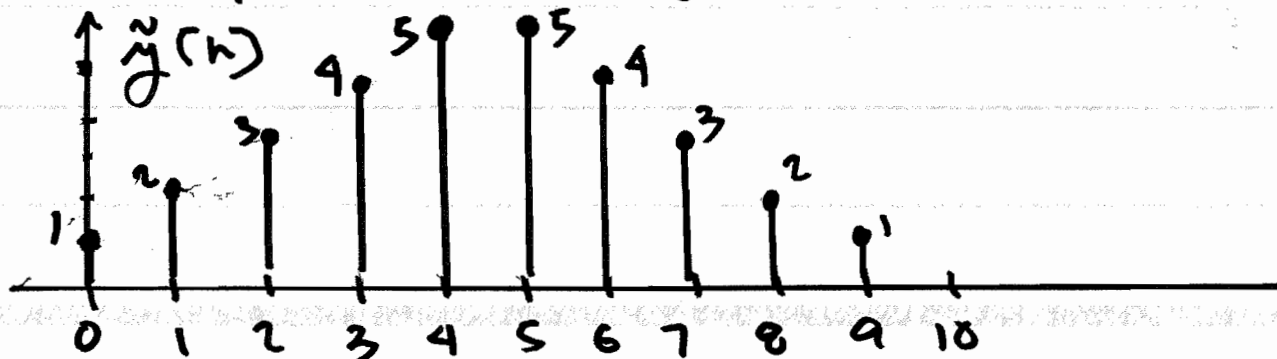
$$\text{Then: } h[n] = \tilde{h}[n-2] + \tilde{h}[n-11]$$

$$\text{It follows: define } \tilde{y}[n] = x[n] * \tilde{h}[n]$$

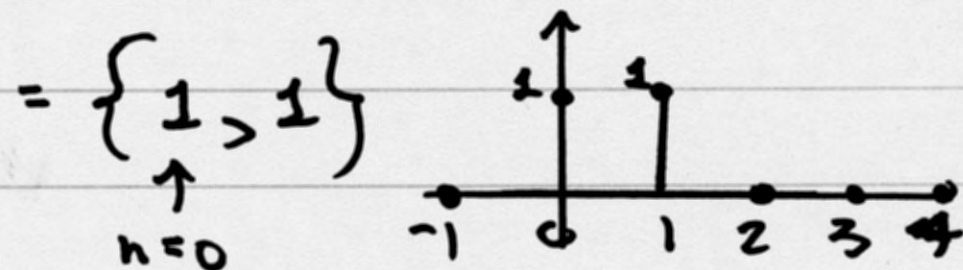
Then:

$$y[n] = \tilde{y}[n-2] + \tilde{y}[n-11]$$

where: $\tilde{y}[n]$ determined from bottom of pg. 2
with $N_1 = 4$ and $N_2 = 5$



Prob. 2.24 $h_2[n] = u[n] - u[n-2]$

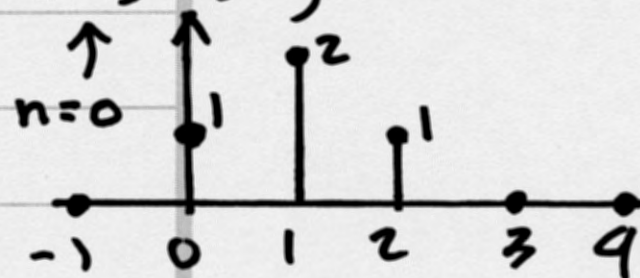


Three LTI systems in series:

$$h_0[n] = h_1[n] * h_2[n] * h_2[n]$$

$$= h_1[n] * \left\{ (u[n] - u[n-2]) * (u[n] - u[n-2]) \right\}$$

$$h_3[n] = \{1, 2, 1\}$$



Recall DT convolution

method discussed in

class notes

$$h_0[n] = h_1[n] * \overbrace{\{1, 2, 1\}}^{h_3[n]}$$

$$= \sum_{k=0} h_1[k] h_3[n-k]$$

First: how "long" is $h_1[n]$? call it N

Given: $h_0[n]$ is of "length" 7

$$= \left\{ \underset{\substack{\uparrow \\ n=0}}{1}, 5, 10, 11, 8, 4, \underset{\substack{\uparrow \\ n=6}}{1} \right\}$$

Thus:

$$N + 3 - 1 = 7 \Rightarrow N = 5$$

which means it's only nonzero over $0 \leq n \leq 4$

n : 0 1 2 3 4 5 6

$h_1[0] \times$	1	2	1	0	0	0	0
$h_1[1] \times$	0	1	2	1	0	0	0
$h_1[2] \times$	0	0	1	2	1	0	0
$h_1[3] \times$	0	0	0	1	2	1	0
$h_1[4] \times$	0	0	0	0	1	2	1
$h_0[n]$	1	5	10	11	8	4	1

$n=0$:

$$h, [0] \cdot 1 = 1 \Rightarrow \boxed{h, [0] = 1}$$

$n=1$:

$$2 h, [0] + 1 \cdot h, [1] = 5 \quad \left| \quad h, [1] = 5 - 2 h, [0] = 3 \right.$$
$$\boxed{h, [1] = 3}$$

$n=2$:

$$1 \cdot h, [0] + 2 \cdot h, [1] + h, [2] = 10$$

$$\Rightarrow h, [2] = 10 - 1 - 2(3) = 3$$

$$\boxed{h, [2] = 3}$$

$n=3$:

$$1 \cdot h, [1] + 2 \cdot h, [2] + 1 \cdot h, [3] = 11$$

$$\Rightarrow h, [3] = 11 - 3 - 2(3) = 2$$

$$\boxed{h, [3] = 2}$$

$n=4$:

$$h, [2] + 2 h, [3] + h, [4] = 8$$

$$h, [4] = 8 - 3 - 2(2) = 1$$

$$\boxed{h, [4] = 1}$$

$$h, [n] = \left\{ \underset{\substack{\uparrow \\ n=0}}{1}, 3, 3, 2, \underset{\substack{\uparrow \\ n=4}}{1} \right\}$$

(6)

Example. Prob. 2.26 $y[n] = x_1[n] * x_2[n] * x_3[n]$

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \quad x_2[n] = u[n+3] \quad x_3[n] = \delta[n] - \delta[n-1]$$

(a) $x_1[n] * x_2[n] = ? = z[n]$

$\alpha = \frac{1}{2} \quad \beta = 1$ Example 2.3 in text

$$r_1 = \frac{\beta}{\beta - \alpha} = \frac{1}{1 - \frac{1}{2}} = 2 \quad r_2 = \frac{\alpha}{\alpha - \beta} = \frac{1/2}{1/2 - 1} = -1$$

$$x_1[n] * x_2[n] = 2 u[n+3] - \left(\frac{1}{2}\right)^{n+3} u[n+3]$$

(b) $z[n] * x_3[n] \Rightarrow$ easiest way

$$z[n] * x_3[n] = z[n] * \{\delta[n] - \delta[n-1]\}$$

$$= z[n] - z[n-1]$$

$$= 2 u[n+3] - \left(\frac{1}{2}\right)^{n+3} u[n+3] - 2 u[n+2] + \left(\frac{1}{2}\right)^{n+2} u[n+2]$$

$$= \delta[n+3] + \left(-\frac{1}{8} + \frac{1}{4}\right) \left(\frac{1}{2}\right)^n u[n+2]$$

$$= \delta[n+3] + \frac{1}{8} \left(\frac{1}{2}\right)^n u[n+2]$$

you do parts
(c) and (d)

Prob. 2.26 (cont.)

Ans to (b) further simplifies as

$$z[n] = \frac{1}{8} \left(\frac{1}{2}\right)^n u[n+3] = \left(\frac{1}{2}\right)^{n+3} u[n+3]$$

Now (c) and (d):

First convolve $x_2[n]$ and $x_3[n]$

$$\begin{aligned} & u[n+3] * (\delta[n] - \delta[n-1]) \\ &= u[n+3] * \delta[n] - u[n+3] * \delta[n-1] \\ &= u[n+3] - u[n+2] = \delta[n+3] \end{aligned}$$

Next, convolve this with $x_1[n]$

$$\left(\frac{1}{2}\right)^n u[n] * \delta[n+3] = \left(\frac{1}{2}\right)^{n+3} u[n+3]$$

• Demonstrates associativity of DT convolution:

$$(x_1[n] * x_2[n]) * x_3[n] = x_1[n] * (x_2[n] * x_3[n])$$

Prob. 2.28

Causality: $h[n] = 0$ for $n < 0$

Stability: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

(a) $h[n] = \left(\frac{1}{3}\right)^n u[n]$ causal

$$\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{1 - \frac{1}{3}} < \infty \quad \text{stable}$$

(b) $h[n] = (.8)^{-2} \delta[n+2] + (.8)^{-1} \delta[n+1] + .8^n u[n]$

$h[n] \neq 0$ for $n < 0$ not causal

$$\sum_{n=-2}^{\infty} |h[n]| = \frac{1}{.64} + \frac{1}{.8} + \frac{1}{1 - .8} < \infty \quad \text{stable}$$

(c) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$ not causal

$$= (2)^{|n|} u[-n] \quad \text{not stable}$$

Approaching infinity as $n \rightarrow -\infty$

$$\begin{aligned}
 (d) \quad h[n] &= (5)^n u[3-n] = (5)^n u[-(n-3)] \\
 &= \left(\frac{1}{5}\right)^{|n|} u[-n] + 5\delta[n] + 5\delta[n-1] + 5^2\delta[n-2] + 5^3\delta[n-3]
 \end{aligned}$$

not causal

stable

$$(e) \quad h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$$

causal

not stable

↑ diverges
as $n \rightarrow \infty$

$$(f) \quad h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[-(n-1)]$$

not causal

$$= \left(-\frac{1}{2}\right)^n u[n] + \left(\frac{1}{1.01}\right)^{|n|} u[-n] + 1.01\delta[n-1]$$

$$\sum = \frac{1}{1 - \frac{1}{2}}$$

(take absolute
value)

$$\sum = \frac{1}{1 - 1.01}$$

$$= \frac{1}{1 - \frac{1}{1.01}}$$

stable!

$$(g) \quad h[n] = n \left(\frac{1}{3}\right)^n u[n]$$

can include $n=0$
since $h[0]=0$

causal!

$$\sum_{n=0}^{\infty} n \left(\frac{1}{3}\right)^n = \frac{1/3}{\left(1 - \frac{1}{3}\right)^2} < \infty \quad \text{Stable!}$$

See part (c) of Prob. 1.59 from Hmuk. 1