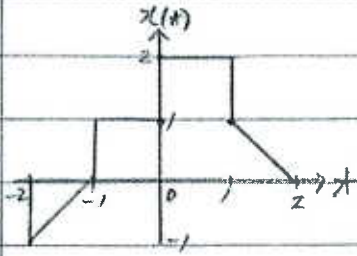


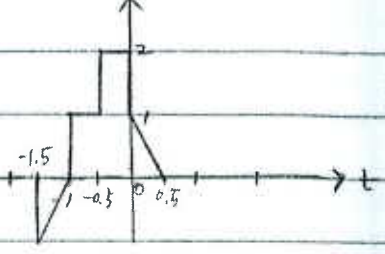
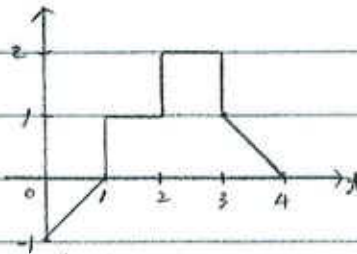
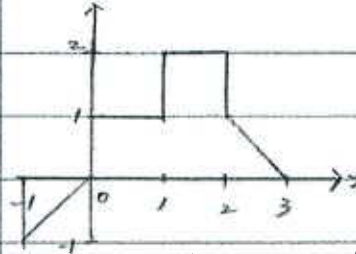
Text Prob 1.21.



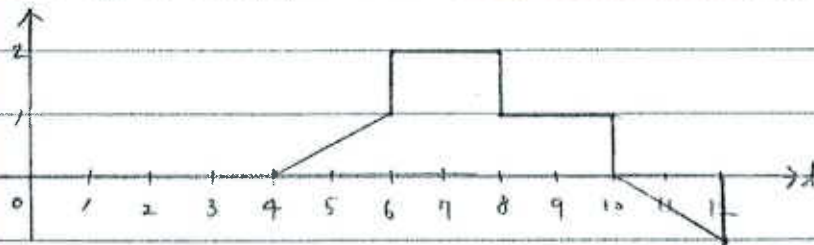
(a) $x(t-1)$

(b) $x(t-2)$

(c) $x(2t+1) = x(2(t+0.5))$

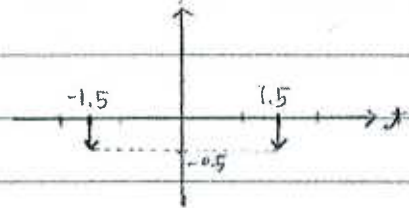
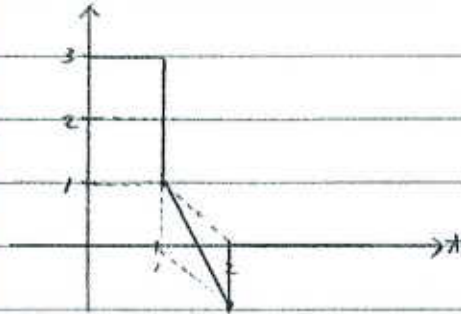


(d) $x(4 - \frac{t}{2}) = x(-\frac{1}{2}(t-8))$

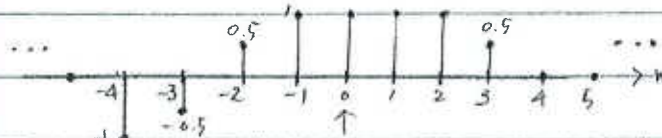


(e) $[x(t) + x(-t)]u(t)$

(f) $x(t) [\delta(t + \frac{3}{2}) - \delta(t - \frac{3}{2})]$

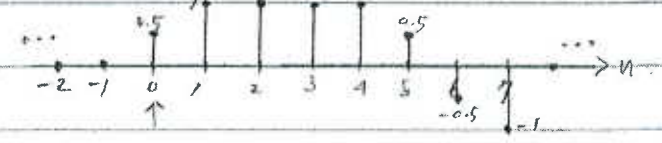
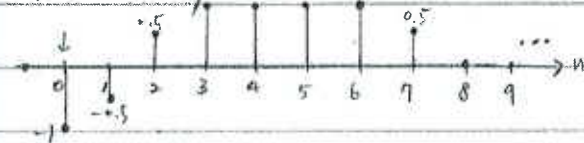


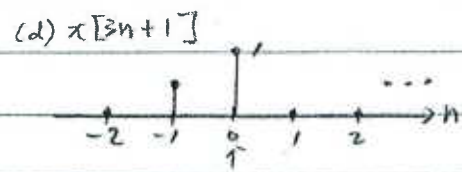
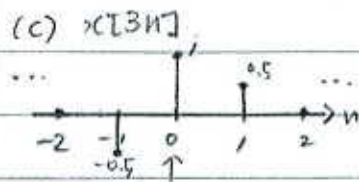
Text Prob 1.22.



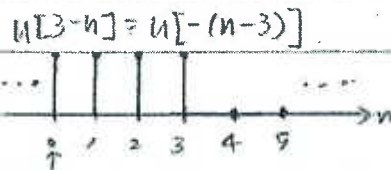
(a) $x[n-4]$

(b) $x[3-n] = x[-(n-3)]$





(e) $x[n] u[3-n] = x[n] u[-(n-3)]$

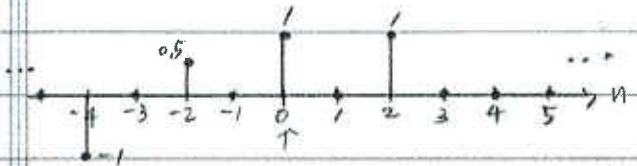


$\therefore x[n] u[3-n] = x[n]$

(f) $x[n-2] \delta[n-2]$



(g) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n] = 0.5(1 + (-1)^n) x[n] = \begin{cases} x[n], & n = \text{even} \# \\ 0, & n = \text{odd} \# \end{cases}$



Text Prob 1.34. (a) $\sum_{n=-\infty}^{\infty} x[n] = \sum_{n=-\infty}^{-1} x[n] + x[0] + \sum_{n=1}^{\infty} x[n] = x[0] + \sum_{n=1}^{\infty} \{x[n] + x[-n]\}$

Recall) If $x[n]$ is odd signal,

then $x[n] + x[-n] = 0$ and $x[0] = 0$

$\therefore \sum_{n=-\infty}^{\infty} x[n] = 0$

(b) Let $y[n] = x_1[n] x_2[n]$.

Then $y[-n] = x_1[-n] x_2[-n]$

$= -x_1[n] x_2[n]$

$= -y[n]$

Recall) If $x[n]$ is even signal,

then $x[n] = x[-n]$.

since $x_1[n]$: odd & $x_2[n]$: even,

\therefore If $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal,

$x_1[n] x_2[n]$ is an odd signal.

Prob. 1.34 (c)

2a

$$x[n] = x_e[n] + x_o[n]$$

$$x^2[n] = x_e^2[n] + 2x_e[n]x_o[n] + x_o^2[n]$$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + 2 \sum_{n=-\infty}^{\infty} x_e[n]x_o[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

Product of even and odd signal is odd

$$\text{Thus: } \sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0$$

$$\text{Thus: } \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

Prob. 1.34 (d) Similarly for CT:

$$\begin{aligned} \int_{-\infty}^{\infty} x^2(t) dt &= \int_{-\infty}^{\infty} (x_e(t) + x_o(t))^2 dt \\ &= \int_{-\infty}^{\infty} x_e^2(t) dt + \int_{-\infty}^{\infty} 2x_e(t)x_o(t) dt + \int_{-\infty}^{\infty} x_o^2(t) dt \end{aligned}$$

$x_e(t)x_o(t)$ is an odd function such that

$$\int_{-\infty}^{\infty} x_e(t)x_o(t) dt = 0$$

$$\text{Thus: } \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} x_e^2(t) dt + \int_{-\infty}^{\infty} x_o^2(t) dt$$

Text Prob 1.26. (a) $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$ do not have any factors in common.

Note) $x[n]$ is periodic if $\omega_0 = 2\pi m/N$ for m and $N =$ positive integers

$$N = \frac{2\pi m}{\omega_0} = \frac{7}{3} m$$

\Rightarrow When $m=3$ (makes N the smallest integer),

$$N = \boxed{7}$$

\therefore The signal is periodic with fundamental period $N = \boxed{7}$

$$(b) x[n] = \cos\left(\frac{\pi}{8}n - \pi\right)$$

$$N = \frac{2\pi m}{\omega_0} = 16\pi m$$

\Rightarrow There is NO integer m that can make N an integer.

\therefore The signal is NOT periodic.

(c) $x[n] = \cos\left(\frac{\pi}{8}n^2\right)$ Grader: don't grade part (c) of this problem: don't take any pts off

$$x[n+N] = \cos\left(\frac{\pi}{8}(n+N)^2\right) = \cos\left(\frac{\pi}{8}n^2 + \frac{\pi}{8}N^2 + \frac{\pi}{4}nN\right)$$

To show $x[n] = x[n+N]$,

$$\frac{\pi}{8}N^2 + \frac{\pi}{4}nN = 2\pi k, \quad \forall n \ \& \ k \in \mathbb{Z}$$

$$= 2\pi(l+m) = 2\pi l + 2\pi m, \quad l, m \in \mathbb{Z}$$

$$i) \frac{\pi}{8}N^2 = 2\pi l$$

$$\Rightarrow N^2 = 16l, \quad N = 4\sqrt{l}$$

When $\sqrt{l} \in \mathbb{Z}$, $N =$ multiple of 4. - (*)

$$ii) \frac{\pi}{4}nN = 2\pi m$$

$$\Rightarrow N = 8 \frac{m}{n}$$

When $\frac{m}{n} \in \mathbb{Z}$, $N =$ multiple of 8. - (**)

$\Rightarrow N=8$ to satisfy both condition (*) and (**)

\therefore The signal is periodic with fundamental period $N=8$.

$$(d) x[n] = \cos\left(\frac{\pi}{2}n\right) \cos\left(\frac{\pi}{4}n\right)$$

$$\text{Recall) } \cos A \cos B = 0.5 (\cos(A-B) + \cos(A+B))$$

$$x[n] = \frac{1}{2} \left\{ \underbrace{\cos\left(\frac{\pi}{4}n\right)}_{(1)} + \underbrace{\cos\left(\frac{3\pi}{4}n\right)}_{(2)} \right\}$$

i) Regarding term - (1)

$$N_1 = \frac{8}{3} m_1, \quad m_1, N_1 = \text{positive integers}$$

ii) Regarding term - (2)

$$N_2 = 8 m_2, \quad m_2, N_2 = \text{positive integers}$$

⇒ The positive integers, $m_1 = 3$ and $m_2 = 1$, makes

the positive integer $N = N_1 = N_2 = 8$ the smallest.

∴ The signal is periodic with fundamental period $N = 8$.

$$(e) x[n] = 2 \underbrace{\cos\left(\frac{\pi}{4}n\right)}_{(1)} + \underbrace{\sin\left(\frac{\pi}{8}n\right)}_{(2)} - 2 \underbrace{\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)}_{(3)}$$

i) Regarding term - (1)

$$N_1 = 8 m_1, \quad \text{for } m_1, N_1 = \text{positive integers.}$$

ii) Regarding term - (2)

$$N_2 = 16 m_2, \quad \text{for } m_2, N_2 = \text{positive integers.}$$

iii) Regarding term - (3)

$$N_3 = 4 m_3, \quad \text{for } m_3, N_3 = \text{positive integers.}$$

⇒ The positive integers, $m_1 = 2$, $m_2 = 1$, $m_3 = 4$, makes

the positive integer $N = N_1 = N_2 = N_3 = 16$ the smallest.

∴ The signal is periodic w/ fundamental period $N = 16$.

Text Prob 1.36.

(a) Note) If $x[n]$ is periodic with period N ,

$$\text{then } x[n+N] = x[n], \quad \forall n \in \mathbb{Z} \text{ \& } N \in \mathbb{Z}$$

If $x[n]$ is periodic, $e^{j\omega_0(n+N)T} = e^{j\omega_0 n T}$, where $\omega_0 = 2\pi/T_0$, $\forall n \in \mathbb{Z}$.
 - (*)

From (*), we obtain

$$e^{j\omega_0 N T} = 1 = e^{j2\pi k}, \quad k \in \mathbb{Z}.$$

This implies that

$$\frac{2\pi}{T_0} N T = 2\pi k \Rightarrow \frac{T}{T_0} = \frac{k}{N} = \text{a rational number.}$$

(b) If $T/T_0 = p/q$ where $p, q \in \mathbb{Z}$,

$$\text{then } x[n] = e^{j2\pi n (p/q)}$$

Since $x[n]$ is periodic with period N ,

$$\exp(j2\pi (p/q)(n+N)) = \exp(j2\pi (p/q)n), \quad \forall n \in \mathbb{Z}.$$

- (**)

From (**), we can get

$$\exp(j2\pi (p/q)N) = 1 = \exp(j2\pi k), \quad k \in \mathbb{Z}.$$

Note) N_0 : the smallest N satisfying $x[n+N] = x[n] \quad \forall n \in \mathbb{Z}$.

$\therefore N_0 = \frac{q}{\text{gcd}(p, q)} \Rightarrow$ [when $\frac{T}{T_0} = \frac{p}{q} = \text{rational}$, q is one period of $x[n]$, but not necessarily the fundamental period, and since p and q may have a common divisor, we need to divide q by the greatest common divisor $\text{gcd}(p, q)$ to

Now, the fundamental frequency of $x[n]$ is ... find fundamental period

$$\frac{2\pi}{N_0} = \frac{2\pi}{q} \text{gcd}(p, q)$$

$$= \frac{2\pi}{p} \frac{p}{q} \text{gcd}(p, q) = \frac{2\pi}{p} \frac{T}{T_0} \text{gcd}(p, q)$$

$$= \frac{\omega_0 T}{p} \text{gcd}(p, q)$$

(c) $p/\text{gcd}(p, q)$ periods of $x(t)$ are needed.

1.36.

(c) From part (b), we have that $N = \frac{q}{\gcd(p, q)}$

In other words, there are N samples in a period of $x[n]$.

These samples are equally spaced in time (T units of time apart).

thus the time it takes to obtain the N samples forming a

single period of $x[n]$ is: $N \cdot T = \frac{q}{\gcd(p, q)} \cdot T$,

Corresponding to $\frac{N \cdot T}{T_0}$ periods of $x(t)$.

$$\Rightarrow \frac{N \cdot T}{T_0} = \frac{q}{\gcd(p, q)} \cdot \frac{T}{T_0} \quad \text{where } \frac{T}{T_0} = \frac{p}{q}$$

$$\text{So } \frac{N \cdot T}{T_0} = \frac{\cancel{q}}{\gcd(p, q)} \cdot \frac{p}{\cancel{q}} = \underline{\underline{\frac{p}{\gcd(p, q)}}}$$

Hence $\frac{p}{\gcd(p, q)}$ periods of $x(t)$ are needed!

Text Prob. 1.54

(a) i) For $\alpha=1$,

$$\sum_{n=0}^{N-1} \alpha^n = \sum_{n=0}^{N-1} 1 = \underbrace{1 + \dots + 1}_N = N.$$

ii) For $\alpha \neq 1$ & $\alpha \in \mathbb{C}$,

we can write

$$1 - \alpha^N = (1 + \alpha^1 + \dots + \alpha^{N-1}) - (\alpha^1 + \dots + \alpha^{N-1} + \alpha^N).$$

$$= \sum_{n=0}^{N-1} \alpha^n - \sum_{n=0}^{N-1} \alpha^{N+1}$$

$$= \sum_{n=0}^{N-1} \alpha^n - \alpha \sum_{n=0}^{N-1} \alpha^n$$

$$= (1 - \alpha) \sum_{n=0}^{N-1} \alpha^n$$

$$\text{Therefore, } \sum_{n=0}^{N-1} \alpha^n = \frac{1 - \alpha^N}{1 - \alpha}.$$

(b) For $|\alpha| < 1$, $\lim_{N \rightarrow \infty} \alpha^N = 0$

$$\text{From (a), } \sum_{n=0}^{N-1} \alpha^n = \frac{1 - \alpha^N}{1 - \alpha}.$$

$$\text{Therefore, } \sum_{n=0}^{\infty} \alpha^n = \lim_{N \rightarrow \infty} \sum_{n=0}^{N-1} \alpha^n$$

$$= \lim_{N \rightarrow \infty} \left(\frac{1 - \alpha^N}{1 - \alpha} \right) = \frac{1}{1 - \alpha}.$$

Prob. 1.54 (c) Show: $\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2}$

for $|\alpha| < 1$

$$\frac{d}{d\alpha} \left\{ \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} = (1-\alpha)^{-1} \right\}$$

take derivative wrt α on both sides,
using chain rule on right hand side

$$\sum_{n=0}^{\infty} n\alpha^{n-1} = (-1)(1-\alpha)^{-2}(-1) = \frac{1}{(1-\alpha)^2}$$

multiply by α on both sides yields result

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2}$$

Prob. 1.54 (d)

$$\begin{aligned} \sum_{n=k}^{\infty} \alpha^n &= \sum_{n=0}^{\infty} \alpha^n - \sum_{n=0}^{k-1} \alpha^n \\ &= \frac{1}{1-\alpha} - \frac{1-\alpha^k}{1-\alpha} \\ &= \frac{\alpha^k}{1-\alpha} \end{aligned}$$