Section	Property	Aperiodic signal	Fourier transform
		$x(t)$	$X(\omega)$
		$y(t)$	$Y(\omega)$
4.3.0	Duality	$X(t)$	$2\pi\, x(-\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
4.3.3	Conjugation	$x^*(t)$	$X^*(-\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(\omega)Y(\omega)$
4.5	Multiplication	$x(t)y(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * Y(\omega) =$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\theta)Y(\omega - \theta)\, d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(\omega) = X^*(-\omega) \\ \mathcal{Re}\{X(\omega)\} = \mathcal{Re}\{X(-\omega)\} \\ \mathcal{Im}\{X(\omega)\} = -\mathcal{Im}\{X(-\omega)\} \\ \lvert X(\omega)\rvert = \lvert X(-\omega)\rvert \\ \sphericalangle X(\omega) = -\sphericalangle X(-\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(\omega)$ purely imaginary and od
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{Ev}\{x(t)\}$ [$x(t)$ real]	$\mathcal{Re}\{X(\omega)\}$
		$x_o(t) = \mathcal{Od}\{x(t)\}$ [$x(t)$ real]	$j\mathcal{Im}\{X(\omega)\}$

Initial Value Theorems: $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, d\omega$

$X(0) = \int_{-\infty}^{\infty} x(t)\, dt$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega$$

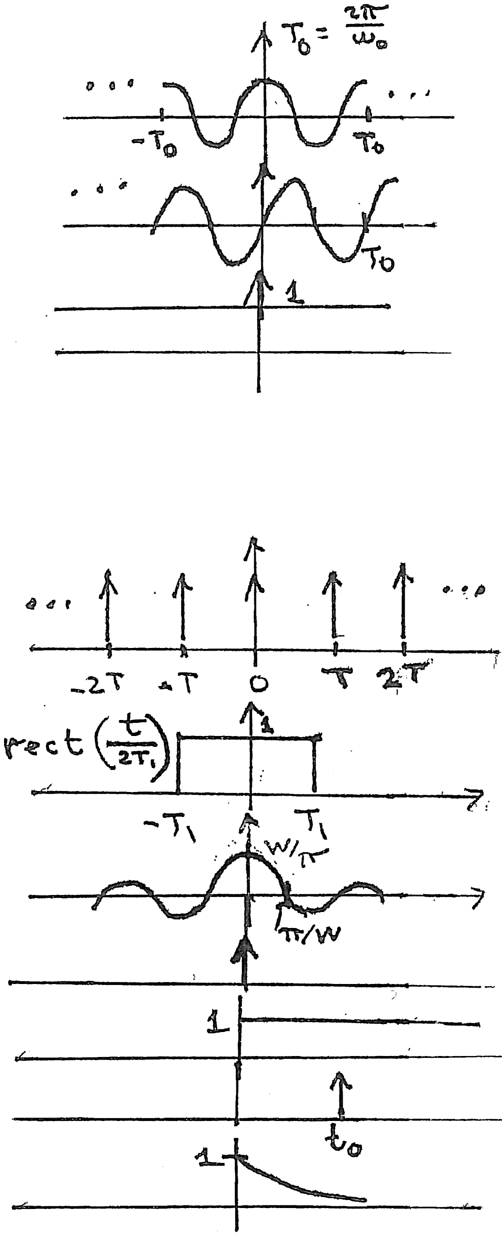
4.3.8 Frequency Shift Variants

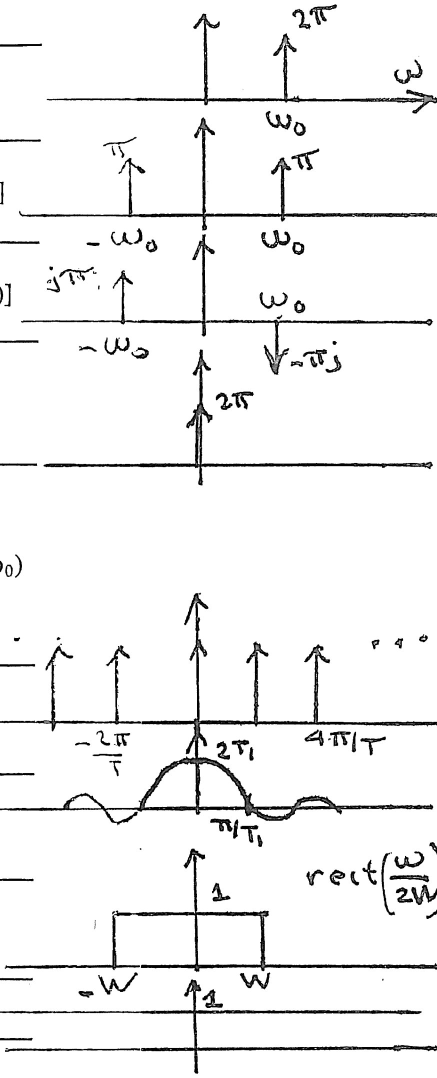
$$x(t)\cos(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}X(\omega-\omega_0) + \frac{1}{2}X(\omega+\omega_0)$$

$$x(t)\sin(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2j}X(\omega-\omega_0) - \frac{1}{2j}X(\omega+\omega_0)$$

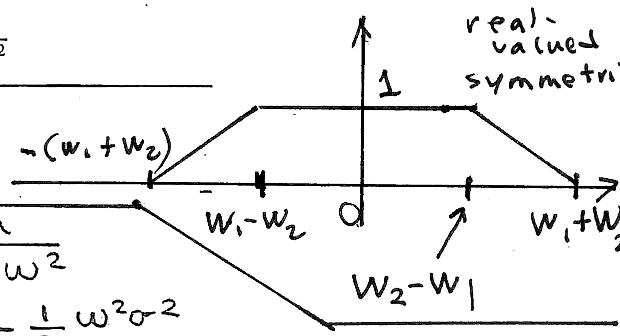
TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$x(t) = 1$	$2\pi\,\delta(\omega)$
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$
$x(t) \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$e^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$
$te^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$





$$\frac{\pi}{W_1} \cdot \frac{\sin(W_1 t)}{\pi t} \cdot \frac{\sin(W_2 t)}{\pi t}$$



$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$
$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$	$e^{-\frac{1}{2}\omega^2\sigma^2}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(\omega) = j$ for $\omega<0$; $-j$ for $\omega>0$