EE301 Signals and Systems — Spring 2019
Final Exam — Wednesday, May 1, 2019

Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive
There are FIVE problems.
Closed Book but Closed Notes.
Three 8.5 in. x 11 in. crib sheets allowed
Calculators NOT allowed.
All work should be done on the sheets provided.
NOTE: the answer to one part may be relevant to the answer to another part.
You are always welcome to use the answer from a previous part, to save time.
Make it clear and briefly explain if you use the answer from a previous part.
You must show all work for each problem to receive full credit.

Problem 1. Short answer questions.

(a) Briefly write and explain one of the main advantages of digital over analog, in terms of storage, transmission, and/or processing, in a coherent sentence.

This problem is on many old exams AND in the answers to one of the Problems on our Exam 3 from this semester

(b) If you sample at a rate ω_s in terms of radians/sec, what discrete-time frequency is the analog frequency $\frac{\omega_s}{2}$ mapped to? Then, briefly explain why π is the highest discrete-time frequency.

answer = π
This kind of problem on many old exams

(c) The DT signal $x[n]$ is obtained by sampling the sinewave $x_{a_1}(t) = \cos(25t)$ at a rate of $\omega_s = 40$ radians/sec. Specify the frequency ω_{a_2} of another analog sinewave $x_{a_2}(t) = \cos(\omega_{a_2} t)$ at a lower frequency that will yield the exact same DT signal $x[n]$ when sampled at the same rate, $\omega_s = 40$ radians/sec.

answer = 15 rads/sec
This kind of problem on many old exams

Problem 2 (a). The signal $x_a(t) = \{u(t+2) - u(t-2)\}$ is sampled every $T_s = 0.25$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a quarter of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. Assume that the signal is turned on equal to 1 at both ends (edges), i.e., at both $t = -2$ secs and $t = +2$ secs.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.25 \text{ secs} \quad \text{and} \quad x_a(t) = \{u(t+2) - u(t-2)\}$$

See Prob 2See Prob 2 for Exam 3 for SP 2015. Modify for T_s = 0.25 for Exam 3 for SP 2015. Modify for T_s = 0.25

Problem 2 (b). The signal $x_a(t) = t\,\{u(t+2) - u(t-2)\}$ is sampled every $T_s = 0.25$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a quarter of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. Assume that the signal is turned on equal to 1 at both ends (edges), i.e., at both $t = -2$ secs and $t = +2$ secs.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.25 \text{ secs} \quad \text{and} \quad x_a(t) = t\,\{u(t+2) - u(t-2)\}$$

See Prob 2(b) Exam 3 for SP 2015. Modify for T_s =0.25

Yes or No Questions with Explanation. Circle Yes or No for each question below, and briefly **explain** your answer in the space provided. You must cite at least one Fourier Transform Property in your answer for each part. Explanation is more important than your Yes or No answer.

Yes No Consider a signal $x(t)$ that is real-valued and non-negative for all time, i.e., $x(t) > 0$ for $-\infty < t < \infty$. Under these conditions, is it possible that the Fourier Transform $X(\omega)$ at $\omega = 0$ can be 0? That is, under these conditions, can $X(0) = 0$?

For this whole page, see solution to Exam 2 from SP 2013

Yes No Consider the signal $x(t) = \sqrt{2}\,\mathrm{rect}(2t) + \frac{sin(\pi t)}{\pi t} + \sqrt{\pi}\, e^{-t^2}$. Does the Fourier Transform of this signal, $X(\omega)$, have an imaginary part?

Yes No Consider a signal $y(t) = t\, e^{-t^2}$ with Fourier Transform $Y(\omega)$. Is the area under $Y(\omega)$ integrated over all frequencies equal to zero?

Yes No Consider $x(t)$ to be real-valued and having odd symmetry, $x(-t) = -x(t)$. Will the value of the Fourier Transform $X(\omega)$ at $\omega = 0$ always be zero, i.e., $X(0) = 0$ for a signal with odd symmetry?

Yes No Is the energy distribution as a function of frequency for the signal at the output of an LTI filter the same as the energy distribution as a function of frequency for the corresponding input signal?

Yes No Supposed that $X(\omega) = 0$ for $|\omega| > 10$ rads/sec. Will the Fourier Transform of $y(t) = a\, x(t-t_1) + b\, x(t-t_2) + c\, x(t-t_3)$ also be equal to 0 for $|\omega| > 10$ rads/sec, regardless of the values of the time-delays t_1, t_2, and t_3 and the amplitude scalings a, b, and c?

For this whole page, see solution to Exam 2 SP 2013. Prob. 1

Yes No Let $y(t)$ denote the output when $x(t)$ is the signal input to an LTI system with impulse response $h(t)$. Supposed that $X(\omega) = 0$ for $|\omega| > 10$ rads/sec. Is it possible to design an LTI system such that $Y(\omega) = 0$ for $|\omega| > 5$ rads/sec? That is, is it possible to lower the bandwidth (the max frequency) of signal through LTI filtering?

Yes No Consider the signal $y(t) = \frac{d}{dt}x(t)$ Is the Fourier Transform $Y(\omega)$ guaranteed to be 0 at at $\omega = 0$, i.e., $Y(0) = 0$, for any signal $x(t)$?

Yes No Is ALL of the energy of a FINITE-duration sinewave totally concentrated at the frequency of the sinewave?

Yes No Let $y(t)$ denote the output when a periodic signal $x(t)$ with period T is the signal input to an LTI system with impulse response $h(t)$. Will the output $y(t)$ be periodic with the same period T for any impulse response $h(t)$?

Problem 4. For each part, show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer. ALSO: if for any part, you use your answer from a previous part, clearly justify your answer.

Problem 4 (a). The 2-sided exponential pulse $x(t) = e^{-2|t|}$, is time-shifted to form $y(t) = x(t-5)$. Find the numerical value of $E = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$.

I will provide solution for this whole problem shortly BUT this problem is very similar to Prob 2 on Exam 2 from SP2014

Problem 4 (b). The 2-sided exponential pulse $x(t) = e^{-2|t|}$ is multiplied by a complex-valued sinewave to form $y(t) = e^{j5t}x(t)$. Find the numerical value of $E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$.

Problem 4 (c). The 2-sided exponential pulse $x(t) = e^{-2|t|}$ is compressed in time by a factor of 2 to form $y(t) = x(2t)$. Find the numerical value of the energy $E = \int_{-\infty}^{\infty} y^2(t)dt$.

Problem 4 (d). The 2-sided exponential pulse $x(t) = e^{-2|t|}$ is multiplied by t to form $y(t) = tx(t)$. Find the numerical value of $A = \int_{-\infty}^{\infty} y(t)dt$.

Problem 4 (e). The 2-sided exponential pulse $x(t) = e^{-2|t|}$ is differentiated to form $y(t) = \frac{d}{dt}x(t)$. Find the numerical value of $A = \int_{-\infty}^{\infty} y(t)dt$.

Problem 4 (f). The 2-sided exponential pulse $x(t) = e^{-2|t|}$ is input to an LTI system with a 2-sided exponential impulse response $h(t) = e^{-3|t|}$. Determine the numerical value of the area, $A = \int_{-\infty}^{\infty} y(t)dt$, under the output $y(t) = x(t) * h(t)$.

Problem 4 (g). The sinc pulse $x(t) = \dfrac{\sin(4t)}{\pi t}$ is input to an LTI sytem impulse response $h(t) = \dfrac{\pi}{2}\dfrac{\sin(2t)}{\pi t}\dfrac{\sin(6t)}{\pi t}$ Determine a simple, closed-form expression for the output $y(t) = x(t) * h(t)$.

Problem 4 (h). The sinc pulse $x(t) = \dfrac{\sin(4t)}{\pi t}$ is input to an LTI sytem impulse response $h(t) = \dfrac{\pi}{2}\dfrac{\sin(2t)}{\pi t}\dfrac{\sin(6t)}{\pi t}$ Determine the numerical value of the energy $E = \int_{-\infty}^{\infty} y^2(t)dt$ of the output $y(t) = x(t) * h(t)$.

Problem 5. The signal

$$x(t) = \frac{1}{t^2+1}\cos(25t) - \left\{\frac{1}{t^2+1} * \frac{1}{\pi t}\right\}\sin(25t)$$

is input to an LTI system with impulse response

$$h(t) = \left\{\frac{\pi}{5}\,\frac{\sin(5t)}{\pi t}\,\frac{\sin(20t)}{\pi t}\right\}$$

Determine the output $y(t) = x(t) * h(t)$. Show all work, state which Fourier Transform pairs and/or properties you are using, and clearly indicate your final answer.

See solution to Prob. 3 on Exam 2 for SP 2015

Additional Space for Problem 5 answer and work.