EE301 Signals and Systems **Final Exam**

Final Exam **Friday, May 7, 2010**

Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **four** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

NOTES:

- You need only plot the magnitude of a DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period 2π.
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of T_s is seconds for all parts.
- $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}cos(2\theta)$

Note:

Final Grades: The final grades are due next Tuesday, May 11.
It takes a while to grade the finals, get them recorded in the database,
rank order every one, determine grade cut-offs, etc.
Thus, the final grades may not be done until Monday possibly.
Note: We are not allowed to post grades, not even with a partial SSN.

Problem 1.

(a) Consider the continuous-time signal $x_1(t)$ below. A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{40}$. Plot the DTFT of $x_1[n]$, $X_1(\omega)$, over $-\pi < \omega < \pi$.

$$x_1(t) = T_s\ \frac{2\pi}{5}\ \left\{\frac{\sin(2.5t)}{\pi t}\right\}^2\ 2\ \cos(5t)$$

(b) Repeat part (a) for $T_s = \frac{2\pi}{15}$.

(c) Consider the continuous-time signal $x_1(t)$ below. A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{30}$. Plot the DTFT of $x_1[n]$, $X_1(\omega)$, over $-\pi < \omega < \pi$.

$$x_1(t) = T_s\ \frac{2\pi}{5}\ \left\{\frac{\sin(2.5t)}{\pi t}\right\}^2\ 2\ \sin(5t)$$

(d) Repeat part (c) for $T_s = \frac{2\pi}{15}$.

Problem 2. For this problem, the signal $x(t)$ is defined below.

$$x(t) = \frac{4}{4+t^2}$$

(a) Determine and write a closed-form expression for the Fourier Transform, $X(\omega)$, of $x(t)$.

(b) Plot $X(\omega)$ as a function of frequency, showing as much detail as possible.

(c) Compute the energy of the signal $x(t)$ defined below. Your final answer should be a number. Show all work.

$$E_x = \int_{-\infty}^{\infty} x^2(t)dt$$

Problem 3.

(a) Determine and plot the magnitude of the Fourier Transform $X_1(\omega)$ of the real-valued signal $x_1(t)$ defined below.

$$x_1(t) = \frac{1}{2}\left\{\frac{\sin(10(t-\frac{\pi}{10}))}{\pi(t-\frac{\pi}{10})} - \frac{\sin(10(t+\frac{\pi}{10}))}{\pi(t+\frac{\pi}{10})}\right\}$$

(b) The signal $x_1(t)$ is input to an LTI system with impulse response given below, creating the output $y_1(t) = x_1(t) * h(t)$

$$h(t) = 2\pi t \left\{\frac{\sin(5t)}{\pi t}\right\}^2$$

We create a complex-valued signal as below, where the original signal $x_1(t)$ is the real part and the filter output $y_1(t)$ is the imaginary part.

$$z_1(t) = x_1(t) + jy_1(t)$$

Plot the magnitude of the Fourier Transform $Z_1(\omega)$ of the signal $z_1(t)$. *HINT:* $z_1(t) = x_1(t) + jy_1(t) = x_1(t) * \{\delta(t) + jh(t)\}$.

(c) Determine and plot the magnitude of the Fourier Transform $X_2(\omega)$ of the real-valued signal $x_2(t)$ defined below.

$$x_2(t) = \frac{1}{10}\frac{d}{dt}\left\{\frac{\sin(10t)}{\pi t}\right\}$$

(d) The signal $x_2(t)$ is input to the same LTI system with impulse response given below, creating the output $y_2(t) = x_2(t) * h(t)$

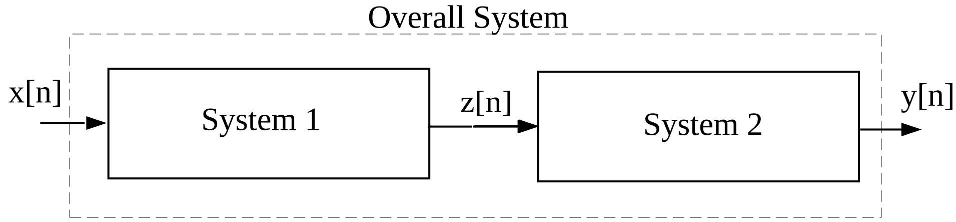
$$h(t) = 2\pi t \left\{\frac{\sin(5t)}{\pi t}\right\}^2$$

A second complex-valued signal is created as below, where the original signal $x_2(t)$ is the real part and the filter output $y_2(t)$ is the NEGATIVE of the imaginary part.

$$z_2(t) = x_2(t) - jy_2(t)$$

Plot the magnitude of the Fourier Transform $Z_2(\omega)$ of the signal $z_2(t)$. *HINT:* $z_2(t) = x_2(t) - jy_2(t) = x_2(t) * \{\delta(t) - jh(t)\}$.

Problem 4. Consider two discrete-time LTI systems in series.



(a) System 1 is described by the following difference equation

$$z[n] = z[n-1] + x[n] - x[n-6] \tag{1}$$

Determine and plot (stem-plot) the impulse response $h_1(n)$ of System 1.

(b) The frequency response $|H_1(\omega)|$ of System 1 is the DTFT of the impulse response $h_1[n]$. Plot the magnitude, $|H_1(\omega)|$, of the frequency response of System 1 over $-\pi < \omega < \pi$.

(i) Explicitly list all frequencies within the range $-\pi < \omega \leq \pi$ for which $H(\omega) = 0$.

(ii) Explicitly state the numerical value of $H(0)$.

(c) The input signal is obtained from sampling a continuous-time signal as

$$x[n] = x_a(nT_s), \qquad x_a(t) = u(t) - u(t-10) \quad \text{and} \quad T_s = 4$$

Determine and plot (stem-plot) the intermediate output $z(n)$ obtained with this input.

(d) The second system is described by the following difference equation. (The input is $z[n]$ according to the block diagram above, but for purposes of analyzing what the system is doing, you can think of $z[n]$ as $x[n]$ in the difference equation below.)

$$y[n] = \frac{1}{4}y[n-1] + z[n] - 4z[n-1] \tag{2}$$

Determine and plot the magnitude, $|H_2(\omega)|$, of the frequency response of System 2 over $-\pi < \omega < \pi$.

(e) Determine and plot the magnitude, $|Y(\omega)|$, of the DTFT of the output $y[n]$ obtained with the input $x[n]$ defined in part (c). Clearly indicate the frequencies for which $Y(\omega) = 0$ over $-\pi \leq \omega \leq \pi$.

(f) Compute the numerical value of $\mathcal{E}_y = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} |Y(\omega)|^2 d\omega$.