Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains five problems.
All work should be done in the blue books provided.
Do not return this test sheet, just return the blue books.

NOTES:

• You need only plot the magnitude of a DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period $2\pi$.

• You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.

• You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.

• The unit of $T_s$ is seconds for all parts.

Notes:

Great semester! it was a pleasure having each of you in class this semester!
I really enjoyed teaching 301 this semester. Good luck on the Final!
Have a great summer!

Final Grades: The final grades are due next Monday, May 8.
It takes a while to grade the finals, get them recorded in the database, rank order every one, determine grade cut-offs, etc.
Thus, the final grades will not be done until Sunday night at the earliest.
Note: We are not allowed to post grades, not even with a partial SSN.
Problem 1. Consider the discrete-time LTI system described by the difference equation
\[ y[n] = x[n + 2] + 2x[n + 1] + 3x[n] + 2x[n - 1] + x[n - 2] \]  
(1)

(a) Plot the magnitude \(|H(e^{j\omega})|\) and phase \(\angle H(e^{j\omega})\) (two separate plots) of the frequency response of this system (equal to the DTFT \(H(e^{j\omega})\) of the impulse response \(h[n]\)) as a function of frequency over \(-\pi < \omega < \pi\). Show as much detail as possible.

(b) Let \(x[n] = x_a(nT_s)\) where \(T_s = 0.5\) seconds and \(x_a(t) = \cos(2\pi t)\{u(t) - u(t - 0.75)\}\), which is the product of a sinewave with a frequency of 1 Hertz and a rectangular pulse of duration three-fourths of a second. Do the following:

(i) Plot the magnitude \(|X(e^{j\omega})|\) of the DTFT of the resulting sampled signal \(x[n]\).

(ii) Plot the time-domain output \(y[n]\) obtained when \(x[n]\) described above is input to the LTI system described by Equation (1).

(iii) Plot the magnitude \(|Y(e^{j\omega})|\) of the DTFT of the output signal \(y[n]\).

(iv) Determine the numerical value of \(\sum_{n=-\infty}^{\infty} y^2[n]\). Show all work.

Problem 2. A discrete-time signal is created by sampling a continuous-time signal as
\(x[n] = x_a(nT_s)\) where \(T_s = \frac{2\pi}{6}\) and \(x_a(t) = T_s \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{3\pi t} \right\} \).

(i) Plot the magnitude of the DTFT of \(x[n]\) over \(-\pi < \omega < \pi\).

(ii) \(x[n]\) is passed through the DT linear system drawn below with impulse response \(h_a[n] = 2(-1)^n \left\{ \frac{\sin\left(\frac{2\pi n}{3}\right)}{\pi n} \right\} \) AND \(h_b[n] = \left\{ \frac{\sin\left(\frac{\pi n}{3}\right)}{\pi n} \right\} \) yielding the output denoted \(y_{ab}[n]\). Plot magnitude of the DTFT of \(y_{ab}[n]\) over \(-\pi < \omega < \pi\).
Problem 3.

Let \( x[n] = x_a(nT_s) \) where \( T_s = \frac{2\pi}{12} \) and \( x_a(t) = 2T_s \left\{ \frac{\sin(t)}{\pi t} + \cos(5t) \right\}^2 \).

(a) Plot the magnitude of the DTFT of \( x[n] \) over \(-\pi < \omega < \pi\).

(b) \( x[n] \) is passed through a DT linear system with impulse response \( h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{3}n)}{\pi n} \right\} \) yielding the output \( y_a[n] \). Plot magnitude of the DTFT of \( y_a[n] \).

(c) \( x[n] \) is passed through a DT linear system with impulse response \( h_b[n] = 2 \cos \left( \frac{\pi}{2} n \right) \left\{ \frac{\sin(\frac{\pi}{6}n)}{\pi n} \right\} \) yielding the output \( y_b[n] \). Plot magnitude of the DTFT of \( y_b[n] \) over \(-\pi < \omega < \pi\).

(d) \( x[n] \) is passed through a DT linear system with impulse response \( h_c[n] = \frac{\sin(\frac{\pi}{3}n)}{\pi n} \) yielding the output \( y_c[n] \). Plot magnitude of the DTFT of \( y_c[n] \) over \(-\pi < \omega < \pi\).

Problem 4.

Consider the continuous-time signal below equal to a sum of four sinewaves.

\[
x(t) = \sin(2t) + \sin(4t + 45^\circ) + \sin(6t + 90^\circ) + \sin(8t + 135^\circ)
\]

A discrete-time signal is created by sampling \( x(t) \) according to \( x[n] = x(nT_s) \) for \( T_s = \frac{2\pi}{8} \).

(i) Plot the magnitude of the DTFT of \( x[n] \) over \(-\pi < \omega < \pi\).

(ii) \( x[n] \) is passed through a DT linear system described by the difference equation

\[
y[n] = \frac{1}{3}y[n-1] + x[n] - 3x[n-1]
\]

Plot magnitude of the DTFT of the output \( y[n] \) over \(-\pi < \omega < \pi\).
Problem 5.

Consider the continuous-time signal \( x(t) = x_a(t) \ast \left\{ \frac{\sin(2.5t)}{\pi t} \right\} \), where \( \ast \) denotes convolution and \( x_a(t) \) is the periodic signal \( x(t) \) below with period \( T = 3\pi \).

A discrete-time signal is created by sampling \( x(t) \) according to \( x[n] = x(nT_s) \) for \( T_s = \frac{2\pi}{4} \).

(i) Plot the magnitude of the DTFT of \( x[n] \) over \(-\pi < \omega < \pi\).

(ii) \( x[n] \) is passed through a DT linear system with impulse response \( h[n] = 2 \frac{\sin(\frac{\pi n}{4})}{\pi n} \cos \left( \frac{\pi}{2} n \right) \) yielding the output \( y[n] \). Plot magnitude of the DTFT of \( y[n] \) over \(-\pi < \omega < \pi\).