

**Notes:**

**Great semester!** it was a pleasure having each of you in class this semester!

I really enjoyed teaching 301 this semester. **Good luck on the Final!**

**Have a great summer!**

”

**Final Grades:** The final grades are due next Tuesday, May 10.

It takes a while to grade the finals, get them recorded in the database,  
rank order every one, determine grade cut-offs, etc.

Thus, the final grades will not be done until Sunday night at the earliest.

*Note:* We are not allowed to post grades, not even with a partial SSN.

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**Cheating.** I have received two reports of cheating after EACH of the 3 mid-terms.

Thus, I will be watching very carefully tonight. Since it was  
reported for Exam 3 that the cheating went on when I was answering questions,

I will not answer questions tonight unless you are  
absolutely confident that there is a serious mistake or error in the exam.

**KEEP YOUR EYES ON YOUR OWN EXAM AND NO TALKING!**

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## Cover Sheet

Test Duration: 120 minutes.

Coverage: Comprehensive.

Open Book but Closed Notes.

Calculators NOT allowed.

This test contains **six** problems.

All work should be done in the blue books provided.

**DUE TO THE FAST TURNAROUND NEEDED TO TURN YOUR FINAL LETTER GRADES IN ON TIME and AS THESE PROBLEMS ARE SIMILAR TO PROBLEMS ON PREVIOUS EXAMS (2004 and 2005) THERE WILL NOT BE AS MUCH PARTIAL CREDIT FOR WRONG ANSWERS AS GIVEN IN THE MIDTERMS.**

**THEREFORE, CHECK YOUR ANSWERS CAREFULLY.**

Do **not** return this test sheet, just return the blue books.

For EACH of the part of this problem:

- You need only plot the magnitude of the DTFT over  $-\pi < \omega < \pi$ , but it is very important to keep in mind that a DTFT is always periodic with period  $2\pi$ .
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over  $-\pi < \omega < \pi$  for which the DTFT is zero.
- You **MUST** show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of  $T_s$  is seconds for all parts.

**Problem 1.** Consider the discrete-time LTI system described by the difference equation below (which is **different** from the one in Problem 2 of Exam 3.)

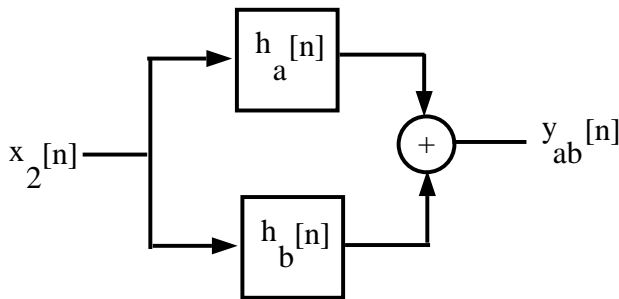
$$y[n] = x[n + 1] + x[n] + x[n - 1] \quad (1)$$

- (a) Plot the magnitude  $|H(e^{j\omega})|$  and phase  $\angle H(e^{j\omega})$  (two separate plots) of the frequency response of this system (equal to the DTFT  $H(e^{j\omega})$  of the impulse response  $h[n]$ ) as a function of frequency over  $-\pi < \omega < \pi$ . Show as much detail as possible.
- (b) Consider  $x[n] = x_a(nT_s)$  where  $T_s = 2$  and  $x_a(t) = u(t+5) - u(t-5)$ . Do the following:
- Plot the magnitude  $|X(e^{j\omega})|$  of the DTFT of the resulting sampled signal  $x[n]$ .
  - Plot the time-domain output  $y[n]$  obtained when  $x[n]$  described above is input to the LTI system described by Equation (1).
  - Determine the numerical value of  $\sum_{n=-\infty}^{\infty} y^2[n]$ . Show all work.

**Problem 2.**

A discrete-time signal is created by sampling the continuous-time signal  $x(t)$  according to  $x[n] = x(nT_s)$  for  $T_s = \frac{3\pi}{20}$ , where  $x(t) = T_s \frac{\sin(10t)}{\pi t}$ .

- Plot the magnitude of the DTFT of  $x[n]$  over  $-\pi < \omega < \pi$ .
- $x[n]$  is passed through the DT linear system drawn below with impulse response  $h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\}$  AND  $h_b[n] = 2 \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\}$  yielding the output denoted  $y_{ab}[n]$ . Plot magnitude of the DTFT of  $y_{ab}[n]$  over  $-\pi < \omega < \pi$ .



**Problem 3.**

Consider the continuous-time signal  $x_a(t) = \left\{ \frac{\sin(4t)}{\pi t} \right\}^2 \cos(6t)$ . A DT signal is obtained by sampling  $x(t)$  according to  $x[n] = x_a(nT_s)$  for  $T_s = \frac{2\pi}{28}$ .

(a) Plot the magnitude of the DTFT of  $x[n]$  over  $-\pi < \omega < \pi$ .

(b)  $x[n]$  is passed through a DT linear system with impulse response  $h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{4\pi}{7}n)}{\pi n} \right\}$  yielding the output  $y_a[n]$ . Plot magnitude of the DTFT of  $y_a[n]$ .

(c)  $x[n]$  is passed through a DT linear system with impulse response  $h_b[n] = 2 \cos\left(\frac{2\pi}{7}n\right) \left\{ \frac{\sin(\frac{\pi}{7}n)}{\pi n} \right\}$  yielding the output  $y_b[n]$ . Plot magnitude of the DTFT of  $y_b[n]$  over  $-\pi < \omega < \pi$ .

(d)  $x[n]$  is passed through a DT linear system with impulse response  $h_c[n] = \frac{\sin(\frac{\pi}{7}n)}{\pi n}$  yielding the output  $y_c[n]$ . Plot magnitude of the DTFT of  $y_c[n]$  over  $-\pi < \omega < \pi$ .

**Problem 4.**

Consider the continuous-time signal below equal to a sum of four sinewaves.

$$x(t) = \sin(6t) + \sin(12t + 45^\circ) + \sin(18t + 90^\circ) + \sin(24t + 135^\circ)$$

A discrete-time signal is created by sampling  $x(t)$  according to  $x[n] = x(nT_s)$  for  $T_s = \frac{2\pi}{24}$ .

(i) Plot the magnitude of the DTFT of  $x[n]$  over  $-\pi < \omega < \pi$ .

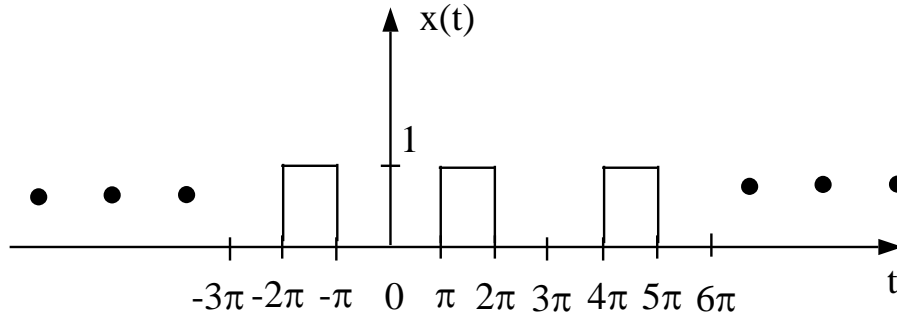
(ii)  $x[n]$  is passed through a DT linear system described by the difference equation

$$y[n] = \frac{1}{4}y[n-1] + x[n] - 4x[n-1]$$

Plot magnitude of the DTFT of the output  $y[n]$  over  $-\pi < \omega < \pi$ .

**Problem 5.**

Consider the continuous-time signal  $x(t) = x(t) * \left\{ \frac{\sin(3t)}{\pi t} \right\}$ , where  $*$  denotes convolution and  $x(t)$  is the periodic signal  $x(t)$  below with period  $T = 3\pi$ .



A discrete-time signal is created by sampling  $x(t)$  according to  $x[n] = x(nT_s)$  for  $T_s = \frac{3\pi}{8}$ .

- (i) Plot the magnitude of the DTFT of  $x[n]$  over  $-\pi < \omega < \pi$ .
- (ii)  $x[n]$  is passed through a DT linear system with impulse response  $h[n] = \frac{\sin(\frac{2\pi}{3}n)}{\pi n}$  yielding the output  $y[n]$ . Plot magnitude of the DTFT of  $y[n]$  over  $-\pi < \omega < \pi$ .

**Problem 6.** Consider the discrete-time LTI system described by the difference equation below.

$$y[n] = x[n] - x[n - 1] + x[n - 2] - x[n - 3] \quad (2)$$

- (a) Plot the magnitude  $|H(e^{j\omega})|$  and phase  $\angle H(e^{j\omega})$  (two separate plots) of the frequency response of this system (equal to the DTFT  $H(e^{j\omega})$  of the impulse response  $h[n]$ ) as a function of frequency over  $-\pi < \omega < \pi$ .

**(HINT:** The impulse response of the system described by Eqn. 2 is the product of  $(-1)^n = \cos(\pi n)$  with the impulse response of the system in Problem 2 of Exam 3.)

- (b) Consider  $x[n] = x_a(nT_s)$  where  $T_s = 3$  and  $x_a(t) = [u(t) - u(t - 10)] \cos\left(\frac{\pi}{3}t\right)$ . That is,  $x[n]$  is a sampled version of an analog signal which is the product of a rectangular pulse and a sinewave. Do the following:

- (i) Plot the magnitude  $|X(e^{j\omega})|$  of the DTFT of the resulting sampled signal  $x[n]$ .
- (ii) Plot the time-domain output  $y[n]$  obtained when  $x[n]$  described above is input to the LTI system described by Equation (2).

- (iii) Determine the numerical value of  $\sum_{n=-\infty}^{\infty} y^2[n]$ . Show all work.