EE301 Signals and Systems | **Final Exam**
Final Exam | **Tuesday, May 4, 2004**

Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **six** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

For EACH of the part of this problem:

- You need only plot the magnitude of the DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period 2π.
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of T_s is seconds for all parts.

Problem 1.

Consider the continuous-time signal $x_1(t) = \left\{ \frac{\sin(3t)}{\pi t} \right\}^2 \cos(4t)$. A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{16}$.

(i) Plot the magnitude of the DTFT of $x_1[n]$ over $-\pi < \omega < \pi$.

(ii) $x_1[n]$ is passed through a DT linear system with impulse response $h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{4}n)}{\pi n} \right\}$ yielding the output $y_{a1}[n]$. Plot magnitude of the DTFT of $y_{a1}[n]$.

(iii) $x_1[n]$ is passed through a DT linear system with impulse response $h_b[n] = 2\cos\left(\frac{\pi}{2}n\right) \left\{ \frac{\sin(\frac{\pi}{4}n)}{\pi n} \right\}$ yielding the output $y_{b1}[n]$. Plot magnitude of the DTFT of $y_{b1}[n]$ over $-\pi < \omega < \pi$.

(iv) $x_1[n]$ is passed through a DT linear system with impulse response $h_c[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ yielding the output $y_{c1}[n]$. Plot magnitude of the DTFT of $y_{c1}[n]$ over $-\pi < \omega < \pi$.

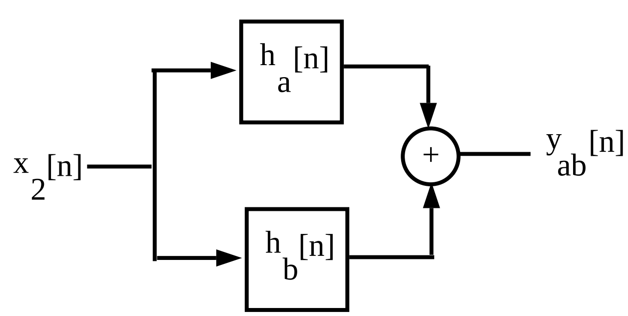
Problem 2.

Consider the continuous-time signal $x_2(t) = \frac{\sin(4t)}{\pi t}$. A discrete-time signal is created by sampling $x_2(t)$ according to $x_2[n] = x_2(nT_s)$ for $T_s = \frac{2\pi}{6}$.

(i) Plot the magnitude of the DTFT of $x_2[n]$ over $-\pi < \omega < \pi$.

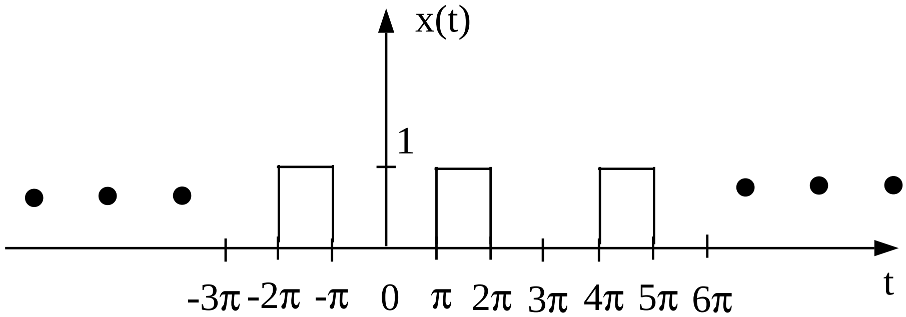
(ii) $x_2[n]$ is passed through a DT linear system with impulse response $h[n] = (-1)^n \left\{ \frac{\sin(\frac{2\pi}{3}n)}{\pi n} \right\}$ yielding the output $y_2[n]$. Plot magnitude of the DTFT of $y_2[n]$ over $-\pi < \omega < \pi$.

(iii) $x_2[n]$ is passed through the DT linear system drawn below with impulse response $h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{3}n)}{\pi n} \right\}$ AND $h_b[n] = 2\left\{ \frac{\sin(\frac{2\pi}{3}n)}{\pi n} \right\}$ yielding the output denoted $y_{ab}[n]$. Plot magnitude of the DTFT of $y_{ab}[n]$ over $-\pi < \omega < \pi$.



Problem 3.

Consider the continuous-time signal $x_3(t) = x(t) * \left\{ \frac{\sin(2t)}{\pi t} \right\}$, where $*$ denotes convolution and $x(t)$ is the periodic signal $x(t)$ below with period $T = 3\pi$.



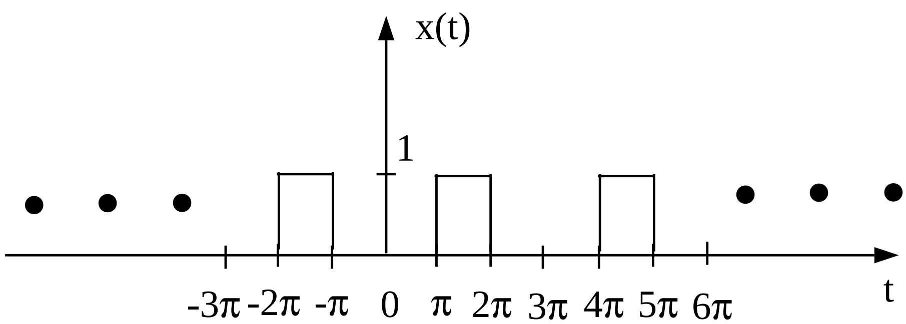
A discrete-time signal is created by sampling $x_3(t)$ according to $x_3[n] = x_3(nT_s)$ for $T_s = \frac{2\pi}{4}$.

(i) Plot the magnitude of the DTFT of $x_3[n]$ over $-\pi < \omega < \pi$.

(ii) $x_3[n]$ is passed through a DT linear system with impulse response $h[n] = \frac{\sin(\frac{3\pi}{4}n)}{\pi n}$ yielding the output $y_3[n]$. Plot magnitude of the DTFT of $y_3[n]$ over $-\pi < \omega < \pi$.

Problem 4.

The CT signal $y(t)$ is obtained by passing the periodic input signal (period 3π)



through the system $y(t) = \int_{t-\pi}^{t+\pi} x(\tau) d\tau$. NEXT, consider the CT signal $x_4(t)$ defined in terms of $y(t)$ as $x_4(t) = y(t) * \left\{ \frac{\sin(2t)}{\pi t} \right\}$, where $*$ denotes convolution. A DT signal is obtained by sampling $x_4(t)$ according to $x_4[n] = x_4(nT_s)$ for $T_s = \frac{2\pi}{4}$.

(i) Plot the magnitude of the DTFT of $x_4[n]$ over $-\pi < \omega < \pi$.

(ii) $x_4[n]$ is passed through a DT linear system with impulse response $h[n] = \frac{\sin(\frac{3\pi}{4}n)}{\pi n}$ yielding the output $y_4[n]$. Plot magnitude of the DTFT of $y_4[n]$ over $-\pi < \omega < \pi$.

Problem 5.

Consider the continuous-time signal below equal to a sum of four sinewaves.

$$x(t) = \cos(3t) + \cos(6t) + \cos(9t) + \cos(12t)$$

A discrete-time signal is created by sampling $x(t)$ according to $x[n] = x(nT_s)$ for $T_s = \dfrac{2\pi}{12}$.

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(ii) $x[n]$ is passed through a DT linear system described by the difference equation

$$y[n] = \frac{1}{2}y[n-1] + x[n] - 2x[n-1]$$

Plot magnitude of the DTFT of the output $y[n]$ over $-\pi < \omega < \pi$.

Problem 6.

Consider the continuous-time signal $x_6(t) = \left\{\dfrac{\sin(4t)}{\pi t}\right\}\left\{\dfrac{\sin(8t)}{\pi t}\right\}$. A DT signal is obtained by sampling $x_6(t)$ according to $x_6[n] = x_6(nT_s)$ for $T_s = \dfrac{2\pi}{36}$.

(i) Plot the magnitude of the DTFT of $x_6[n]$ over $-\pi < \omega < \pi$.

(ii) $x_6[n]$ is passed through a DT linear system with impulse response $h_a[n] = (-1)^n \left\{\dfrac{\sin(\frac{\pi}{3}n)}{\pi n}\right\}$ yielding the output $y_{a1}[n]$. Plot magnitude of the DTFT of $y_{a1}[n]$.

(iii) $x_6[n]$ is passed through a DT linear system with impulse response $h_b[n] = 2\cos\left(\dfrac{4\pi}{9}n\right)\left\{\dfrac{\sin(\frac{2\pi}{9}n)}{\pi n}\right\}$ yielding the output $y_{b1}[n]$. Plot magnitude of the DTFT of $y_{b1}[n]$ over $-\pi < \omega < \pi$.

(iv) $x_6[n]$ is passed through a DT linear system with impulse response $h_c[n] = \dfrac{\sin(\frac{2\pi}{9}n)}{\pi n}$ yielding the output $y_{c1}[n]$. Plot magnitude of the DTFT of $y_{c1}[n]$ over $-\pi < \omega < \pi$.