For EACH of the part of this problem:

• You need only plot the magnitude of the DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period $2\pi$.

• You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.

• You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.

• The unit of $T_s$ is seconds for all parts.
Problem 1.

Consider the continuous-time signal $x_1(t) = \left(\frac{\sin(3t)}{\pi t}\right)^2 \cos(4t)$. A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{16}$.

(i) Plot the magnitude of the DTFT of $x_1[n]$ over $-\pi < \omega < \pi$.

(ii) $x_1[n]$ is passed through a DT linear system with impulse response $h_a[n] = (-1)^n \left\{\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}\right\}$ yielding the output $y_{a1}[n]$. Plot magnitude of the DTFT of $y_{a1}[n]$.

(iii) $x_1[n]$ is passed through a DT linear system with impulse response $h_b[n] = 2 \cos\left(\frac{\pi}{2}n\right) \left\{\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}\right\}$ yielding the output $y_{b1}[n]$. Plot magnitude of the DTFT of $y_{b1}[n]$ over $-\pi < \omega < \pi$.

(iv) $x_1[n]$ is passed through a DT linear system with impulse response $h_c[n] = \frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}$ yielding the output $y_{c1}[n]$. Plot magnitude of the DTFT of $y_{c1}[n]$ over $-\pi < \omega < \pi$.

Problem 2.

Consider the continuous-time signal $x_2(t) = \frac{\sin(4t)}{\pi t}$. A discrete-time signal is created by sampling $x_2(t)$ according to $x_2[n] = x_2(nT_s)$ for $T_s = \frac{2\pi}{6}$.

(i) Plot the magnitude of the DTFT of $x_2[n]$ over $-\pi < \omega < \pi$.

(ii) $x_2[n]$ is passed through a DT linear system with impulse response $h[n] = (-1)^n \left\{\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}\right\}$ yielding the output $y_{2}[n]$. Plot magnitude of the DTFT of $y_{2}[n]$ over $-\pi < \omega < \pi$.

(iii) $x_2[n]$ is passed through the DT linear system drawn below with impulse response $h_a[n] = (-1)^n \left\{\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}\right\}$ AND $h_b[n] = 2 \left\{\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}\right\}$ yielding the output denoted $y_{ab}[n]$. Plot magnitude of the DTFT of $y_{ab}[n]$ over $-\pi < \omega < \pi$.

![Diagram](attachment:image.png)
Problem 3.
Consider the continuous-time signal
\[ x_3(t) = x(t) * \left\{ \frac{\sin(2t)}{\pi t} \right\}, \]
where \(*\) denotes convolution and \(x(t)\) is the periodic signal \(x(t)\) below with period \(T = 3\pi\).

A discrete-time signal is created by sampling \(x_3(t)\) according to \(x_3[n] = x_3(nT_s)\) for \(T_s = \frac{2\pi}{4}\).

(i) Plot the magnitude of the DTFT of \(x_3[n]\) over \(-\pi < \omega < \pi\).

(ii) \(x_3[n]\) is passed through a DT linear system with impulse response \(h[n] = \frac{\sin(3\pi n)}{\pi n}\) yielding the output \(y_3[n]\). Plot magnitude of the DTFT of \(y_3[n]\) over \(-\pi < \omega < \pi\).

Problem 4.
The CT signal \(y(t)\) is obtained by passing the periodic input signal (period 3\pi)

through the system \(y(t) = \int_{t-\pi}^{t+\pi} x(\tau)d\tau\). NEXT, consider the CT signal \(x_4(t)\) defined in terms of \(y(t)\) as \(x_4(t) = y(t) * \left\{ \frac{\sin(2t)}{\pi t} \right\}\), where \(*\) denotes convolution. A DT signal is obtained by sampling \(x_4(t)\) according to \(x_4[n] = x_4(nT_s)\) for \(T_s = \frac{2\pi}{4}\).

(i) Plot the magnitude of the DTFT of \(x_4[n]\) over \(-\pi < \omega < \pi\).

(ii) \(x_4[n]\) is passed through a DT linear system with impulse response \(h[n] = \frac{\sin(3\pi n)}{\pi n}\) yielding the output \(y_4[n]\). Plot magnitude of the DTFT of \(y_4[n]\) over \(-\pi < \omega < \pi\).
Problem 5.
Consider the continuous-time signal below equal to a sum of four sinewaves.

\[ x(t) = \cos(3t) + \cos(6t) + \cos(9t) + \cos(12t) \]

A discrete-time signal is created by sampling \( x(t) \) according to \( x[n] = x(nT_s) \) for \( T_s = \frac{2\pi}{12} \).

(i) Plot the magnitude of the DTFT of \( x[n] \) over \(-\pi < \omega < \pi\).

(ii) \( x[n] \) is passed through a DT linear system described by the difference equation

\[ y[n] = \frac{1}{2}y[n - 1] + x[n] - 2x[n - 1] \]

Plot magnitude of the DTFT of the output \( y[n] \) over \(-\pi < \omega < \pi\).

Problem 6.
Consider the continuous-time signal \( x_6(t) = \left\{ \frac{\sin(4t)}{\pi t} \right\} \left\{ \frac{\sin(8t)}{\pi t} \right\} \). A DT signal is obtained by sampling \( x_6(t) \) according to \( x_6[n] = x_6(nT_s) \) for \( T_s = \frac{2\pi}{36} \).

(i) Plot the magnitude of the DTFT of \( x_6[n] \) over \(-\pi < \omega < \pi\).

(ii) \( x_6[n] \) is passed through a DT linear system with impulse response \( h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi n}{9})}{\pi n} \right\} \)

yielding the output \( y_{a1}[n] \). Plot magnitude of the DTFT of \( y_{a1}[n] \).

(iii) \( x_6[n] \) is passed through a DT linear system with impulse response \( h_b[n] = 2\cos\left(\frac{4\pi n}{9}\right) \left\{ \frac{\sin(\frac{2\pi n}{3})}{\pi n} \right\} \)

yielding the output \( y_{b1}[n] \). Plot magnitude of the DTFT of \( y_{b1}[n] \) over \(-\pi < \omega < \pi\).

(iv) \( x_6[n] \) is passed through a DT linear system with impulse response \( h_c[n] = \frac{\sin(\frac{2\pi n}{9})}{\pi n} \)

yielding the output \( y_{c1}[n] \). Plot magnitude of the DTFT of \( y_{c1}[n] \) over \(-\pi < \omega < \pi\).