SOLUTION

NAME: **NAME**
EE301 Signals and Systems **In-Class Exam**
Exam 3 **Thursday, Apr. 20, 2017**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 5,7
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
For Problem 4, plot your answers on the graphs provided.

VIP Note Regarding DTFT Plots: *The abscissa in each plot is the frequency axis. For each plot, the abscissa goes from -2π to 2π with tic marks every $\pi/8$. There is a dashed vertical line at $\omega = -\pi$ and another dashed vertical line at $\omega = +\pi$. You only have to plot any DTFT over $-\pi < \omega < \pi$.*

Problem 1: True False Questions. Circle the True (T) or False (F) for each part below.

(T) (F) The Nyquist rate for the square of a signal is twice the Nyquist rate of the original signal.

(T) (F) The highest frequency for the DTFT is effectively π, corresponding to half of the sampling rate, whereas the highest frequency for the CTFT is ∞ .

(T) (F) When you multiply a signal by a real-valued sinewave, the Nyquist rate of the resulting modulated signal is greater than the Nyquist rate for the original signal.

(T) (F) When you pass an analog signal through an LTI filter, the Nyquist rate for the output signal is greater than the Nyquist rate for the input signal.

(T) (F) The inverse DTFT is a summation over frequency whereas the inverse CTFT is an integration over frequency.

(T) (F) The DTFT is always periodic with period 2π whereas the CTFT is generally not periodic except under special conditions.

(T) (F) The time-domain variable for the DTFT is a discrete (integer-valued) variable for the DTFT whereas the time-domain variable for the CTFT is a continuous-valued variable.

(T) (F) When you form the product of two analog signals, the Nyquist rate for the product signal is the sum of the respective Nyquist rates for the two individual signals.

(T) (F) The DTFT is a summation over time whereas the CTFT is an integration over time.

(T) (F) The Nyquist rate for the derivative of a signal is greater than the Nyquist rate for the original signal.

Problem 2. Short answer questions.

(a) Briefly write and explain **ONE** of the main advantages of digital over analog, in terms of storage, transmission, and/or processing, in a coherent sentence.

1. Encoding sample amplitudes as 1's and 0's allows for error control/correction coding to automatically correct for bit errors due to noise and imperfections

2. Regeneration for digital communications

3. Flexibility and precision of digital signal processing after A/D conversion

(b) If one samples at a rate ω_s in radians/sec, what analog frequency is the frequency $\frac{\omega_s}{2} + \Delta\omega$ aliased to? Assume $0 < \Delta\omega < \frac{\omega_s}{2}$.

$$\frac{\omega_s}{2} + \Delta\omega - \omega_s = -\frac{\omega_s}{2} + \Delta\omega$$

If the signal is real-valued, e.g.,

$$\cos\left(\left(-\frac{\omega_s}{2} + \Delta\omega\right)t\right) = \cos\left(\left(\frac{\omega_s}{2} - \Delta\omega\right)t\right)$$

Thus: accept either

$$-\frac{\omega_s}{2} + \Delta\omega \quad \text{or} \quad \frac{\omega_s}{2} - \Delta\omega$$

Alternatively: aliasing starts at $\omega_s - \omega_M$

For a sinewave, $\omega_M =$ frequency of sinewave $= \frac{\omega_s}{2} + \Delta\omega$

Thus, aliased to:

$$\omega_s - \left(\frac{\omega_s}{2} + \Delta\omega\right) = \frac{\omega_s}{2} - \Delta\omega$$

Additional Note on Aliasing

$$\cos\left(\left(\frac{\omega_s}{2}+\Delta\omega\right)t\right)\Bigg|_{t=nT_s}$$

$$= \cos\left(\frac{\omega_s T_s}{2} n + \Delta\omega T_s n\right)$$

$$= \cos\left(\pi n + \Delta\omega T_s n\right)$$

$$= \cos\left(\pi n - 2\pi n + \Delta\omega T_s n\right)$$

$$= \cos\left(-\pi n + \Delta\omega T_s n\right)$$

$\cos(-\theta) = \cos(\theta)$

$$= \cos\left(\pi n - \Delta\omega T_s n\right)$$

now reconstruct ideal A/D: $n = \frac{t}{T_s}$

$$x_r(t) = x[n]\Big|_{n=\frac{t}{T_s}}$$

$$= \cos\left(\frac{\pi}{T_s} t - \Delta\omega T_s \, t/T_s\right)$$

$$= \cos\left(\left(\frac{\omega_s}{2} - \Delta\omega\right)t\right)$$

Problem 3. Consider the input signal $x_0(t)$ below.

$$x_0(t) = e^{-j35t}+e^{-j30t}+e^{-j25t}+e^{-j20t}+e^{-j15t}+e^{-j10t}+e^{-j5t}+1+e^{j5t}+e^{j10t}+e^{j15t}+e^{j20t}+e^{j25t}+e^{j30t}+e^{j35t}$$

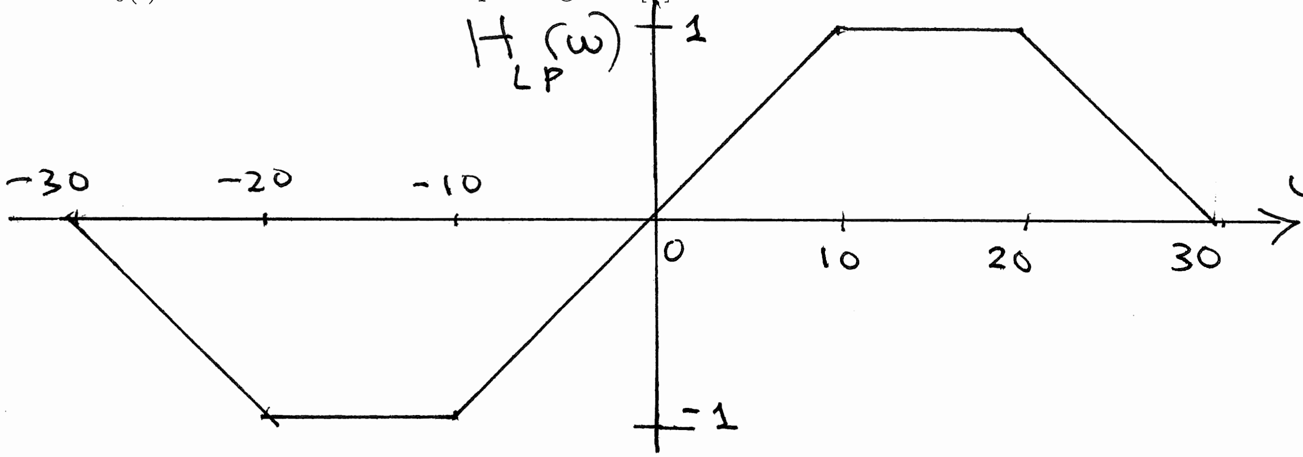
This signal is first input to an analog filter with impulse response

$$h_{LP}(t) = \frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\frac{\sin(10t)}{\pi t}\right\} 2j\sin(15t)$$

to form $x(t) = x_0(t) * h_{LP}(t)$, and then $x(t)$ is sampled at a rate of $\omega_s = 60$ to form $x[n]$, so that the time between samples is $T_s = \frac{2\pi}{60}$. The DT signal $x[n]$ thus obtained is then input to a DT LTI system with impulse response

$$h[n] = \left\{\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\right\} 2\cos\left(\frac{\pi}{2}n\right) \tag{1}$$

Show all work. Write your expression for the output $y[n] = x[n] * h[n]$ in the space below. Plot both the Fourier Transform of $h_{LP}(t)$ and the DTFT of $h[n]$ to help solve the problem, but since the input is a sum of sinewaves, it is not necessary to plot the Fourier Transform of $x_0(t)$ or the DTFT of the sampled signal $x[n]$.



$$e^{j\omega_0 t} \longrightarrow \boxed{H_{LP}(\omega)} \longrightarrow H(\omega_0)\, e^{j\omega_0 t}$$

$$x(t) = 0.1 + \frac{1}{2}e^{j5t} + e^{j10t} + e^{j15t} + e^{j20t} + \frac{1}{2}e^{j25t}$$
$$-\frac{1}{2}e^{-j5t} - e^{-j10t} - e^{-j15t} - e^{-j20t} - \frac{1}{2}e^{-j25t}$$

The frequencies ± 30 and ± 35 are zeroed out

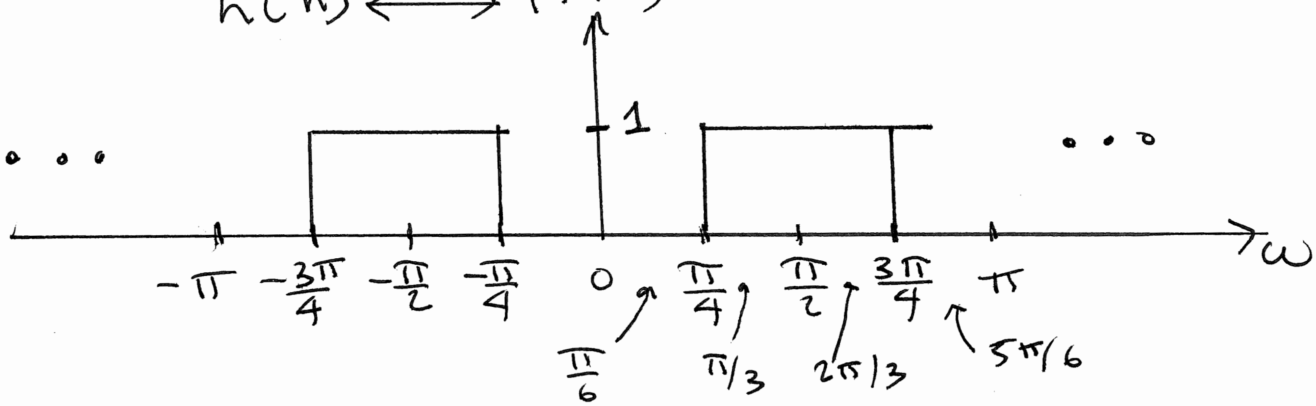
$$T_s = \frac{2\pi}{60} \Rightarrow \omega_d = \omega_a T_s \qquad \omega_d = k\,5\,\frac{2\pi}{60} = k\frac{\pi}{6}$$
$$|k| \le 5, \quad k \ne 0$$

Problem 3. You can continue your work for 3 here.

$$x[n] = \frac{1}{2} e^{j\frac{\pi}{6}n} + e^{j\frac{2\pi}{6}n} + e^{j\frac{3\pi}{6}n} + e^{j\frac{4\pi}{6}n} + \frac{1}{2} e^{j\frac{5\pi}{6}n}$$
$$- \frac{1}{2} e^{-j\frac{\pi}{6}n} - e^{-j\frac{\pi}{3}n} - e^{-j\frac{\pi}{2}n} - e^{-j\frac{2\pi}{3}n} - \frac{1}{2} e^{-j\frac{5\pi}{6}n}$$

$$e^{j\omega_0 n} \longrightarrow \boxed{h[n]} \longrightarrow H(\omega_0) e^{j\omega_0 n}$$

$$h[n] \xleftrightarrow{DTFT} H(\omega)$$



$$y[n] = e^{j\frac{\pi}{3}n} + e^{j\frac{\pi}{2}n} + e^{j\frac{2\pi}{3}n}$$
$$- e^{-j\frac{\pi}{3}n} - e^{-j\frac{\pi}{2}n} - e^{-j\frac{2\pi}{3}n}$$

Workout Problem 4. Consider the continuous-time signal $x_a(t)$ below. Note that the multiplication by the scalar j is included to make the Fourier Transform $X_a(\omega)$ be purely real-valued, and the multiplication by the scalar T_s is intended to offset the amplitude-scaling by the sampling rate $F_s = \frac{1}{T_s}$ that inherently occurs in the process of sampling.

$$x_a(t) = -jT_s \frac{1}{10}\frac{d}{dt}\left\{\frac{\sin(20t)}{\pi t}\right\}$$

(a) A discrete-time signal is created by sampling $x_a(t)$ according to $x[n] = x_a(nT_s)$ for $T_s = \frac{2\pi}{50}$. Plot the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show your work on this page and the next page, and do your plot in the space provided on the next page.

(b) Repeat part (a) for $T_s = \frac{2\pi}{25}$. Plot the new DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show your work and do your plot in the space provided on the sheets attached.

$$X_a(\omega) = -j\, T_s \frac{1}{10}\, j\omega \;\text{rect}\left(\frac{\omega}{40}\right)$$

$$= T_s \frac{\omega}{10}\;\text{rect}\left(\frac{\omega}{40}\right)$$

$$X_s(\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_a(\omega - k\omega_s)$$

$$= \sum_{k=-\infty}^{\infty}\left(\frac{\omega - k\omega_s}{\cancel{40}\ \text{ten}}\right)\text{rect}\left(\frac{\omega - k\omega_s}{40}\right)$$

$$X_d(\omega) = X_s(F_s\omega) = X_s\left(\frac{\omega}{T_s}\right)$$ } compression by sampling rate in Hertz

DTFT

$$\omega_d = \frac{\omega_a}{F_s} = \omega_a T_s$$

(a)

For: $T_s = \frac{2\pi}{50} \Rightarrow \omega_s = 50 > 2\omega_M$ where $\omega_M = 20$

$\Rightarrow$ no aliasing $\Rightarrow$ only $k=0$ term contributes in $-\pi < \omega < \pi$

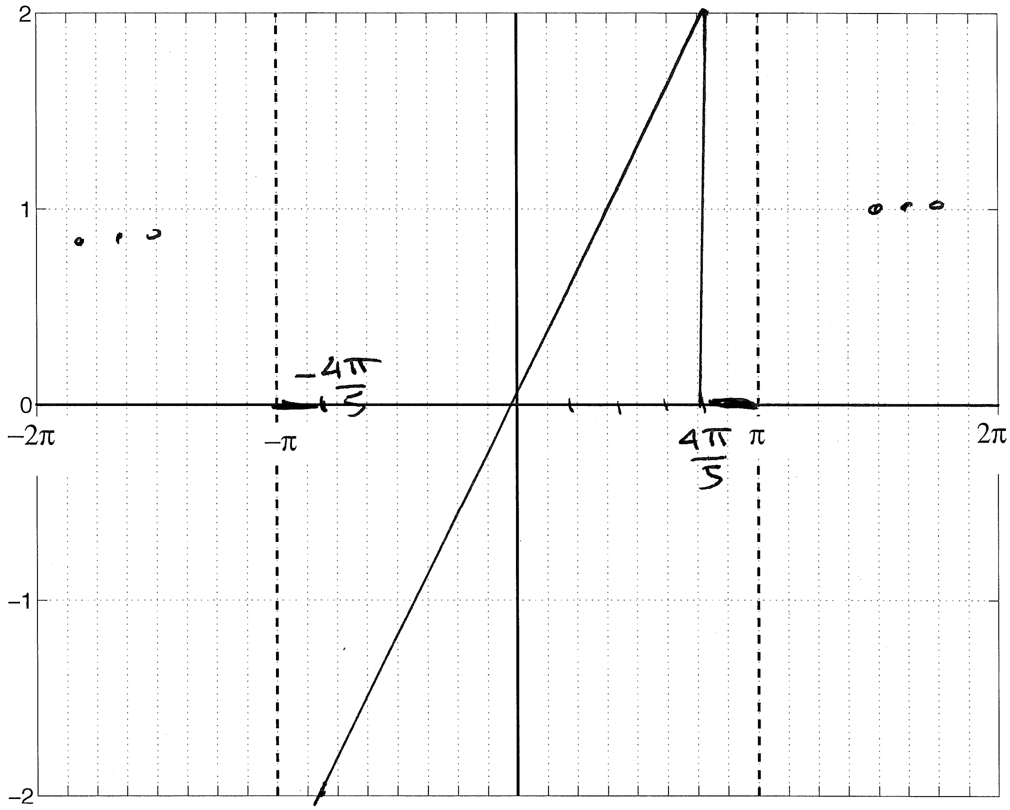
$$X_d(\omega) = \frac{1}{10}\frac{\omega}{T_s}\,\text{rect}\left(\frac{\omega}{T_s 40}\right) = \frac{1}{10}\frac{25}{\pi}\,\omega\;\text{rect}\left(\frac{\omega}{8\pi/5}\right)$$

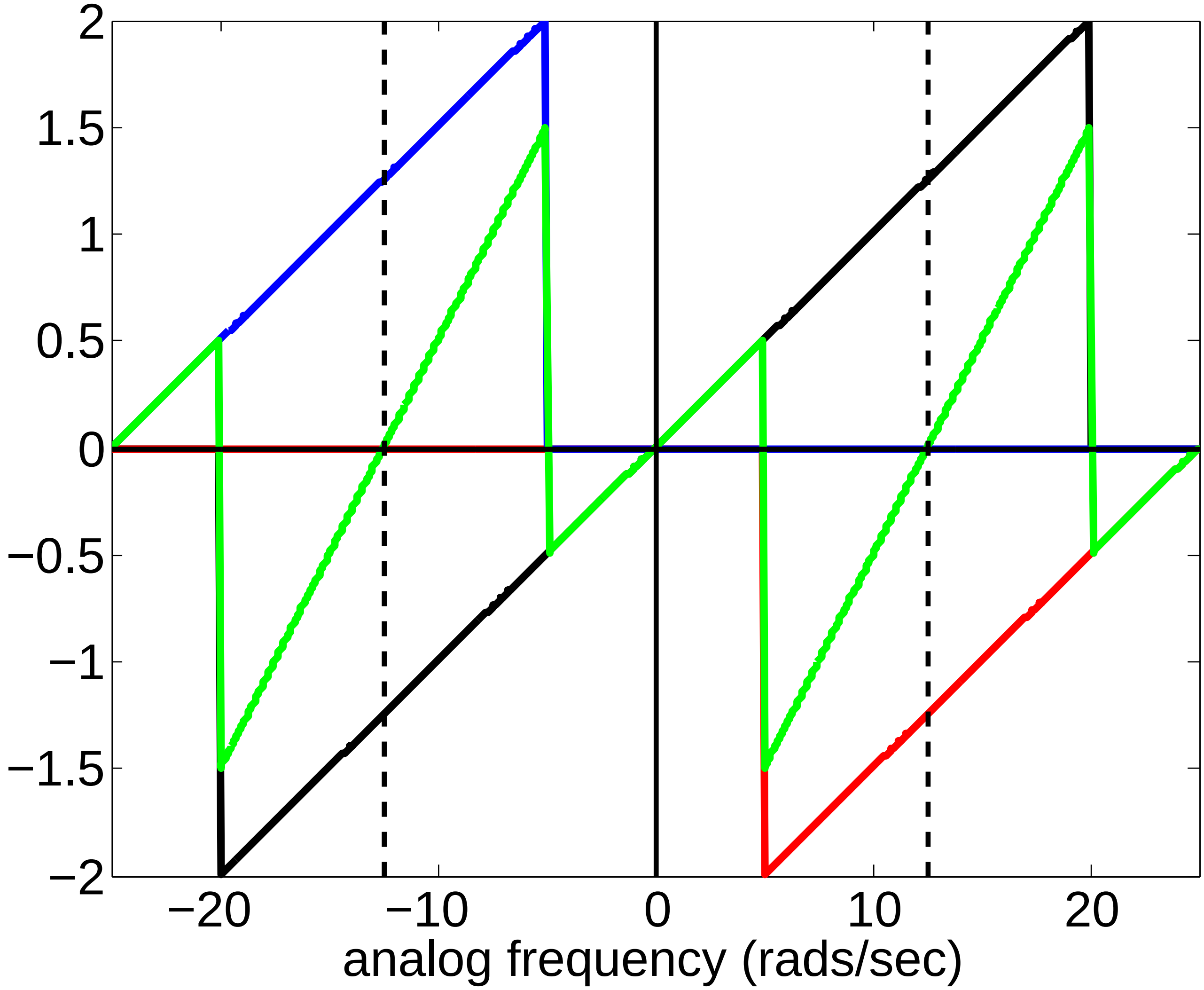
$$\boxed{1/T_s = \frac{50}{2\pi} = \frac{25}{\pi}}\qquad \boxed{40\,\frac{2\pi}{50} = \frac{8\pi}{5}}$$

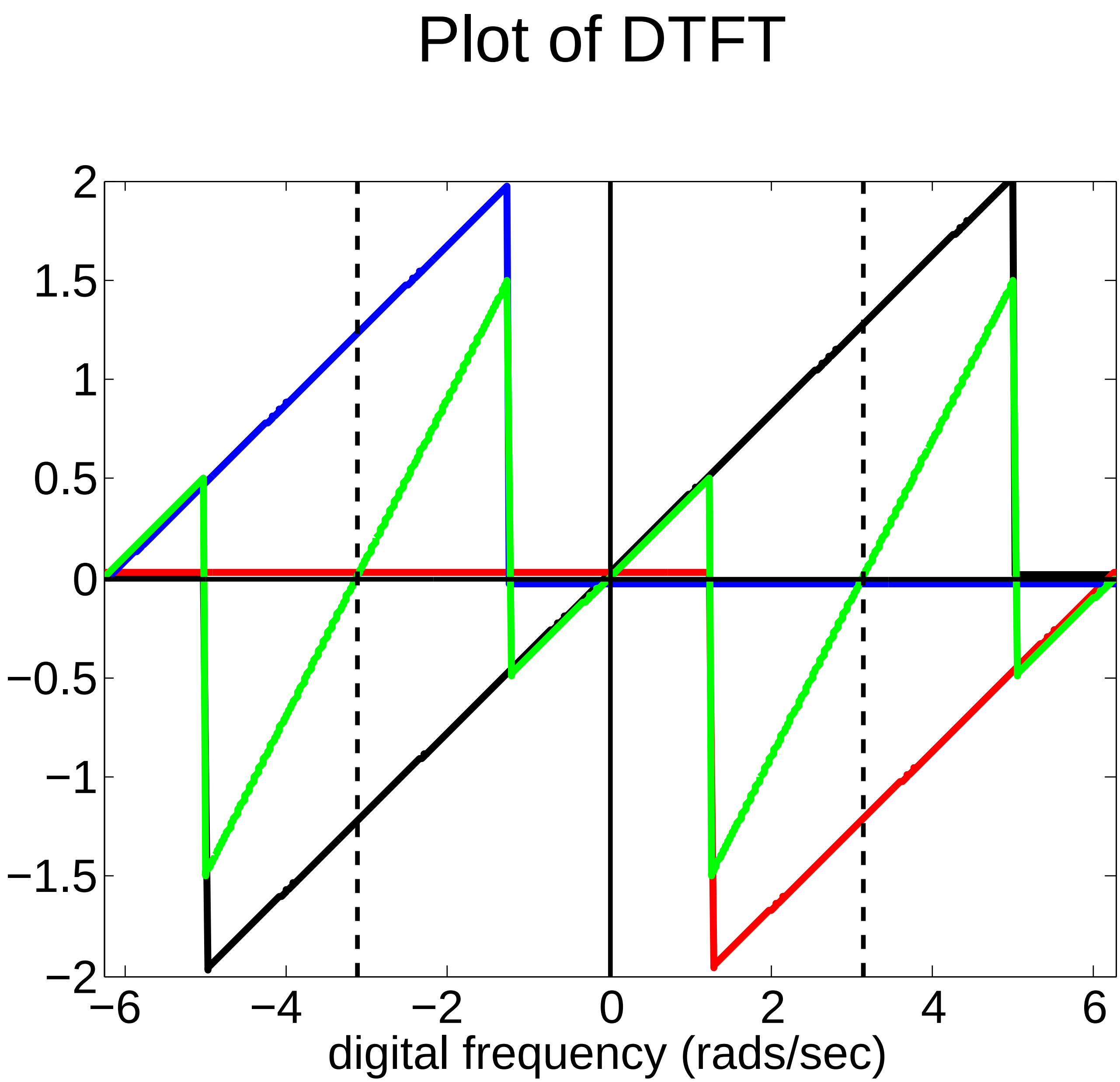
$\omega_M = 20$ mapped to $20T_s = 20\,\frac{2\pi}{50} = \frac{4\pi}{5}$

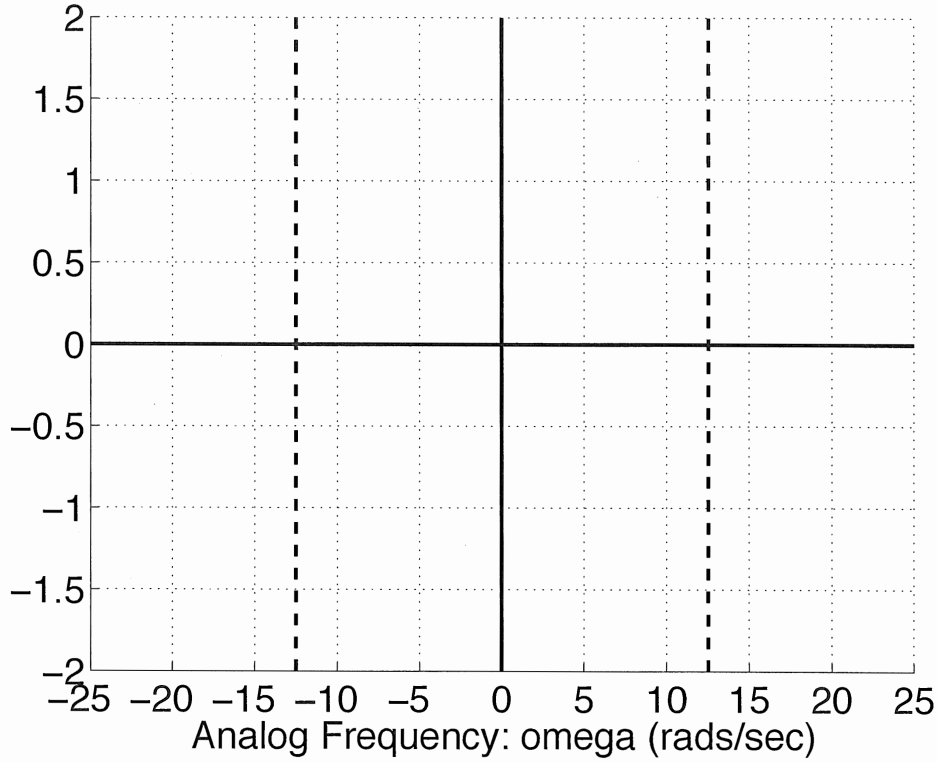
Plot your answer to Problem 4 (a) here. Show work .
Plot your answer to Problem 4 (b) on next page .

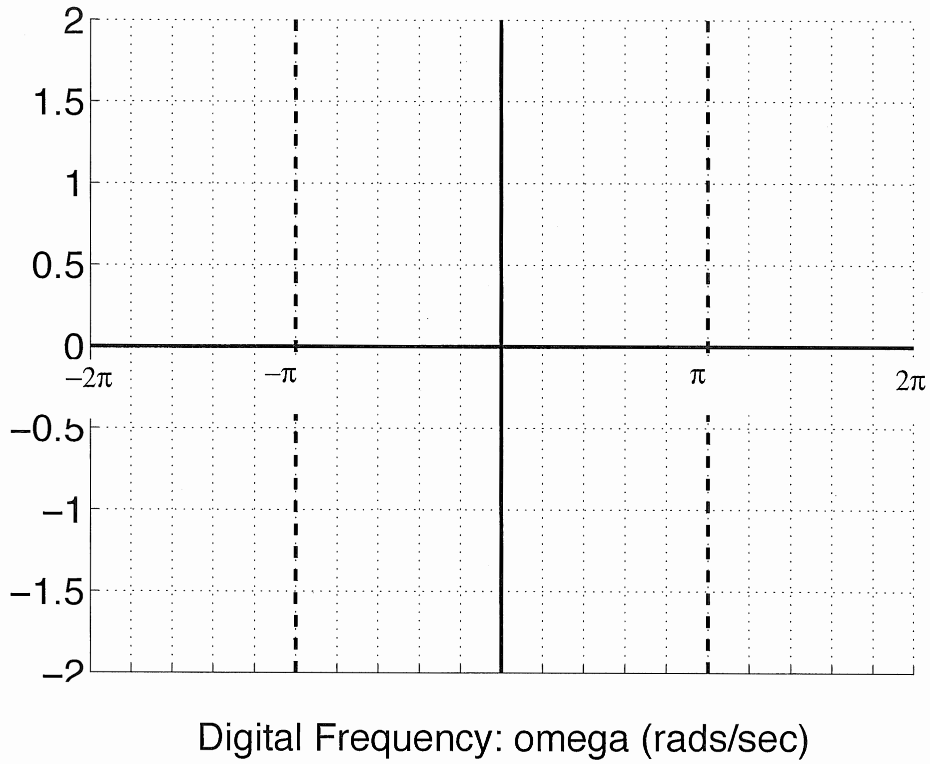
(a)




2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−20
−10
0
10
20
analog frequency (rads/sec)


Plot of DTFT
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−6
−4
−2
0
2
4
6
digital frequency (rads/sec)


2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−25 −20 −15 −10 −5 0 5 10 15 20 25
Analog Frequency: omega (rads/sec)


2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2π
−π
π
2π
Digital Frequency: omega (rads/sec)