

Cover Sheet

Test Duration: 75 minutes.

Coverage: Chaps. 7 and 5, but will need Chap. 4 material
Open Book but Closed Notes. One two-sided handwritten sheet.

Calculators NOT allowed.

This test contains **one** long problem, with many parts.

All work should be done in the blue books provided.

You must show all work for each problem to receive full credit.

Do **not** return this test sheet, just return the blue books.

NOTE: All the signals in this exam have a purely real-valued Fourier Transform, but there may be frequency bands for which the Fourier Transform is negative-valued. Since each Fourier Transform is real-valued, I am asking you to plot the DTFT showing the regions where it is negative. You do NOT have to plot the magnitude.

Problem 1.

- (a) Consider the continuous-time signal $x_1(t)$ below. A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ for $T_s = \frac{2\pi}{40}$. Plot the DTFT of $x_1[n]$, $X_1(\omega)$, over $-\pi < \omega < \pi$.

$$x_1(t) = j T_s 2\pi t \left\{ \frac{\sin(5t)}{\pi t} \right\}^2$$

- (b) Repeat part (a) for $T_s = \frac{2\pi}{15}$.

- (c) Consider the continuous-time signal $x_2(t)$ below. A discrete-time signal is created by sampling $x_2(t)$ according to $x_2[n] = x_2(nT_s)$ for $T_s = \frac{2\pi}{30}$. Plot the DTFT of $x_2[n]$, $X_2(\omega)$, over $-\pi < \omega < \pi$.

$$x_2(t) = -j T_s \frac{d}{dt} \left\{ \frac{\sin(10t)}{\pi t} \right\}$$

- (d) Repeat part (c) for $T_s = \frac{2\pi}{15}$.

- (e) Consider the continuous-time signal $x_3(t)$ below. A discrete-time signal is created by sampling $x_3(t)$ according to $x_3[n] = x_3(nT_s)$ for $T_s = \frac{2\pi}{20}$. Plot the DTFT of $x_3[n]$, $X_3(\omega)$, over $-\pi < \omega < \pi$.

$$x_3(t) = j T_s \frac{1}{2} \left\{ \frac{\sin(10(t - \frac{\pi}{10}))}{\pi(t - \frac{\pi}{10})} - \frac{\sin(10(t + \frac{\pi}{10}))}{\pi(t + \frac{\pi}{10})} \right\}$$

- (f) Repeat part (e) for $T_s = \frac{2\pi}{15}$.

- (g) Consider the continuous-time signal $x_4(t)$ below. A discrete-time signal is created by sampling $x_4(t)$ according to $x_4[n] = x_4(nT_s)$ for $T_s = \frac{2\pi}{60}$. Plot the DTFT of $x_4[n]$, $X_4(\omega)$, over $-\pi < \omega < \pi$.

$$x_4(t) = T_s \frac{\pi}{2} \left\{ \frac{\sin(2t)}{\pi t} \frac{\sin(8t)}{\pi t} \right\}$$

- (h) Repeat part (g) for $T_s = \frac{2\pi}{18}$.

- (i) Repeat part (g) for $T_s = \frac{2\pi}{16}$.

- (j) Consider the continuous-time signal $x_5(t)$ below. A discrete-time signal is created by sampling $x_5(t)$ according to $x_5[n] = x_5(nT_s)$ for $T_s = \frac{2\pi}{48}$. Plot the DTFT of $x_5[n]$, $X_5(\omega)$, over $-\pi < \omega < \pi$.

$$x_5(t) = \cos(6t) + \cos(12t) + \cos(15t) + \cos(18t) + \cos(21t) + \cos(24t)$$

- (k) Repeat part (j) for $T_s = \frac{2\pi}{24}$.