

Cover Sheet

Test Duration: 75 minutes.

Coverage: Chaps. 5 and 7

Open Book but Closed Notes (NO LOOSE SHEETS)

Calculators NOT allowed.

This test contains **two** problems.

All work should be done in the blue books provided.

You must show all work for each problem to receive full credit.

Do **not** return this test sheet, just return the blue books.

Problem 1. For EACH part of this problem, plot the magnitude $|X(e^{j\omega})|$ of the DTFT of the sampled signal $x[n]$ over $-\pi < \omega < \pi$. Show as much detail as possible.

(a) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{9}$ and $x_a(t) = T_s \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\}$.

(b) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{6}$ and $x_a(t) = T_s \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\}$.

(c) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{5}$ and $x_a(t) = T_s \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\}$.

(d) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{4}$ and $x_a(t) = T_s \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\}$.

(e) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{12}$ and $x_a(t) = 2T_s \left\{ \frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t} \right\} \cos(3t)$.

(f) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{5}$ and $x_a(t) = 2T_s \left\{ \frac{\sin(t)}{\pi t} + \cos(2t) \right\}^2$.

(g) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{32}$ and $x_a(t) = 2T_s \left\{ \frac{\sin(t)}{\pi t} \frac{\sin(3t)}{\pi t} \right\} \cos(4t)$.

PROCEED TO NEXT PAGE FOR PROBLEM 2.

Problem 2. Consider the discrete-time LTI system described by the following simple difference equation.

$$y[n] = x[n] - x[n - 1] \quad (1)$$

- (a) Determine the impulse response of this system, $h[n]$. Plot $h[n]$ (stem plot).
- (b) Determine and write a closed-form expression for the DTFT, $H(e^{j\omega})$, of $h[n]$. $H(e^{j\omega})$ is the frequency response of the system.
- (c) Plot the magnitude $|H(e^{j\omega})|$ over $-\pi < \omega < \pi$.
- (d) Plot the phase $\angle H(e^{j\omega})$ over $-\pi < \omega < \pi$.

$$x_a(t) = 3(1 - |t|)\{u(t + 1) - u(t - 1)\} \quad (2)$$

Observe $x_a(t)$ has a triangle shape of height 3 and of duration two seconds center at $t = 0$. Let $x[n] = x_a(nT_s)$ where $T_s = \frac{1}{3}$. That is, $x[n]$ obtained by sampling $x_a(t)$ at a rate of three samples per second.

- (e) Plot $x[n]$ (stem plot).
- (f) Determine and write a closed-form expression for the DTFT, $X(e^{j\omega})$, of $x[n]$.
- (g) Plot the magnitude $|X(e^{j\omega})|$ over $-\pi < \omega < \pi$.
- (h) Determine the output signal $y[n]$ when the sampled signal $x[n]$ is input to the system $y[n] = x[n] - x[n - 1]$. Plot $y[n]$ (stem plot).
- (i) Determine and write a closed-form expression for the DTFT, $Y(e^{j\omega})$, of $y[n]$.
- (j) Plot the magnitude $|Y(e^{j\omega})|$ of the DTFT of $y[n]$ over $-\pi < \omega < \pi$.
- (k) Determine the numerical value of $\sum_{n=-\infty}^{\infty} y^2[n]$. Show all work.