

Cover Sheet

Test Duration: 70 minutes.

Coverage: Chaps. 5 and 7

Open Book but Closed Notes (NO LOOSE SHEETS)

Calculators NOT allowed.

This test contains **two** problems.

All work should be done in the blue books provided.

You must show all work for each problem to receive full credit.

Do **not** return this test sheet, just return the blue books.

Problem 1. Consider the discrete-time LTI system described by the following difference equation.

$$y[n] = x[n] - x[n - 4] \quad (1)$$

- (a) Plot the magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$ (two separate plots) of the frequency response of this system, equal to the DTFT $H(e^{j\omega})$ of the impulse response $h[n]$, as a function of frequency over $-\pi < \omega < \pi$. Show as much detail as possible.

For EACH of the remaining parts of this problem, you must do EACH of the following THREE steps. For all parts, you MUST show all work and explain how you got your answer concisely, but with sufficient detail to receive full credit.

- (i) Plot the magnitude $|X(e^{j\omega})|$ of the DTFT of the sampled signal $x[n]$ which is input to the discrete-time system described by equation (1) above.
- (ii) Plot the magnitude $|Y(e^{j\omega})|$ of the DTFT of the corresponding output signal $y[n]$.
- (iii) Determine the numerical value of $\sum_{n=-\infty}^{\infty} y^2[n]$. Show all work.

(b) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{40}$ and $x_a(t) = T_s \left\{ \frac{\sin(10t)}{\pi t} \right\}$.

(c) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{40}$ and $x_a(t) = 2T_s \left\{ \frac{\sin(5t)}{\pi t} \right\} \cos(15t)$.

(d) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{20}$ and $x(t) = T_s \left\{ \frac{\sin(10t)}{\pi t} \right\}$.

(e) $x[n] = x_a(nT_s)$ where $T_s = \frac{3\pi}{20}$ and $x(t) = T_s \left\{ \frac{\sin(10t)}{\pi t} \right\}$.

(f) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{40}$ and $x_a(t) = 1 + \cos(10t) + \cos(20t)$.

(g) $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{40}$ and

$$x_a(t) = 1 + 2 \cos(5t + 45^\circ) + 3 \cos(10t + 90^\circ) + 4 \cos(15t + 135^\circ) + 5 \cos(20t + 180^\circ)$$

PROCEED TO NEXT PAGE FOR PROBLEM 2.

Problem 2. Consider the discrete-time LTI system described by the following difference equation.

$$y[n] = x[n] + x[n - 1] + x[n - 2] + x[n - 3] \quad (2)$$

- (a) Plot the magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$ (two separate plots) of the frequency response of this system (equal to the DTFT $H(e^{j\omega})$ of the impulse response $h[n]$) as a function of frequency over $-\pi < \omega < \pi$. Show as much detail as possible.
- (b) For $x[n] = x_a(nT_s)$ where $T_s = 3$ and $x_a(t) = u(t) - u(t - 10)$, do the following:
- Plot the magnitude $|X(e^{j\omega})|$ of the DTFT of the sampled signal $x[n]$ which is input to the discrete-time system described by equation (2) above.
 - Plot the magnitude $|Y(e^{j\omega})|$ of the DTFT of the corresponding output signal $y[n]$.
 - Determine the numerical value of $\sum_{n=-\infty}^{\infty} y^2[n]$. Show all work.