SOLUTION

EE301 Signals and Systems — Spring 2017

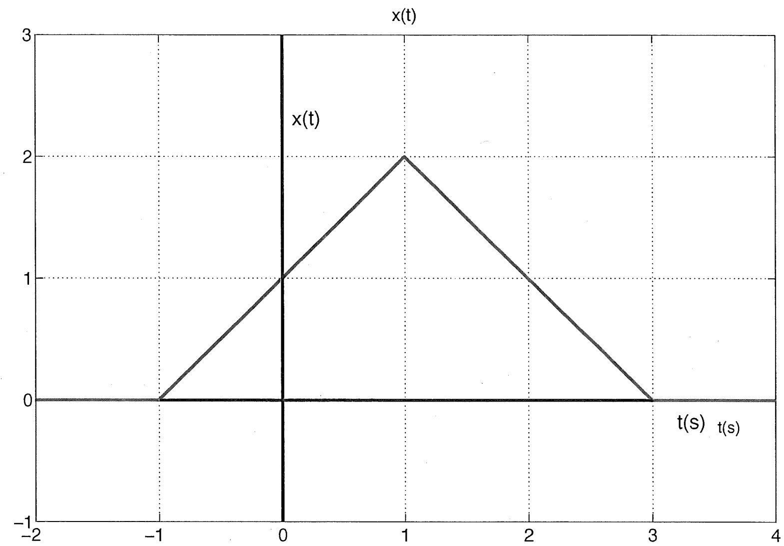
Exam 2 — Thursday, 4:30-5:45 pm, Mar. 30, 2017

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chapter 4, Hmwks 6-7
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet (both sides)
Calculators NOT allowed.
All work should be done on the sheets provided.
You can NOT do work on the back of a page unless permission is granted.
No work on the back of a page will be graded unless permission is granted.
You must show all work for each problem to receive full credit.

Additional Directions

There are four problems, each with multiple parts.
Do each part in order and clearly mark your answer for each part.



Problem 1. Consider the time-domain signal $x(t)$ plotted above. Using the properties of the Fourier Transform, you are able to compute all of the quantities required in each of the parts below without having to determine the corresponding Fourier Transform, $X(\omega)$. Indicate which properties you are using and clearly label your final answer for each part.

(a) Find $X(0)$. (Find the value of the Fourier Transform at $\omega = 0$.)

(b) Evaluate $\int_{-\infty}^{\infty} X(\omega)d\omega$.

(c) Evaluate $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$.

(d) Plot the phase of the Fourier Transform, $\angle X(\omega)$. That is, $X(\omega) = |X(\omega)|e^{\angle X(\omega)}$; plot $\angle X(\omega)$

(e) Find the value below, where $H(\omega) = 2\frac{\sin(\omega)}{\omega}$

$$\int_{-\infty}^{\infty} X(\omega)H(\omega)e^{j2\omega}d\omega = ??$$

(a) $X(0) = \int_{-\infty}^{\infty} x(t)\,dt$

$= 2\,\frac{1}{2}\cdot 2\cdot 2 = 4$

(b) $\int_{-\infty}^{\infty} X(\omega)\,d\omega = x(t)\Big|_{t=0} 2\pi = 1\cdot 2\pi = 2\pi$

(c) $= \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty} x^2(t)\,dt$

$= 2\pi\int_0^2 2x^2(t+1)\,dt = 4\pi\int_0^2 (-(t-2))^2\,dt = 4\pi\left[\frac{(t-2)^3}{3}\right]_0^2$

since $x(t+1)$ is symmetric about $t=0$

$= 4\pi\left\{\frac{8}{3}\right\} = \frac{32\pi}{3}$ ANS

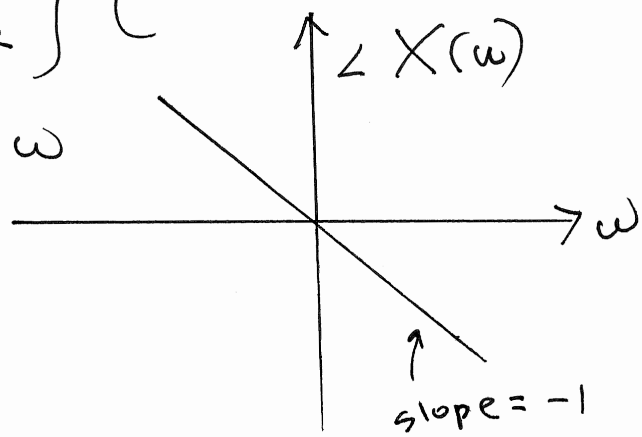
Additional Space for Problem 1 answer and work.

(d) $x(t) = x_e(t-1)$ where $x_e(t)$ is the same triangle centered at $t=0$; $x_e(t)$ is even-symmetric

$$x_e(t) = \text{rect}\left(\frac{t}{2}\right) * \text{rect}\left(\frac{t}{2}\right) \overset{\mathcal{F}}{\longleftrightarrow} \underbrace{\left\{\frac{\sin(\omega)}{\omega/2}\right\}^2}_{\geq 0 \text{ for all } \omega \text{, i.e., strictly non-negative}}$$

$$X(\omega) = \left\{\frac{\sin(\omega)}{\omega/2}\right\}^2 e^{-j\omega}$$

$$\angle X(\omega) = -\omega$$

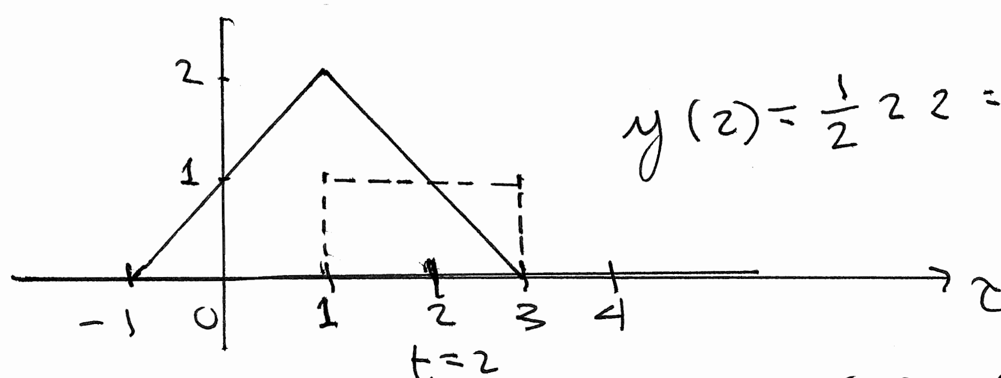


e) $$2\pi\, y(t) = \int_{-\infty}^{\infty} X(\omega) H(\omega) e^{j\omega t}\, d\omega$$

$$2\pi\, y(2) = \int_{-\infty}^{\infty} X(\omega) H(\omega) e^{j2\omega}\, d\omega$$

where:
$y(t) = x(t) * h(t)$
$h(t) = \text{rect}\left(\frac{t}{2}\right)$

$$= x(t) * h(t)\Big|_{t=2} = x(t) * \text{rect}\left(\frac{t}{2}\right)\Big|_{t=2}$$



$$y(2) = \frac{1}{2}\, 2\, 2 = 2$$

$y(2) = 2$ ANS.

final answer is: 2 (2π)= 4 π

Problem 2 You are given the Fourier Transform pair below, which was derived in homework. Show all work and clearly mark your answers for parts (a), (b), and (c) in the space provided.

$$\{1+\cos(\pi t)\}\,\mathrm{rect}\left(\frac{t}{2}\right) \longleftrightarrow 2\frac{\sin(\omega)}{\omega}\,\frac{\pi^2}{\pi^2-\omega^2}$$

(a) Determine the Fourier Transform, $X_a(\omega)$, of the signal below. Indicate which propertie(s) you are using.

$$x_a(t) = \frac{\sin(t)}{\pi t}\,\frac{\pi^2}{\pi^2-t^2}$$

(b) Determine and plot the Fourier Transform, $X_b(\omega)$, of the signal below. Indicate which propertie(s) you are using.

$$x_b(t) = W\frac{\sin(Wt)}{\pi W t}\,\frac{\pi^2}{\pi^2-(Wt)^2} = \frac{\sin(Wt)}{\pi t}\,\frac{\pi^2}{\pi^2-(Wt)^2}$$

(c) Determine and plot the Fourier Transform, $X_c(\omega)$, of the signal below. Indicate which propertie(s) you are using.

$$x_c(t) = \frac{d}{dt}x_b(t) = \frac{d}{dt}\left\{\frac{\sin(Wt)}{\pi t}\,\frac{\pi^2}{\pi^2-(Wt)^2}\right\}$$

(a) Duality Prop:

$$x_a(t) \xleftrightarrow{\mathcal{F}} X_a(\omega) = \frac{2\pi}{2\pi}\{1+\cos(\pi(-\omega))\}\,\mathrm{rect}\left(\frac{-\omega}{2}\right)$$

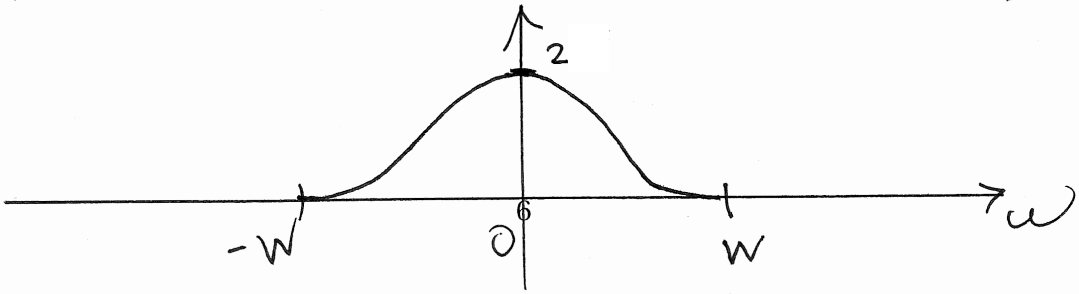
$$= \frac{2\pi}{2\pi}\{1+\cos(\pi\omega)\}\,\mathrm{rect}\left(\frac{\omega}{2}\right)$$

(b) Time-Scaling Prop:

$$\tilde{x}_b(t) = x_a(Wt) = \frac{\sin(Wt)}{\pi W t}\cdot\frac{\pi^2}{\pi^2-(Wt)^2}$$

$$\xleftrightarrow{\mathcal{F}} \tilde{X}_b(\omega) = \frac{1}{W}\left\{1+\cos\left(\frac{\pi\omega}{W}\right)\right\}\mathrm{rect}\left(\frac{\omega}{2W}\right)$$

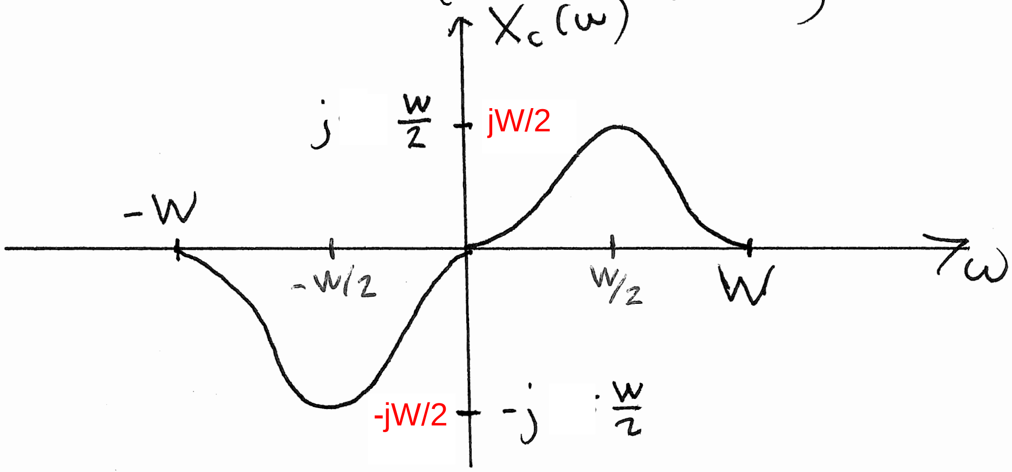
$$x_b(t) = W\tilde{x}_b(t) \xleftrightarrow{\mathcal{F}} X_b(\omega) = \left\{1+\cos\left(\frac{\pi\omega}{W}\right)\right\}\mathrm{rect}\left(\frac{\omega}{2W}\right)$$



Additional Space for Problem 2 answer and work.

(c) $x_c(t) = \frac{d}{dt} x_b(t) \overset{\mathcal{F}}{\longleftrightarrow} X_c(\omega) = j\omega X_b(\omega)$

$$X_c(\omega) = j\omega\left\{1 + \cos\left(\frac{\pi\omega}{W}\right)\right\}\text{rect}\left(\frac{\omega}{2W}\right)$$



Problem 3. The signal

$$x(t) = 2 + \sqrt{2}\cos(5t) + \cos(10t) + \sqrt{2}\cos(15t) + 3\cos(20t + \pi/3) + 7\cos(25t - \pi/6)$$

is input to an LTI system with impulse response

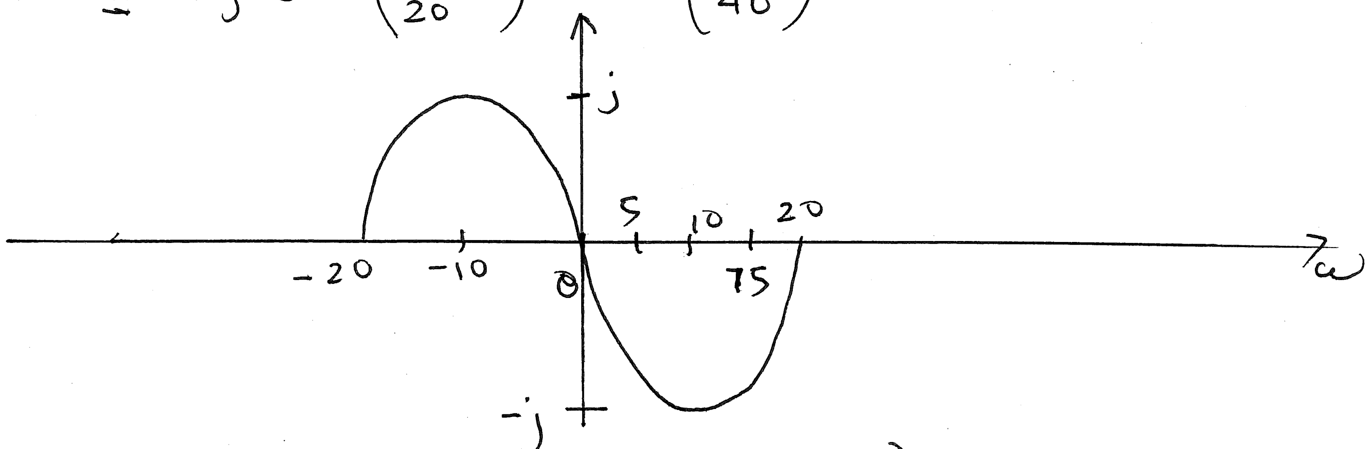
$$h(t) = \frac{1}{2}\left\{ \frac{\sin\left(20\left(t - \frac{\pi}{20}\right)\right)}{\pi\left(t - \frac{\pi}{20}\right)} - \frac{\sin\left(20\left(t + \frac{\pi}{20}\right)\right)}{\pi\left(t + \frac{\pi}{20}\right)} \right\}$$

Determine the output $y(t) = x(t) * h(t)$. Show all work and clearly indicate your final answer for $y(t)$ (in the time domain.) The following values may be useful:

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \qquad \sin\left(\frac{\pi}{2}\right) = 1 \qquad \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} \qquad \sin(-\theta) = -\sin(\theta)$$

$$H(\omega) = \frac{1}{2}\,\text{rect}\left(\frac{\omega}{40}\right)\left\{e^{-j\frac{\pi}{20}\omega} - e^{j\frac{\pi}{20}\omega}\right\}\frac{j}{j}$$

$$= -j\sin\left(\frac{\pi}{20}\omega\right)\text{rect}\left(\frac{\omega}{40}\right)$$



$H(0) = 0$ (DC rejected or notched out)

$$H(-5) = -j\sin\left(-\frac{5\pi}{20}\right) = -j\sin\left(-\frac{\pi}{4}\right) = j\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}e^{j\frac{\pi}{2}}$$

$$H(5) = H^*(-5) = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{2}}$$

$$H(10) = -j\sin\left(10\pi/20\right) = -j\sin\left(\frac{\pi}{2}\right) = -j = 1e^{-j\frac{\pi}{2}}$$

-ten

$$H(-10)\cancel{H(10)} = H(-10) = H^*(+10) = j = e^{j\frac{\pi}{2}}$$

$$H(15) = -j\sin\left(\frac{15}{20}\pi\right) = -j\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{2}}$$

$$H(-15) = H^*(15) = \frac{1}{\sqrt{2}}e^{j\frac{\pi}{2}}$$

$H(20) = 0 \qquad H(25) = 0$

Additional Space for Problem 3 answer and work.

$$y(t) = \sqrt{2}\,\frac{1}{\sqrt{2}}\cos\left(5t - \frac{\pi}{2}\right)$$

$$+ \cos\left(10t - \frac{\pi}{2}\right)$$

$$+ \sqrt{2}\,\frac{1}{\sqrt{2}}\cos\left(15t - \frac{\pi}{2}\right)$$

$$= \cos\left(5t - \frac{\pi}{2}\right) + \cos\left(10t - \frac{\pi}{2}\right) + \cos\left(15t - \frac{\pi}{2}\right)$$

Problem 4.

(a) The signal

$$x(t) = e^{\frac{-t^2}{2}}\cos(10t) - \left\{ e^{\frac{-t^2}{2}} * \frac{1}{\pi t} \right\} \sin(10t)$$

Determine and plot the Fourier Transform, $X(\omega)$, of $x(t)$. Briefly explain your answer.

(b) Consider $y(t)$ formed as:

$$y(t) = 2x(t)\cos(10t)$$

Determine and plot the Fourier Transform, $Y(\omega)$, of $y(t)$. You should use the following trigonometric identities:

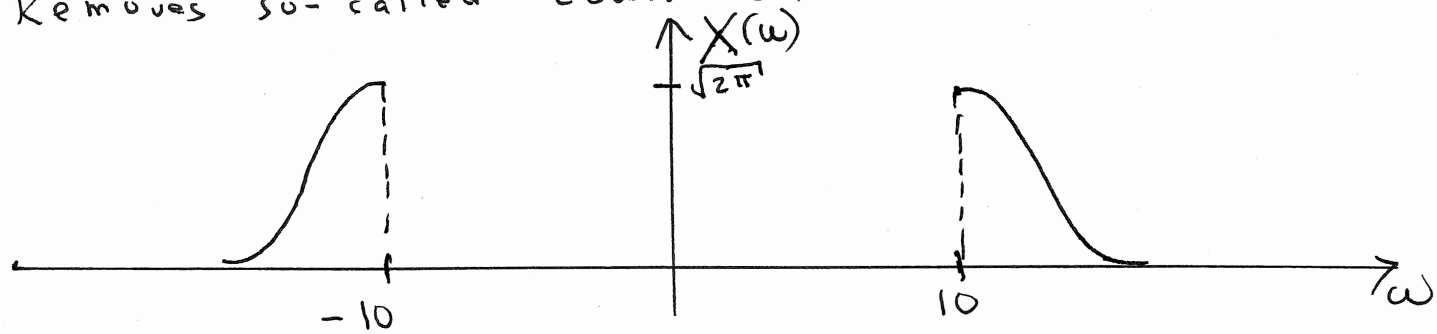
$$2\cos^2(A) = 1 + \cos(2A)$$

$$2\sin(A)\cos(A) = \sin(2A)$$

Briefly explain your answer.

(a) Gaussian signal = standard deviation = 1 = σ

Removes so-called Lower Sideband



(b) Shift to the left and right by 10 OR

use trig - identities

$$y(t) = x_g(t) + x_g(t)\cos(20t) - \underbrace{\hat{x}_g}(t)\sin(20t)$$

$$x_g(t) = e^{-\frac{t^2}{2}} \quad \text{and} \quad \hat{x}_g(t) = e^{-\frac{t^2}{2}} * \frac{1}{\pi t}$$