Solution to Exam 2 Spring 2010

ECE 301

Solution to Problem 1

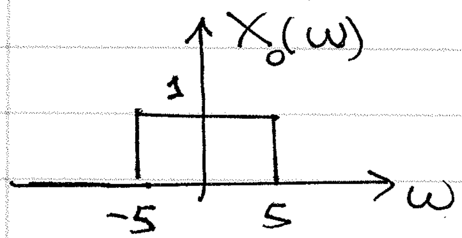
(a) can be solved multiple ways. Easiest way

$$x_1(t) = 2\pi t \left\{ \frac{\sin(5t)}{\pi t} \right\}^2$$

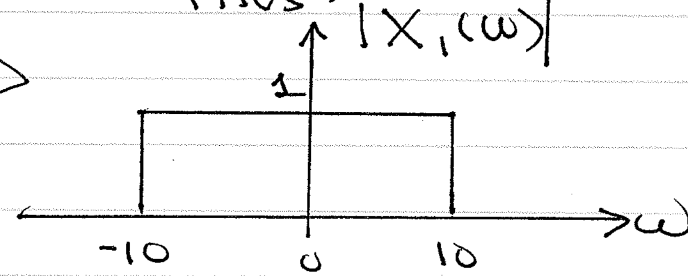
$$= \frac{\sin(5t)}{\pi t} \cdot 2\sin(5t)$$

$$X_1(\omega) = 2\left\{ \frac{1}{2j} X_0(\omega - 5) - \frac{1}{2j} X_0(\omega + 5) \right\}$$

where:



$\Rightarrow$ THUS:



(b) $x_2(t) = \frac{1}{2} z_0\left(t - \frac{\pi}{10}\right) - \frac{1}{2} z_0\left(t + \frac{\pi}{10}\right)$

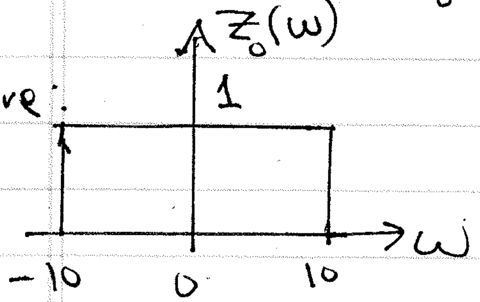
just rewrote $\left\{ = \frac{1}{2} z_0\left(t - \frac{\pi}{10}\right) - \frac{1}{2} z_0\left(t + \frac{\pi}{10}\right) \right.$

$$\boxed{z(t) = \frac{\sin 10t}{\pi t}}$$

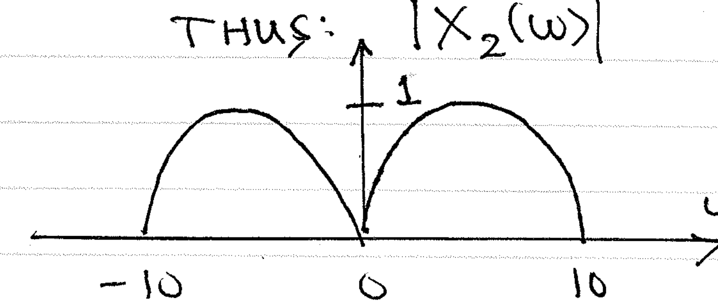
$$X_2(\omega) = Z_0(\omega) \frac{1}{2} \left\{ e^{-j\omega\frac{\pi}{10}} - e^{+j\omega\frac{\pi}{10}} \right\} \frac{j}{j}$$

$$= Z_0(\omega)(-j)\sin\left(\omega \frac{\pi}{10}\right)$$

where:

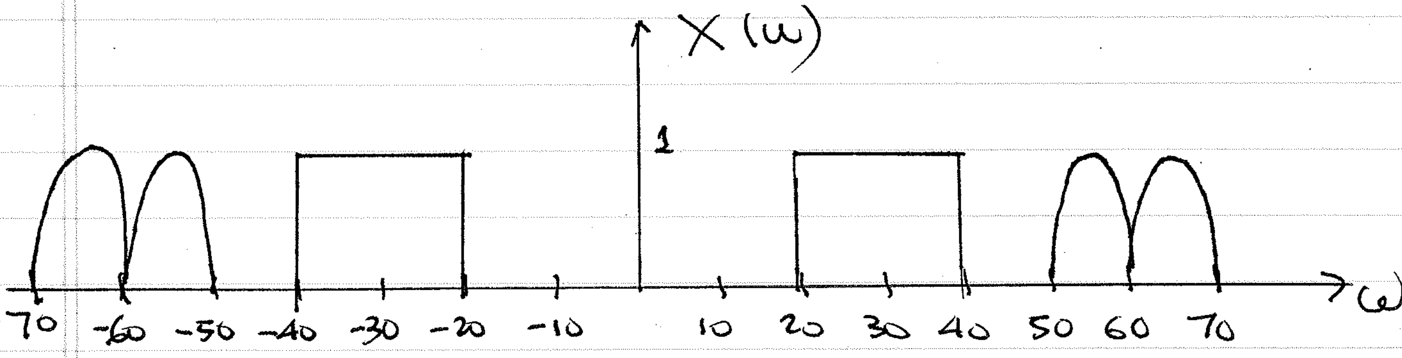


THUS:

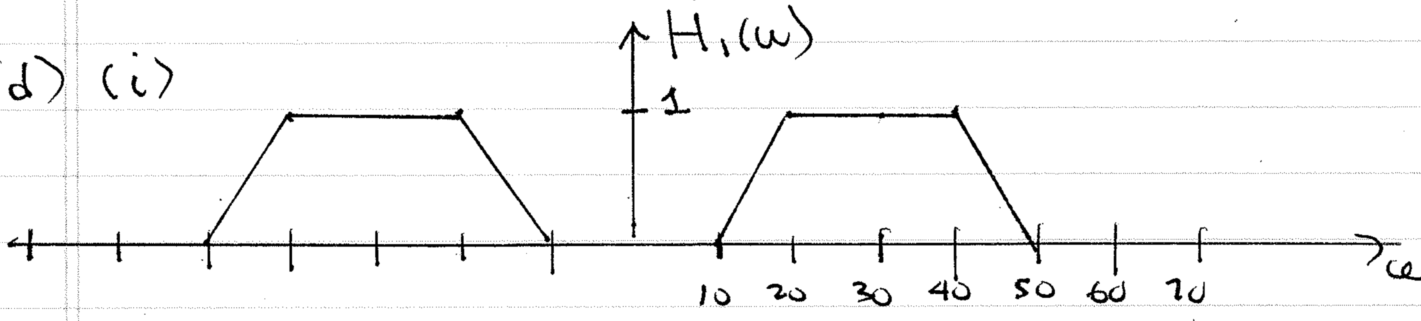


1(c) $x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$

$$X(\omega) = X_1(\omega - 30) + X_1(\omega + 30) + X_2(\omega - 60) + X_2(\omega + 60)$$

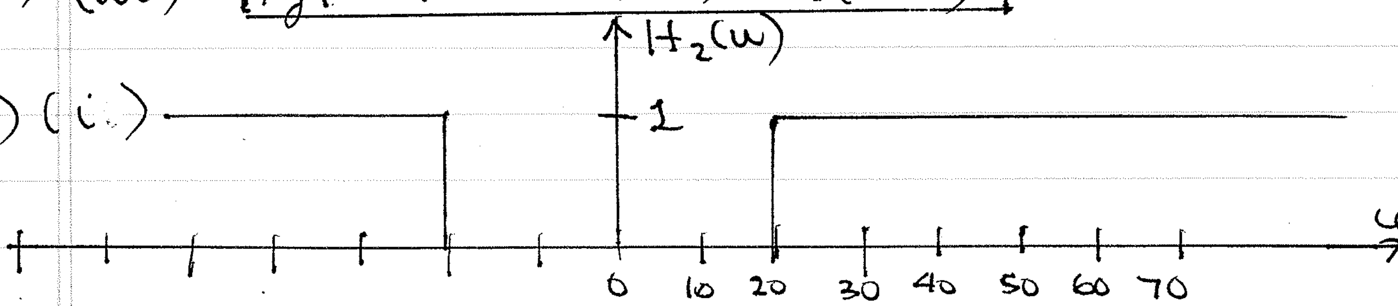


(d) (i)



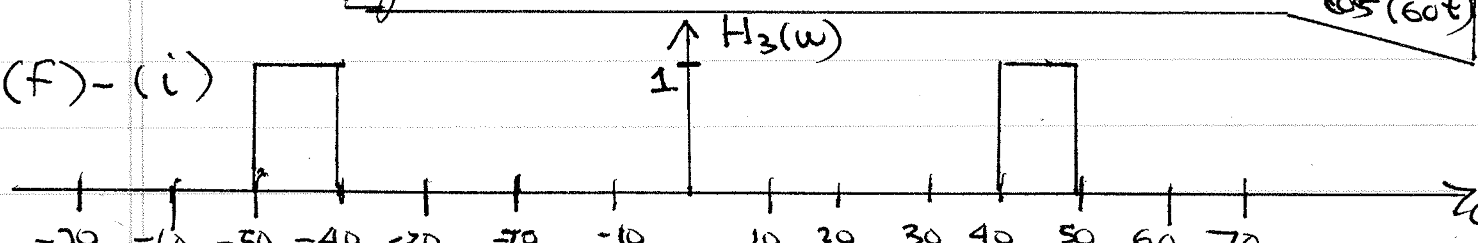
(d) (iii) $\boxed{y_1(t) = 2x_1(t)\cos(30t)}$

(e) (i)



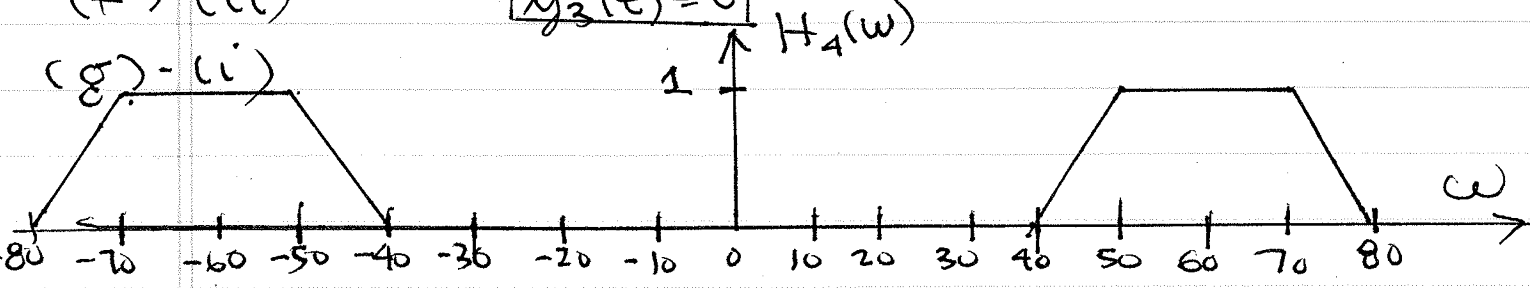
(e) - (iii) $\boxed{y_2(t) = x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)}$

(f) - (i)



(f) - (ii) $\boxed{y_3(t) = 0}$

(g) - (i)

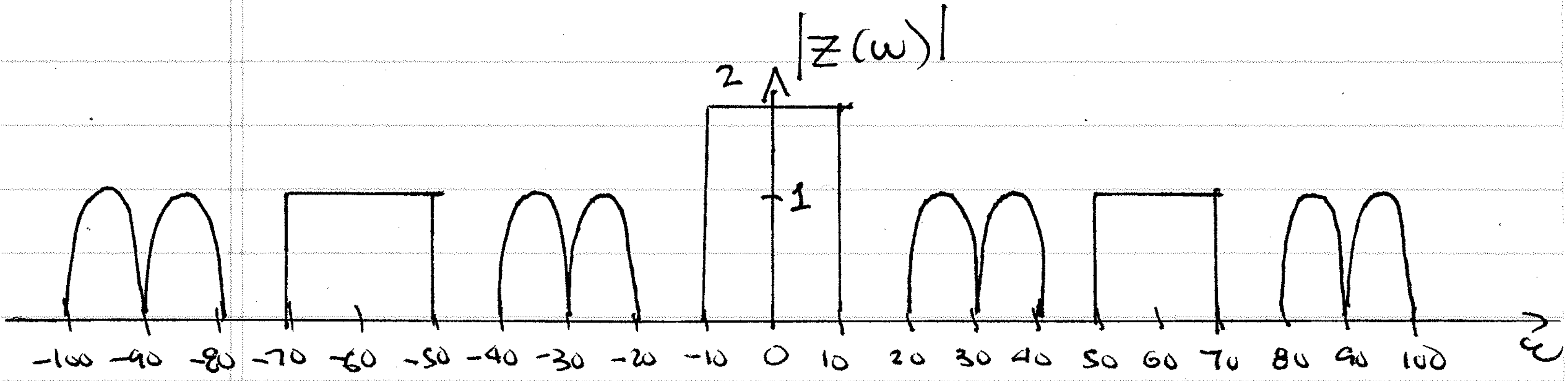


(g)-(iii) $y_4(t) = 2x_2(t)\cos(60t)$

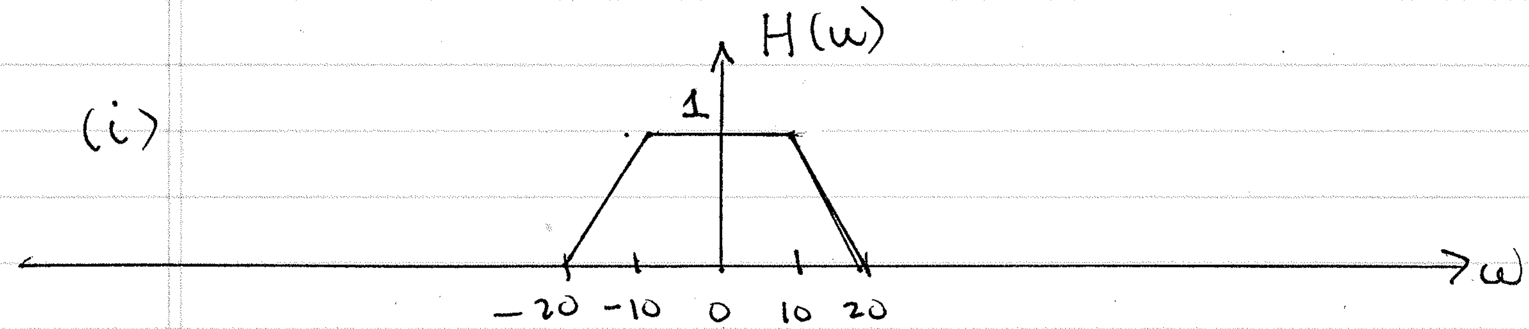
(h) $z(t) = 2x(t)\cos(30t)$

$$= 2\{2x_1(t)\cos^2(30t) + 2x_2(t)\cos(60t)\cos(30t)\}$$

$$= 2\{x_1(t) + x_1(t)\cos(60t) + x_2(t)\cos(30t) + x_2(t)\cos(90t)\}$$



(i)

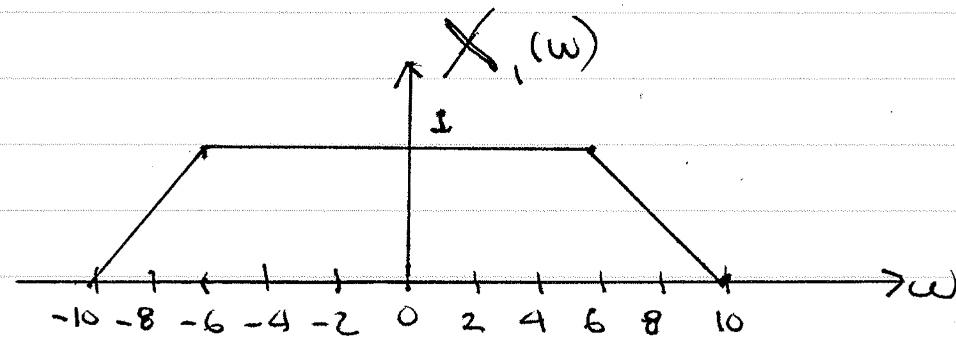


$$w(t) = z(t) * h(t)$$

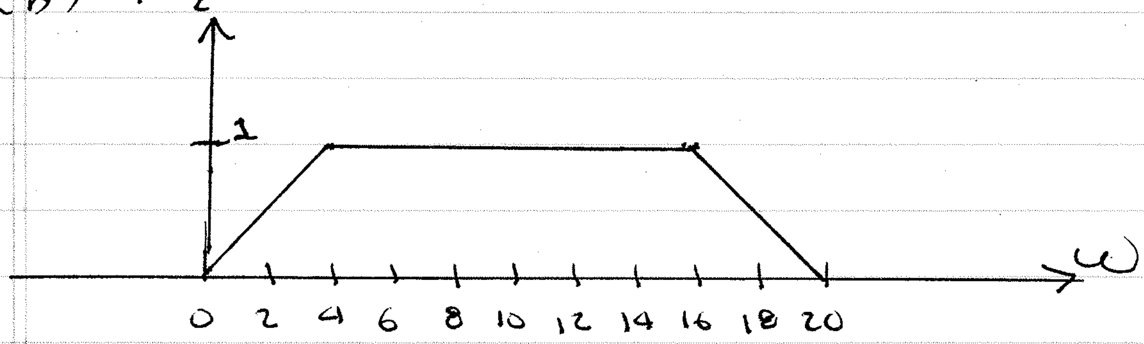
$$= 2x_1(t)$$

Prob. 2 Solution

(a)



(b) $X_2(\omega) = X_1(\omega - 10)$



(c) Note: $x_3(t) = X_2(\omega)\big|_{\omega = t}$

Duality:

$$X_3(\omega) = 2\pi\, x_2(-\omega)$$

$$= 2\pi \frac{\pi}{2} \left\{ \frac{\sin(2\omega)}{\pi\omega} \frac{\sin(8\omega)}{\pi\omega} \right\} e^{-j10\omega}$$

sinc functions are even-symmetric

$$= \frac{\sin(2\omega)}{\omega} \frac{\sin(8\omega)}{\omega} e^{-j10\omega}$$

(d) $x_4(t) = x_3(t)$ } trapezoid is convolution of two rectangles

$$= \frac{1}{4} \operatorname{rect}\left(\frac{t-2}{4}\right) * \operatorname{rect}\left(\frac{t-8}{16}\right)$$

$$X_4(\omega) = \frac{1}{4} \frac{2\sin(2\omega)}{\omega} \frac{2\sin(8\omega)}{\omega} e^{-j2\omega} e^{-j8\omega}$$

$$= \frac{\sin(2\omega)}{\omega} \frac{\sin(8\omega)}{\omega} e^{-j10\omega}$$