SOLUTION

NAME:
EE301 Signals and Systems

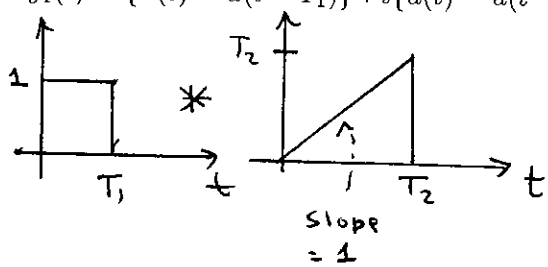
21 February 2019
Exam 1

Cover Sheet

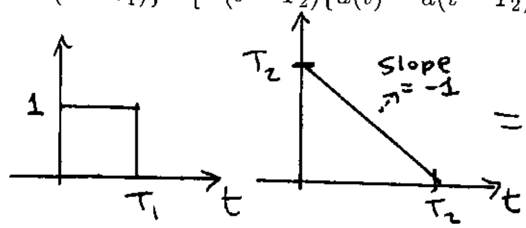
Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on the sheets provided.
You must show work or explain answer for each problem to receive full credit.
Plot your answers on the graphs provided.
WRITE YOUR NAME ON EVERY SHEET.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties	50
2.	Discrete Time Signals and System Properties	50

$$y_1(t) = \{u(t) - u(t-T_1)\} * t\{u(t) - u(t-T_2)\} = \frac{t^2}{2}\{u(t) - u(t-T_1)\} + \left(T_1 t - \frac{T_1^2}{2}\right)\{u(t-T_1) - u(t-T_2)\} + \left(-\frac{t^2}{2} + T_1 t + \frac{T_2^2 - T_1^2}{2}\right)\{u(t-T_2) - u(t-(T_1+T_2))\} \quad (1)$$



$$\{u(t) - u(t-T_1)\} * [-(t-T_2)\{u(t) - u(t-T_2)\}] = \left(-\frac{t^2}{2} + T_2 t\right)\{u(t) - u(t-T_1)\} + \left(-T_1 t + \frac{2T_1T_2 + T_1^2}{2}\right)\{u(t-T_1) - u(t-T_2)\} + \left(\frac{t^2}{2} - (T_1+T_2)t + \frac{(T_1+T_2)^2}{2}\right)\{u(t-T_2) - u(t-(T_1+T_2))\} \quad (2)$$



$$y_2(t) = \{u(t) - u(t-T_1)\} * [-(t-T_2)\{u(t) - u(t-T_2)\}] = y_1(-(t-(T_1+T_2)) \quad (3)$$

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2.	Discrete Time Signals and System Properties	50

VIP If you want to refer to the input signal and output signal for one part of a problem when solving a later part, use that part's letter as a subscript, e.g., you can refer to the input signal and corresponding output signal for part (d) of Prob. 1 as $x_d(t)$ and $y_d(t)$, respectively.

VIP: Solving and part, you can just write: z(t)= Formula A with a=-3 and b =-2 BUT don't have to write out Formula A substituting a=-3 and b=-2. Just use z(t) for remainder of your solution.

Formula A:
$$e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}e^{at}u(t) + \frac{1}{b-a}e^{bt}u(t) \quad (1)$$

Formula B:
$$\alpha^n u[n] * \beta^n u[n] = \frac{\alpha}{\alpha-\beta}\alpha^n u[n] + \frac{\beta}{\beta-\alpha}\beta^n u[n] \quad (2)$$

Formula C: if $x(t) * h(t) = y(t)$ then: $a\,x(t-t_1) * b\,h(t-t_2) = ab\,y(t-(t_1+t_2))$ (3)

Formula D: if $x[n] * h[n] = y[n]$ then: $a\,x[n-n_1] * b\,h[n-n_2] = ab\,y[n-(n_1+n_2)]$ (4)

Formula E:
$$e^{at}u(t) * e^{at}u(t) = te^{at}u(t) \quad (5)$$

Formula F:
$$\alpha^n u[n] * \alpha^n u[n] = (n+1)\alpha^n u[n] \quad (6)$$

Prob. 1. Consider the LTI system characterized by the I/O relationship:

$$\text{System 1:} \quad y(t) = -\int_{t-3}^{t} (t-\tau-3)x(\tau)d\tau$$

(a) Write the impulse response of the system, $h_1(t)$.

$h_1(t) = -(t-3)\{u(t) - u(t-3)\}$

(b) Is the system causal? Justify your answer using the impulse response.

Yes since $h_1(t) = 0$ for $t < 0$

(c) Is the system stable? Justify your answer using the impulse response.

$$\int_{-\infty}^{\infty} |h_1(t)| \, dt = \frac{1}{2} 3(3) = \frac{9}{2} < \infty$$

Yes, system is stable

= area under triangle

(d) Determine and plot the output $y(t)$ when the input is

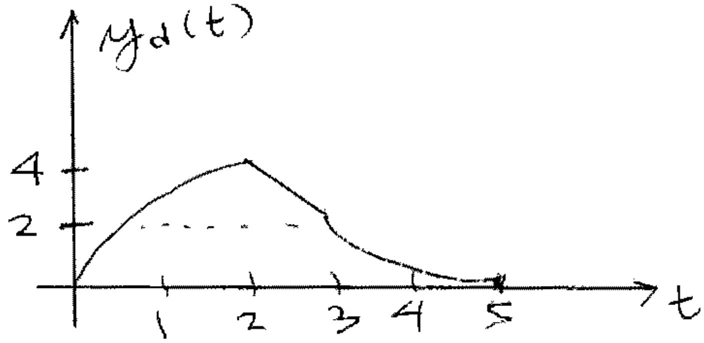
$$x(t) = u(t) - u(t-2)$$

Formula 2 with $T_1 = 2$ and $T_2 = 3$.

$-\frac{4}{2} + 3(2)$

$= 4$

$\frac{1}{2} \cdot 2 \cdot 2 =$



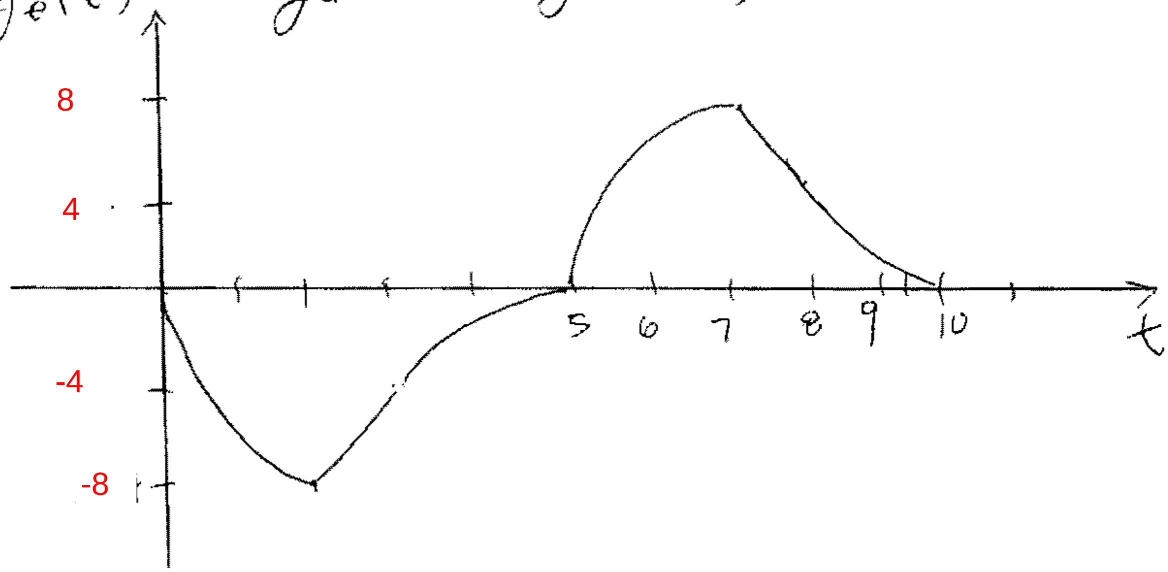
(e) Determine and PLOT an expression for the output of System 1, $y(t)$.

$$x(t) = -2\{u(t) - u(t-2)\} + 2\{u(t-5) - u(t-7)\}$$

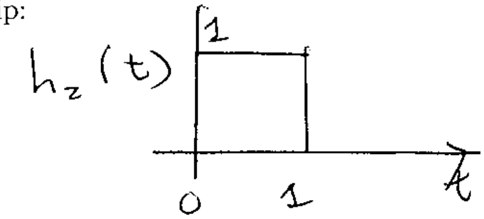
Linearity & time-invariance

no overlap :)

$y_e(t) = -2\, y_d(t) + 2\, y_d(t-5)$



Next: Consider the LTI system characterized by the I/O relationship:

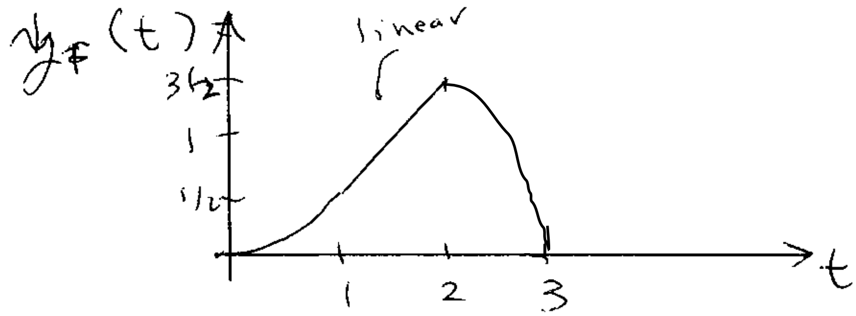


$$\text{System 2:} \qquad y(t) = \int_{t-1}^{t} x(\tau)d\tau$$

(f) Determine and plot the output $y_f(t)$ of System 2 for the input

$$x_f(t) = t\{u(t) - u(t-2)\}$$

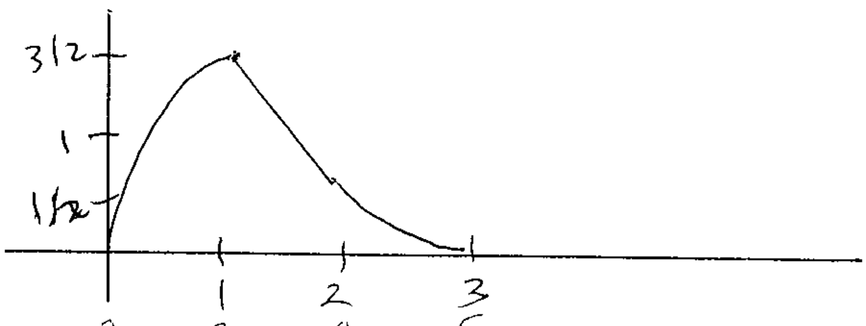
Formula 1 with $T_1 = 1$ and $T_2 = 2$



(g) Determine and plot the output $y_g(t)$ of System 2 for the input

$$x_g(t) = -(t-2)\{u(t) - u(t-2)\}$$

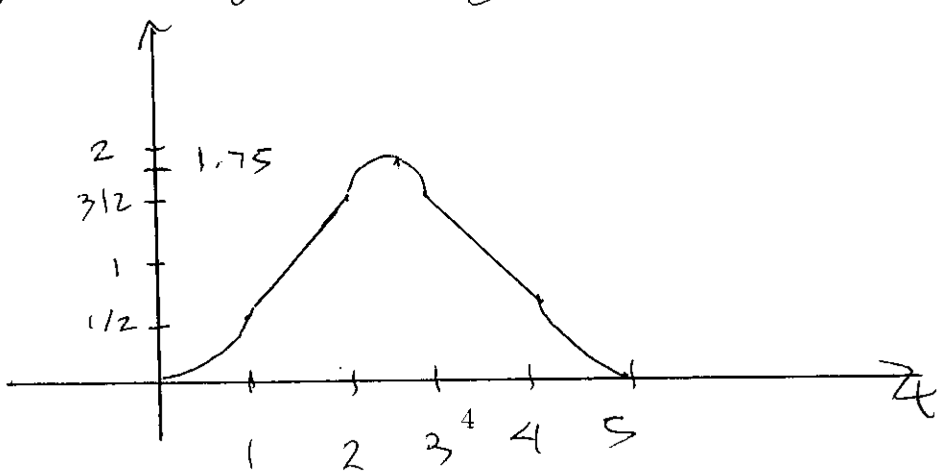
Formula 2 with $T_1 = 1$ and $T_2 = 2$



(h) Determine and plot the output $y_h(t)$ of System 2 for the triangle input below.

$$x_h(t) = x_f(t) + x_g(t-2)$$

$y_h(t) = y_f(t) + y_g(t-2)$



Next: Consider the LTI system characterized by the I/O relationship. NOTE: there are no plots needed for the three parts below.

$$\text{System 3:} \quad y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)} x(\tau) d\tau$$

$h_3(t) = e^{-2t} u(t)$

(i) Determine the output $y_i(t)$ of System 3 for the input

$$x_i(t) = t\{u(t) - u(t-2)\}$$

see page attached

(j) Determine a closed-form expression for the output $y_j(t)$ of System 3 for the input

$$x_j(t) = -(t-2)\{u(t) - u(t-2)\}$$

see page attached

(k) Determine a closed-form expression for the output $y_k(t)$ of System 3 for the triangle input below.

$$x_k(t) = x_i(t) + x_j(t-2)$$

$$y_k(t) = y_i(t) + y_j(t-2)$$

Next: Consider the LTI system characterized by the I/O relationship. NOTE: there are no plots needed for the three parts below.

$$\text{System 3:} \quad y(t) = \int_{-\infty}^{t} e^{-2(t-\tau)} x(\tau) d\tau \qquad h_3(t) = e^{-2t} u(t)$$

(i) Determine the output $y_i(t)$ of System 3 for the input

$$x_i(t) = t\{u(t) - u(t-2)\}$$

$$y_i(t) = x_i'(t) * h_d(t) * u(t) * e^{-2t}u(t)$$

$$= \{-2\,\delta(t-2) + (u(t) - u(t-2))\} * e^{-2t}u(t) * u(t)$$

$$= -2\, z(t-2) + \underbrace{u(t) * e^{-2t}u(t) * u(t)}_{w(t)} - w(t-2)$$

$z(t) =$

$-\frac{1}{2} e^{-2t}u(t) + \frac{1}{2}u(t)$

$w(t) =$

$$\{-\tfrac{1}{2} e^{-2t}u(t) + \tfrac{1}{2}u(t)\} * u(t)$$

$$= -\tfrac{1}{2} z(t) + \tfrac{1}{2} t u(t)$$

Ans:

$$y_i(t) = -2\, z(t-2) + \underbrace{w(t)} - w(t-2)$$

$$= -\tfrac{1}{2} z(t) + \tfrac{1}{2} t u(t)$$

(j) Determine a closed-form expression for the output $y_j(t)$ of System 3 for the input

$$x_j(t) = -(t-2)\{u(t) - u(t-2)\}$$

$$y_j(t) = x_j(t) * h_d(t) * u(t) * e^{-2t}u(t)$$

$$= \{2\delta(t) - (u(t) - u(t-2))\} * u(t) * e^{-2t}u(t)$$

reverse

$$= 2\underbrace{u(t) * e^{-2t}u(t)}_{z(t)} - \underbrace{u(t) * e^{-2t}u(t)} * u(t) + \underbrace{u(t-2) * e^{-2t} * u(t)}_{w(t-2)}$$

$$z(t) = \frac{1}{-2}e^{-2t}u(t) + \frac{1}{2}u(t)$$

$$w(t) = \left\{-\frac{1}{2}e^{-2t}u(t) + \frac{1}{2}u(t)\right\} * u(t)$$

$$= -\frac{1}{2}z(t) + \frac{1}{2}t\,u(t)$$

Answer:

$$y_j(t) = 2z(t) - w(t) + w(t-2)$$

Problem 2. [50 points] Problem 2 is separate from Problem 1; the first DT system is enumerated as System 1.

(a) Consider the causal LTI System characterized by the difference equation below. Write an expression for the impulse response of this system, denoted $h_1[n]$.

$$\text{System 1:} \quad y[n] = -\frac{1}{4}y[n-1] + x[n]$$

$$h_1[n] = \left(-\frac{1}{4}\right)^n u[n]$$

(b) Is the system causal? Justify your answer using the impulse response.

Yes since $h_1[n] = 0$ for $n < 0$

(c) Is the system stable? Justify your answer using the impulse response.

$$\sum_{n=-\infty}^{\infty} |h_1[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} < \infty$$

system is stable

(d) Does the system have memory or is it memoryless? Justify your answer using the impulse response.

$h_1[n] \neq K\delta[n]$ so system has memory

(e) Determine the output $y[n]$ of System 1 for the input

$$x[n] = \left(\frac{1}{3}\right)^n u[n]$$

Formula 2 with $\alpha = \frac{1}{3}$ and $\beta = -\frac{1}{4}$

$$y_e[n] = \left(\frac{1}{3}\right)^n u[n] * \left(-\frac{1}{4}\right)^n u[n]$$

(f) Determine the output $y[n]$ of System 1 when the input is

$$x[n] = \left(\frac{1}{3}\right)^n u[n-3] = \left(\frac{1}{3}\right)^3 \left(\frac{1}{3}\right)^{n-3} u[n-3]$$

$$= \frac{1}{27} x_e[n-3]$$

$$y_f[n] = \frac{1}{27} y_e[n-3]$$

(g) Determine the output $y[n]$ of System 1 for the input below.

$$x[n] = 4\left(\frac{1}{3}\right)^n \{u[n] - u[n-3]\}$$

$$= 4\, x_e[n] - 4\, x_f[n]$$

$$y_g[n] = 4\, y_e[n] - 4\, y_f[n]$$

(h) Determine the output $y[n]$ of System 1 for the input below.

$$x[n] = \{u[n] - u[n-5]\}$$

$$y_h[n] = \left(-\frac{1}{4}\right)^n u[n] * u[n] - \left(-\frac{1}{4}\right)^n u[n] * u[n-5]$$

$$= z[n] - z[n-5]$$

with $z[n] =$ Formula B with $\alpha = -\frac{1}{4}$

and $\beta = 1$

(i) Consider a second LTI system described by the following difference equation. Determine the impulse response for System 2, denoted $h_2[n]$. You can write your answer in sequence form, using an arrow to denote the $n = 0$ value.

$$\text{System 2:} \quad y[n] = y[n-1] + x[n] - x[n-5]$$

$$h_2[n] = u[n) - u[n-5]$$

$$= \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$$

(j) Determine the output of System 2 ,$y[n]$, for the input below. Write answer in sequence form, using an arrow to denote the $n = 0$ value.

$$x[n] = \{u[n] - u[n-3]\}$$

$$y_j[n] = \{1, 2, 3, 3, 3, 2, 1\}$$

$$n = \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

(k) Determine the output of System 2, $y[n]$, for the input below. Write answer in sequence form, using an arrow to denote the $n = 0$ value. So, for this problem, you are required to list the numbers that comprise the output signal in sequence form.

$$x[n] = 2\{u[n] - u[n-3]\} - 2\{u[n-6] - u[n-9]\}$$

x ~~y~~$_k[n] = 2\, x_j[n] - 2\, x_j[n-6]$

$y_k[n] = 2\, y_j[n] - 2\, y_j[n-6]$

$$= \{\underset{\uparrow}{2}, 4, 6, 6, 6, 4, \underset{\uparrow}{0}, -4, -6, -6, -6, -4, -2\}$$

$n=0$ point of overlap

(l) Determine the output $y[n]$ of System 2 for the input below. **HINT:** to save time, observe: relative to part (h), the role of input signal and impulse response are reversed. If you're going to use this observation, state the relevant property of convolution.

$$x[n] = \left(-\frac{1}{4}\right)^n u[n]$$

$y[n] = \left(-\frac{1}{4}\right)^n u[n] * \{u[n] - u[n-5]\}$

commutativity of convolution

$y_l[n] = y_h[n]$

(m) Determine the output, $y[n]$, when $x[n]$ below

$$x[n] = (n+1)\{u[n] - u[n-4]\} = \{1, 2, 3, 4\}$$

input to two LTI systems in SERIES, with respective impulse responses indicated below. Write answer in sequence form, indicating with an arrow the value corresponding to $n = 0$.

$$h_1[n] = 8\left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\} \qquad h_2[n] = \delta[n] - \delta[n-1]$$

$y[n] = x_1[n] * h_1[n] * h_2[n]$

$= h_1[n] * \underbrace{x_1[n] * h_2[n]}$

1	2	3	4	
	-1	-2	-3	-4

$\{8, 4, 2, 1\} * \{1, 1, 1, 1, -4\}$

8	4	2	1	0	0	0	0
0	8	4	2	1	0	0	0
0	0	8	4	2	1	0	0
0	0	0	8	4	2	1	0
0	0	0	0	-32	-16	-8	-4

$\{\underset{\uparrow}{8}, 12, 14, 15, -25, -13, -7, -4\}$

$n=0$