SOLUTION

# EE301 Signals and Systems — In-Class Exam
# Exam 1 — Tuesday, Feb. 22, 2011

## Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 1,2
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
Plot your answers on the graphs provided.

| Prob. No. | Topic(s) | Points |
|---|---|---|
| 1. | Continuous Time Signals and System Properties and CT Fourier Series | ~~50~~ 55 |
| 2. | Discrete Time Signals and System Properties | ~~50~~ 45 |

55

**Problem 1.** [50 points] Consider two LTI systems connected **in parallel** (as in Fig. 2.23 (a) on text page 105) , where each of the two systems in parallel are respectively characterized by the following input-output relationships:

$$\text{System 1:} \quad y_1(t) = 2\int_{t-4}^{t} x(\tau)d\tau$$
$$\text{System 2:} \quad y_2(t) = \int_{t-7}^{t-4} x(\tau)d\tau$$

As is the case with systems in parallel, the two systems have $x(t)$ as a common input and their resective outputs are summed to yield the overall output $y(t) = y_1(t) + y_2(t)$.

5 (a) Determine and plot the impulse response of System 1, denoted $h_1(t)$.

5 (b) Apply a test to System 1's impulse response, $h_1(t)$, to determine if System 1 is stable or not.

10 (c) Determine and write a closed-form expression for the output of System 1, $y_1(t)$, when the input to this system is the exponential signal $x(t) = e^{-2t}u(t)$.

10 (d) Now, determine and plot the impulse response of the OVERALL system, denoted $h(t)$. Plot in the indicated spot on the sheets attached and show as much detail as possible.

10 (e) Determine and plot the output $y(t)$ when the input to the overall system is the rectangular pulse: $x(t) = \text{rect}\left(\frac{t-1}{2}\right) = u(t) - u(t-2)$.

5 (f) Determine and plot the output $y(t)$ when the input to the overall system is the rectangular pulse: $x(t) = \text{rect}\left(\frac{t-3}{2}\right) = u(t-2) - u(t-4)$.

10 (g) Determine and plot the output $y(t)$ when the input to the overall system is the rectangular pulse: $x(t) = 2\text{rect}\left(\frac{t-2}{4}\right) = 2\{u(t) - u(t-4)\}$. *Hint:* You should be able to use your answers to the last two parts, (e) and (f).

**Problem 2.** [45 points]

10 (a) Consider a system whose impulse response is $h[n] = u[n] - u[n-4]$. Determine and plot the output $y[n]$ when the input is $x[n] = 8\left(\frac{1}{2}\right)^n \{u[n] - u[n-4]\}$. Do a stem-plot in the space provided on the sheets attached.

(b) An signal $x[n]$ is a sum of two DT sinewaves with frequencies $3\pi/8$ and $7\pi/8$, respectively.

$$x[n] = 3e^{j\frac{3\pi}{8}n} + 2e^{j\frac{7\pi}{8}n} \tag{1}$$

Consider this signal as the input to each of the four systems described below.

$$\begin{aligned}
&\text{System 1:} & y[n] &= x[n] + (-1)^{n-1}x[n-1] & (2)\\
&\text{System 2:} & y[n] &= (-j)^n x[n] & (3)\\
&\text{System 3:} & y[n] &= x[n]x[n-1] & (4)\\
&\text{System 4:} & y[n] &= x^*[-n] & (5)\\
& & & & (6)
\end{aligned}$$

For EACH of the four systems above, you must answer EACH of the following THREE questions in the Table provided in the sheets attached. **NOTE:** you do not have to determine the numerical values of any multiplicative scalars in the output - just determine what are the frequencies of the complex sinewaves present in $y[n]$.

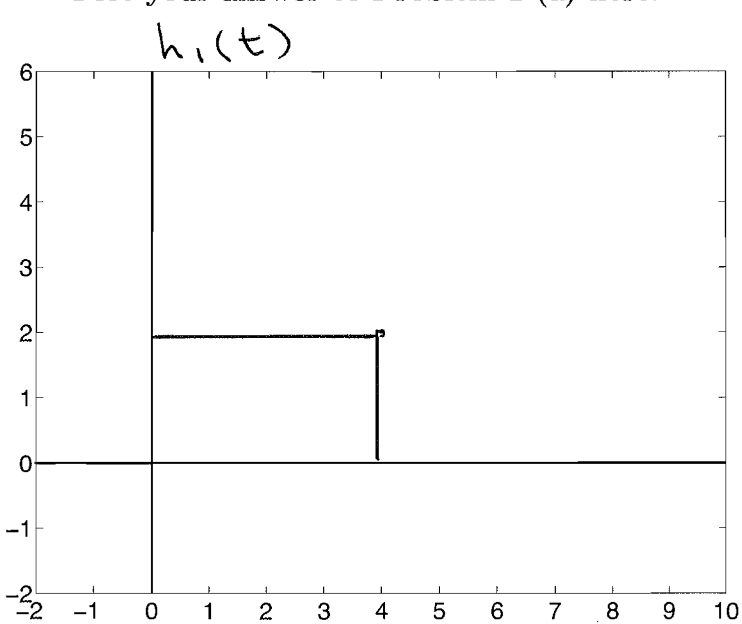
$4\times 2 = 8$ (i) Is the system linear? Yes or No (don't need to justify your answer.)

$4\times 2 = 8$ (ii) Is the system time-invariant? Yes or No (don't need to justify your answer.)

$4\times 5 = 20$ (iii) Determine the frequencies in the output $y[n]$ of each system given the input in Equation 1 above. Each answer should be in the range $[-\pi, \pi]$.

5

**Plot your answer to Problem 1 (a) here.**



Show your work and write your answers to Problem 1, parts (b) and (c) on this page.

1(b) $\int_{-\infty}^{\infty} |h_1(t)| dt \overset{?}{<} \infty$ $\quad h_1(t) = 2\{u(t) - u(t-4)\}$

5

$\int_{-\infty}^{\infty} |h_1(t)| dt = 4(2) = 8 < \infty$ } System 1 is stable

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1(c) $h_1(t) = 2\{u(t) - u(t-4)\}$

10

$$y_1(t) = 2e^{-2t}u(t) * \{u(t) - u(t-4)\}$$
$$= (2e^{-2t}u(t)) * u(t) - (2e^{-2t}u(t)) * u(t-4)$$
$$= 2e^{-2t}u(t) * u(t) - 2e^{-2t}u(t) * u(t-4)$$

From text Example 2.6: $e^{-at}u(t) * u(t)$
$$= \frac{1}{a}\left(1 - e^{-at}\right)u(t)$$

Thus: $a = 2$

$$y_1(t) = \frac{2}{2}\left(1 - e^{-2t}\right)u(t) - \frac{2}{2}\left(1 - e^{-2(t-4)}\right)u(t-4)$$

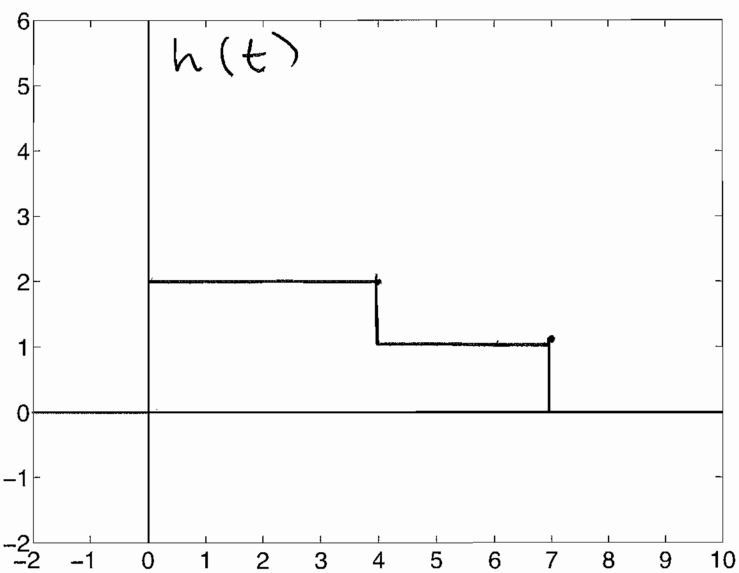
$$= \left(1 - e^{-2t}\right)\left(u(t) - u(t-4)\right)$$
$$+ \underbrace{\left\{1 - 1 - e^{-2t} + e^{8}e^{-2t}\right\}u(t-4)}_{\approx e^{8}e^{-2t}u(t-4)}$$

$$h(t) = h_1(t) + h_2(t)$$

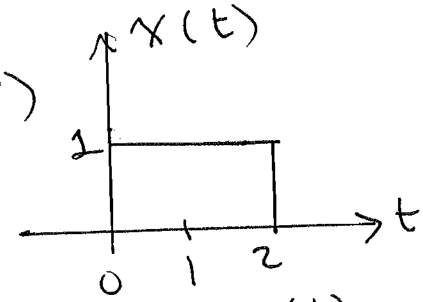
$$h_2(t) = u(t-4) - u(t-7)$$

10

**Plot your answer to Problem 1 (d) here.**

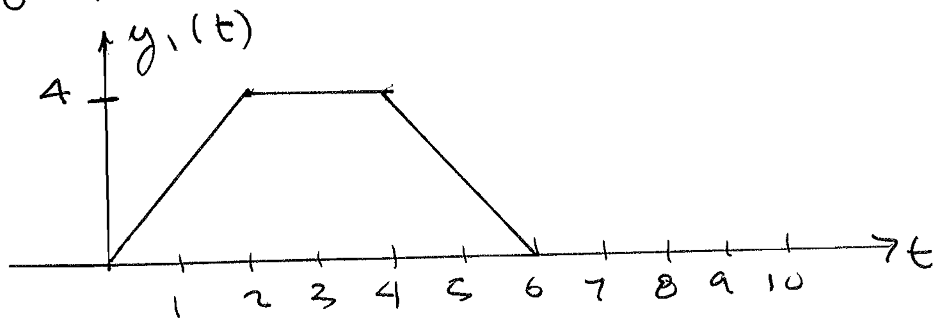


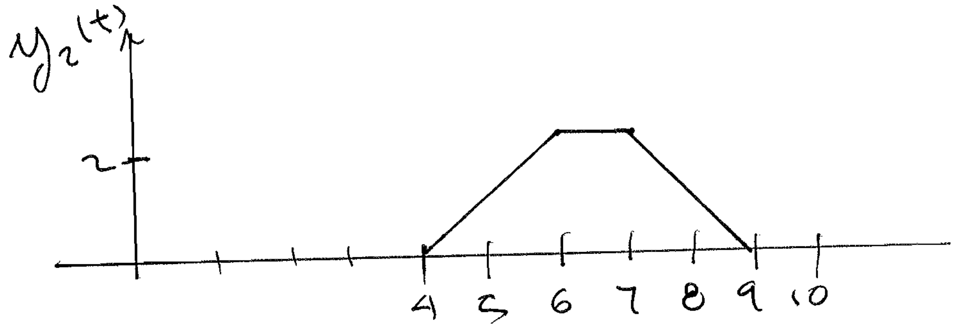
1 (e)



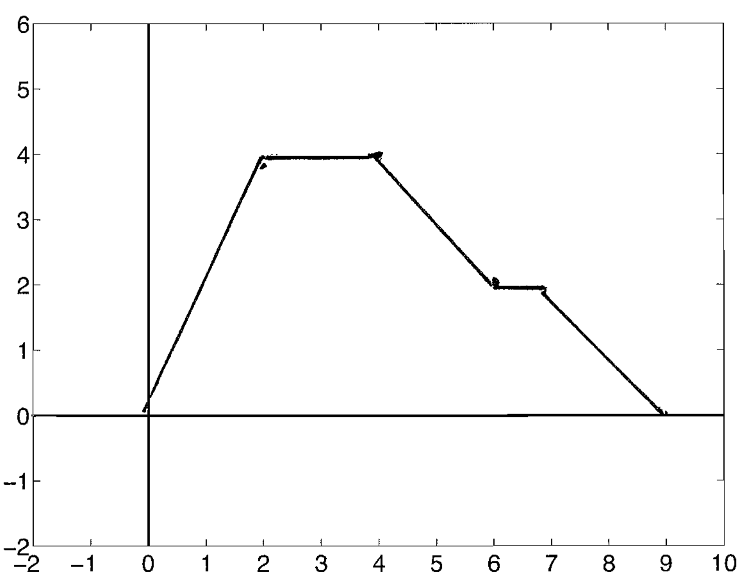

$y(t) = ?$

10



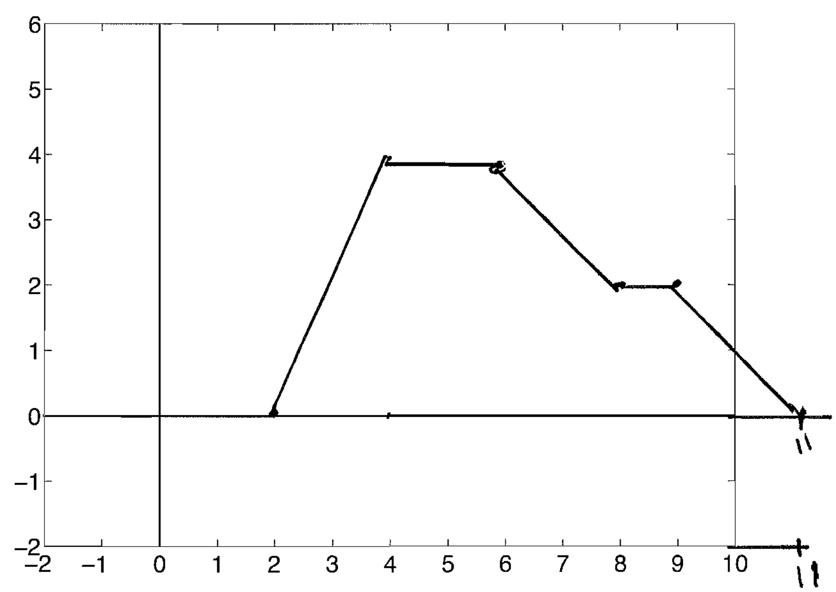

**Plot your answer to Problem 1 (e) here.**



5

1 (f) $\Rightarrow$ answer to (e) shifted to right by 2

Plot your answer to Problem 1 (f) here.



part (e) $x_e(t) \rightarrow \boxed{h(t)} \rightarrow y_e(t)$

part (f) $x_f(t) \rightarrow \boxed{h(t)} \rightarrow y_f(t)$

8) $x(t) = 2\{x_e(t) + x_f(t)\}$

where: $x_e(t)$ is input for (e)

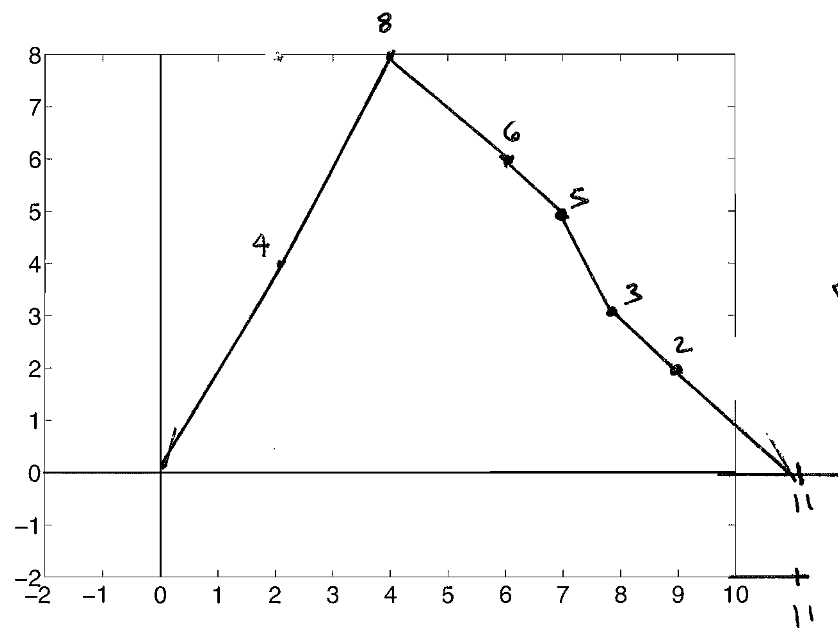
$x_f(t)$ is input for (f)

thus: $y(t) = 2\{y_e(t) + y_f(t)\}$

add answers to 1(e) and (f) and then multiply by 2

10

First, let's do $y_e(t) + y_f(t)$

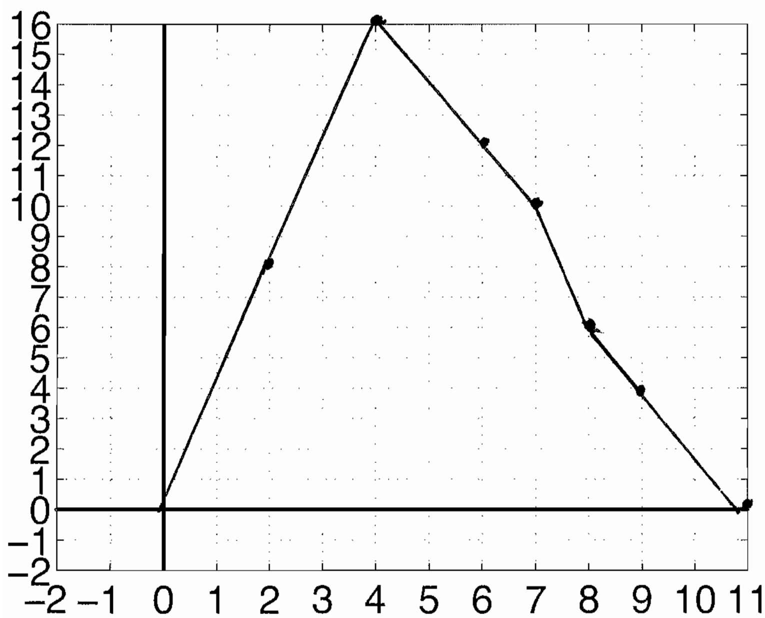



y(t) is 2x times this

see next page

9(a)

**Plot your answer to Problem 1 (g) here.**

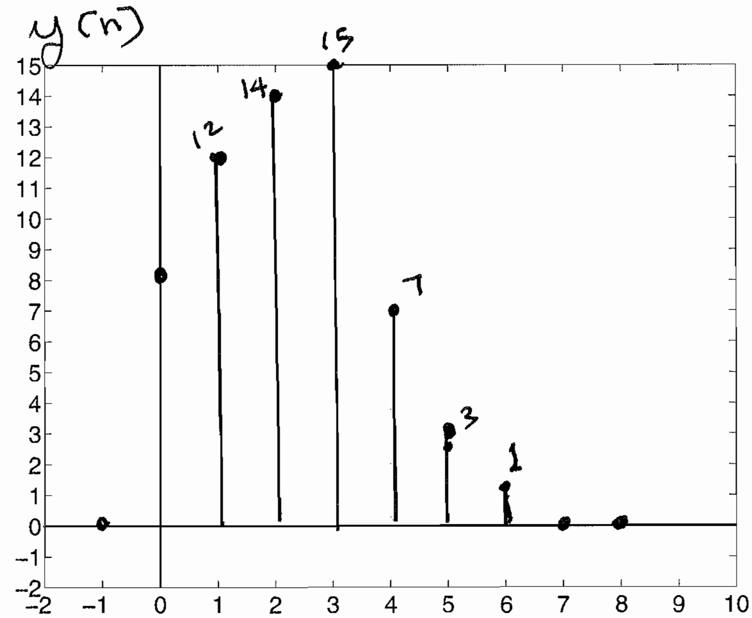


$$x[n] = \{\overset{n=0}{8}, 4, 2, \overset{n=3}{1}\} * h[n] = \{\overset{n=0}{1}, 1, 1, 1\}$$

$$\begin{array}{|l}
8, 4, 2, 1 \\
0, 8, 4, 2, 1 \\
0, 0, 8, 4, 2, 1 \\
0, 0, 0, 8, 4, 2, 1 \\
\hline
\underset{n=0}{8}, 12, 14, 15, 7, 3, 1
\end{array}$$

10

Plot your answer to Problem 2 (a) here.



Show your work and write your answers to Problem 2, part (b) on this page.

System 1 Linear ⇒ so do each sinewave separately:

$$e^{j\frac{3\pi}{8}n} - e^{-j\pi n} e^{j\frac{3\pi}{8}(n-1)} = e^{j\frac{3\pi}{8}n} - e^{-j\frac{3\pi}{8}} e^{\left(\frac{3\pi}{8}-\frac{8\pi}{8}\right)n}$$

$$= e^{j\frac{-3\pi}{8}n} - \alpha e^{-\frac{5\pi}{8}n}$$

$$e^{j\frac{7\pi}{8}n} - e^{-j\pi n} e^{j\frac{7\pi}{8}(n-1)} = e^{j\frac{7\pi}{8}n} + \alpha e^{j\left(\frac{7\pi}{8}-\frac{8\pi}{8}\right)n}$$

$$= e^{j\frac{7\pi}{8}n} + \alpha e^{-j\frac{\pi}{8}n}$$

System 2 Linear ⇒ do each sinewave separately

$$e^{-j\frac{\pi}{2}n} e^{j\frac{3\pi}{8}n} \Rightarrow -\frac{4\pi}{8} + \frac{3\pi}{8} = -\frac{\pi}{8}$$

$$e^{-j\frac{\pi}{2}n} e^{-j\frac{7\pi}{8}n} \Rightarrow -\frac{4\pi}{8} + \frac{7\pi}{8} = \frac{3\pi}{8}$$

System 3 $\left(3e^{j\frac{3\pi}{8}n} + 2e^{j\frac{7\pi}{8}n}\right)\left(3e^{j\frac{3\pi}{8}(n-1)} + 2e^{j\frac{7\pi}{8}(n-1)}\right)$

$$= 9e^{j\frac{6\pi}{8}n} + \alpha_1 e^{j\left(\frac{3\pi}{8}+\frac{7\pi}{8}\right)n} + \alpha_2 e^{j\left(\frac{7\pi}{8}+\frac{3\pi}{8}\right)n} + \alpha_3 e^{j\frac{14\pi}{8}n}$$

$3\pi/4$ $\qquad \frac{10\pi}{8} \Rightarrow \frac{5\pi}{4} - \frac{8\pi}{4} = -\frac{3\pi}{4}$ $\qquad$ same $\qquad \frac{7\pi}{4} - \frac{8\pi}{4} = -\frac{\pi}{4}$

| System | Linear? | Time-Invariant? | Frequencies in $y[n]$ in range $[-\pi, \pi]$ | |
|---|---|---|---|---|
| 1 | Yes 2 | 2 No | $3\pi/8$, $-5\pi/8$, $7\pi/8$, $-\pi/8$ | 5 |
| 2 | Yes 2 | 2 No | $-\pi/8$, $3\pi/8$ | 5 |
| 3 | No 2 | 2 Yes | $3\pi/4$, $-3\pi/4$, $-\pi/4$ | 5 |
| 4 | No 2 | 2 No | $3\pi/8$, $7\pi/8$ | 5 |

System 4 note: if $x[n] = \alpha e^{j\omega_0 n}$

$$y[n] = \alpha^* e^{-j(-\omega_0 n)} = \alpha^* e^{j\omega_0 n}$$

⇒ frequencies unchanged

system is not linear due to homogeneity being violated

⇒ superposition still holds