

## Cover Sheet

Test Duration: 75 minutes.

Coverage: Chaps. 1,2

Open Book but Closed Notes.

One 8.5 in. x 11 in. crib sheet

Calculators NOT allowed.

This test contains **two** problems.

All work should be done on the sheets provided.

You must show all work for each problem to receive full credit.

Plot your answers on the graphs provided.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties and CT Fourier Series	60
2.	Discrete Time Signals and System Properties	40

**Problem 1.** [60 points] Consider two LTI systems connected **in parallel** (as in Fig. 2.23 (a) on text page 105) , where each of the two systems in parallel are respectively characterized by the following input-output relationships:

$$\text{System 1: } y_1(t) = 2 \int_{t-4}^t x(\tau) d\tau$$

$$\text{System 2: } y_2(t) = \int_{t-7}^{t-4} x(\tau) d\tau$$

As is the case with systems in parallel, the two systems have  $x(t)$  as a common input and their respective outputs are summed to yield the overall output  $y(t) = y_1(t) + y_2(t)$ .

- (a) Determine and plot the impulse response of System 1, denoted  $h_1(t)$ .
- (b) Apply a test to System 1's impulse response,  $h_1(t)$ , to determine if System 1 is stable or not.
- (c) Determine and write a closed-form expression for the output of System 1,  $y_1(t)$ , when the input to this system is the exponential signal  $x(t) = e^{-2t}u(t)$ .
- (d) Now, determine and plot the impulse response of the OVERALL system, denoted  $h(t)$ . Plot in the indicated spot on the sheets attached and show as much detail as possible.
- (e) Determine and plot the output  $y(t)$  when the input to the overall system is the rectangular pulse:  $x(t) = \text{rect}\left(\frac{t-1}{2}\right) = u(t) - u(t-2)$ .
- (f) Determine and plot the output  $y(t)$  when the input to the overall system is the rectangular pulse:  $x(t) = \text{rect}\left(\frac{t-3}{2}\right) = u(t-2) - u(t-4)$ .
- (g) Determine and plot the output  $y(t)$  when the input to the overall system is the rectangular pulse:  $x(t) = 2\text{rect}\left(\frac{t-2}{4}\right) = 2\{u(t) - u(t-4)\}$ . *Hint:* You should be able to use your answers to the last two parts, (e) and (f).

**Problem 2.** [40 points]

- (a) Consider a system whose impulse response is  $h[n] = u[n] - u[n - 4]$ . Determine and plot the output  $y[n]$  when the input is  $x[n] = 8 \left(\frac{1}{2}\right)^n \{u[n] - u[n - 4]\}$ . Do a stem-plot in the space provided on the sheets attached.
- (b) An signal  $x[n]$  is a sum of two DT sinewaves with frequencies  $3\pi/8$  and  $7\pi/8$ , respectively.

$$x[n] = 3e^{j\frac{3\pi}{8}n} + 2e^{j\frac{7\pi}{8}n} \quad (1)$$

Consider this signal as the input to each of the four systems described below.

$$\text{System 1: } y[n] = x[n] + (-1)^{n-1}x[n - 1] \quad (2)$$

$$\text{System 2: } y[n] = (-j)^n x[n] \quad (3)$$

$$\text{System 3: } y[n] = x[n]x[n - 1] \quad (4)$$

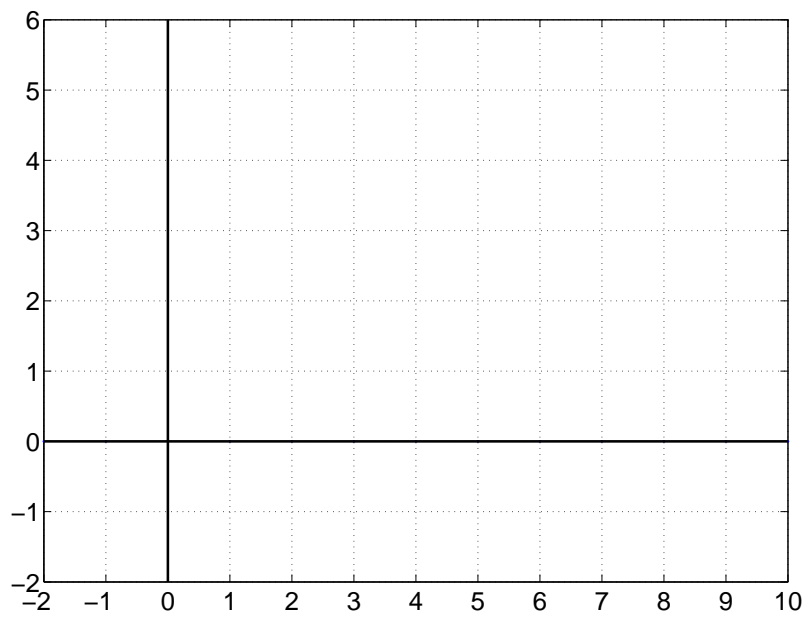
$$\text{System 4: } y[n] = x^*[-n] \quad (5)$$

$$(6)$$

For EACH of the four systems above, you must answer EACH of the following THREE questions in the Table provided in the sheets attached. **NOTE:** you do not have to determine the numerical values of any multiplicative scalars in the output - just determine what are the frequencies of the complex sinewaves present in  $y[n]$ .

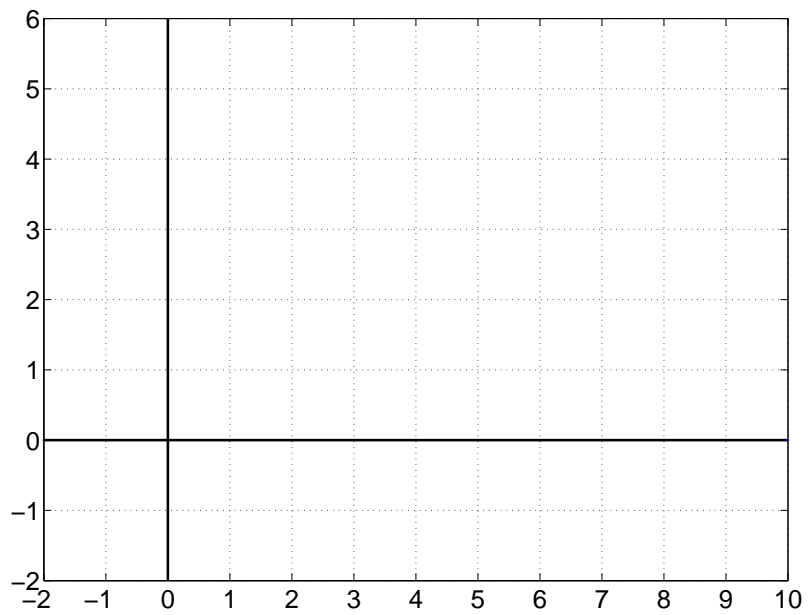
- (i) Is the system linear? Yes or No (don't need to justify your answer.)
- (ii) Is the system time-invariant? Yes or No (don't need to justify your answer.)
- (iii) Determine the frequencies in the output  $y[n]$  of each system given the input in Equation 1 above. Each answer should be in the range  $[-\pi, \pi]$ .

Plot your answer to Problem 1 (a) here.

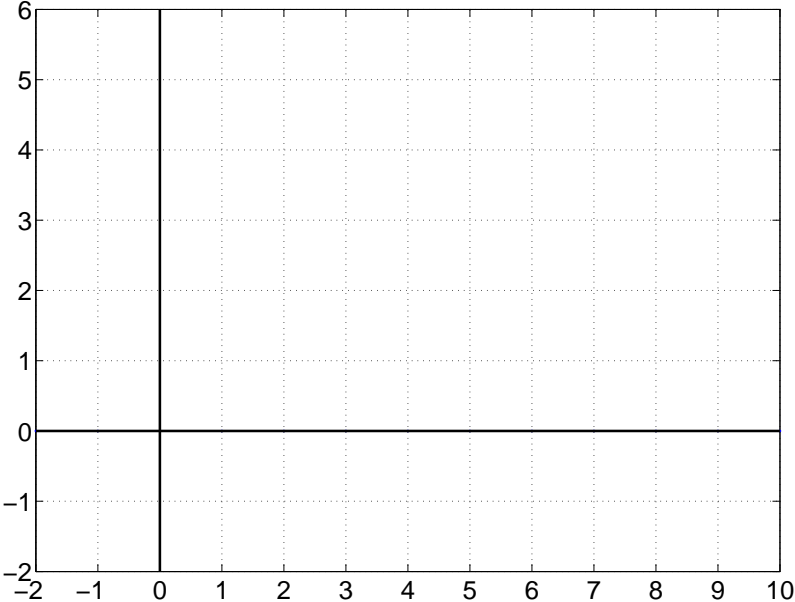


Show your work and write your answers to Problem 1, parts (b) and (c) on this page.

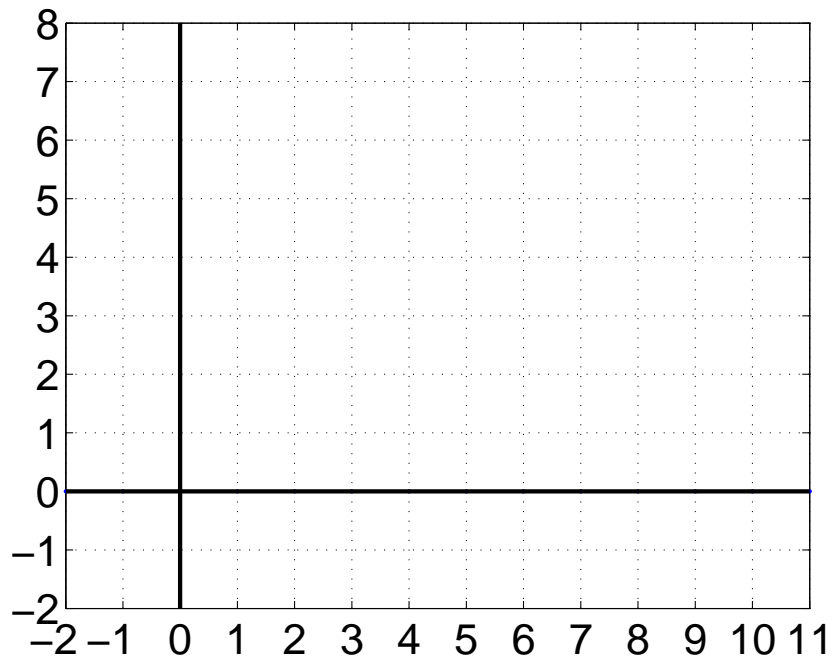
Plot your answer to Problem 1 (d) here.



Plot your answer to Problem 1 (e) here.

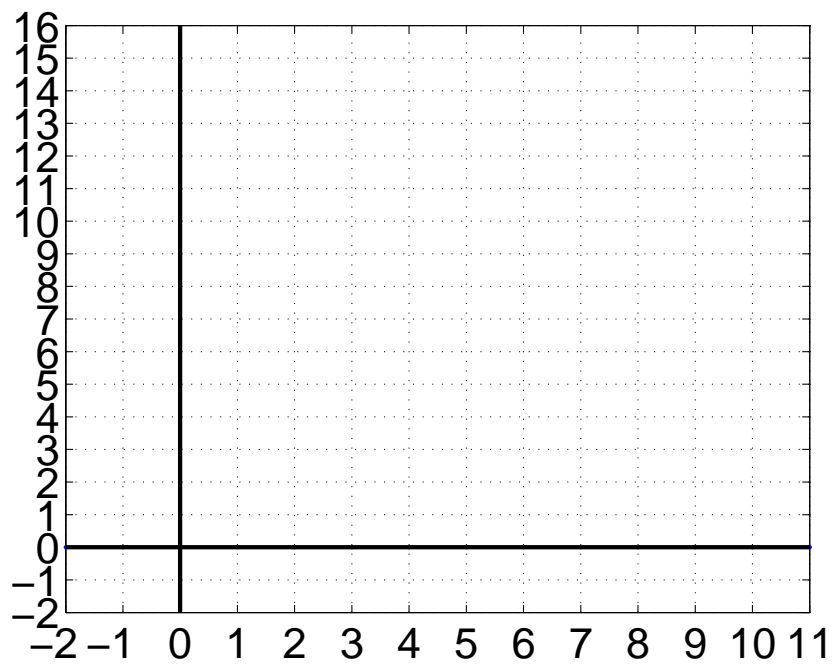


Plot your answer to Problem 1 (f) here.

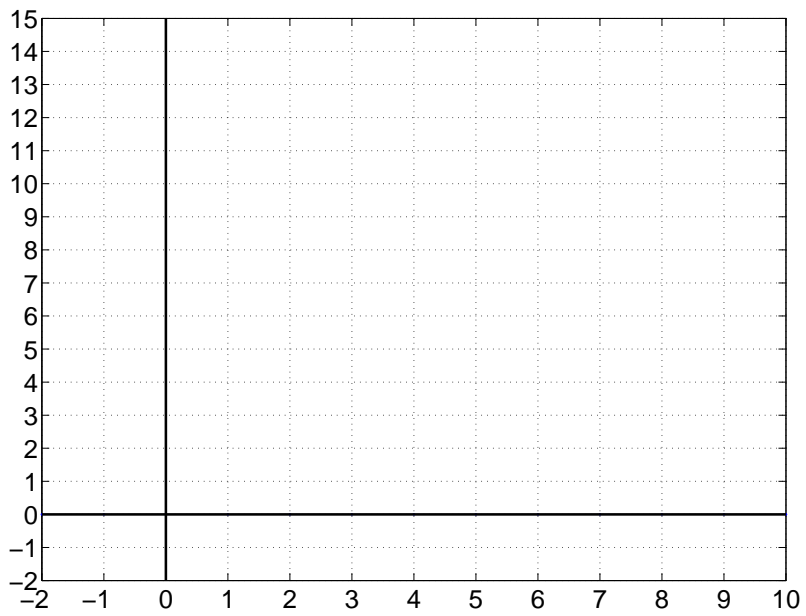




Plot your answer to Problem 1 (g) here.



Plot your answer to Problem 2 (a) here.



Show your work and write your answers to Problem 2, part (b) on this page.

System	Linear?	Time-Invariant?	Frequencies in $y[n]$ in range $[-\pi, \pi]$
1			
2			
3			
4			