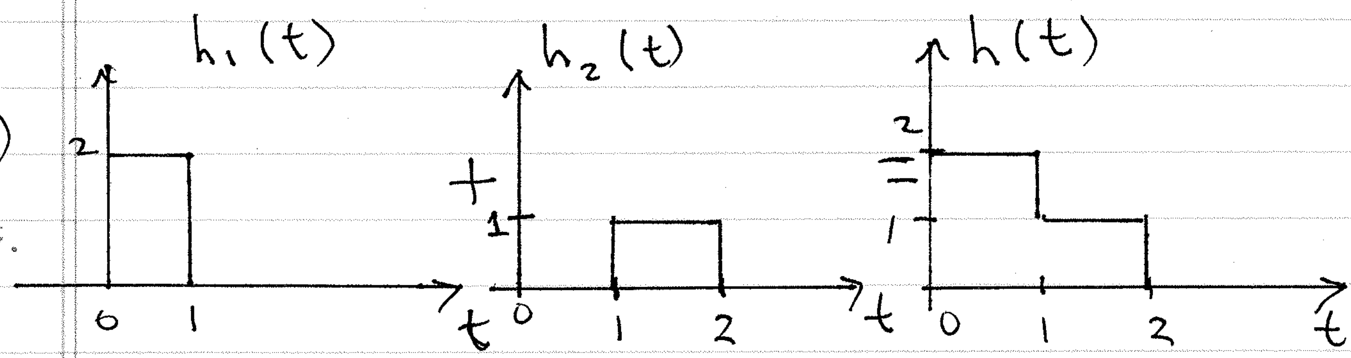
ECE 301 Exam 1 Spring 2010

Solution

Problem 1 (55) parallel $\Rightarrow h(t) = h_1(t) + h_2(t)$

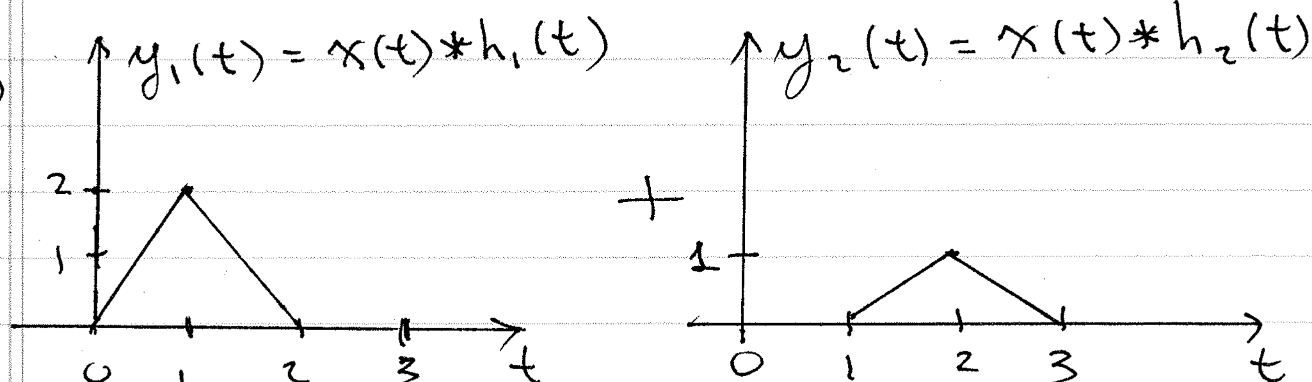
(a) 5 pts.

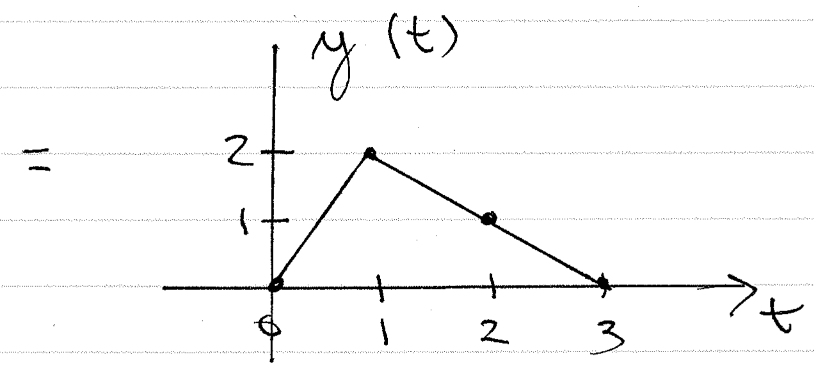


5 pts. (b) $h(t) = 0$ for $t < 0 \Rightarrow$ causal!

5 pts. (c) $\displaystyle\int_{-\infty}^{\infty} |h(t)|\,dt = 2 + 1 = 3 < \infty \Rightarrow$ stable!

5 pts. (d)

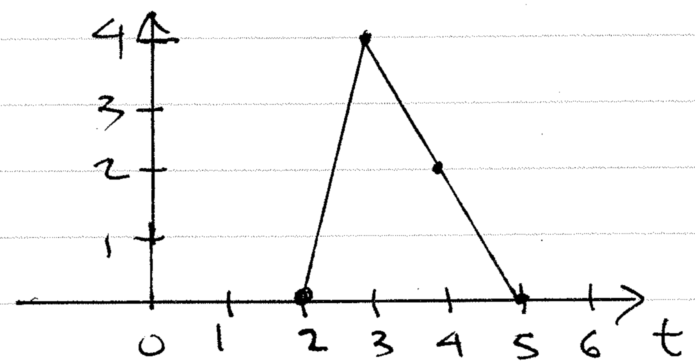




Prob. 1 (e) $x_2(t) = 2x_1(t-2)$

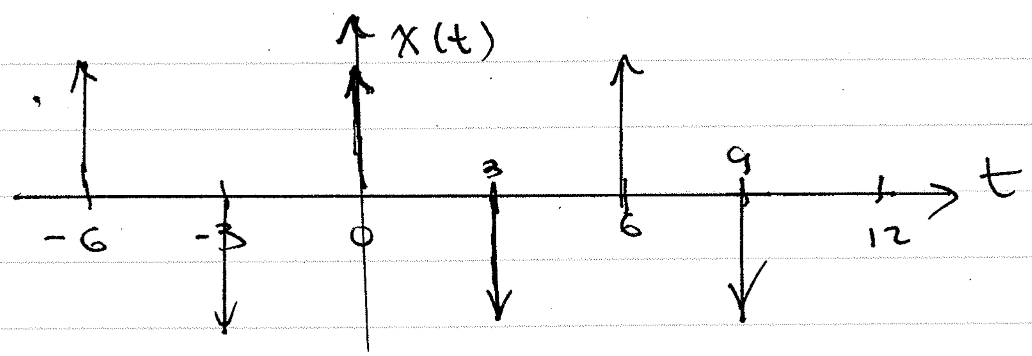
Second input signal scaled by 2 and shifted to right by 2 seconds. Since system is LTI:

$$y_2(t) = 2y_1(t-2)$$



10 pts. total

(f)



5 pts. Period = $T = 6$

5 pts. : $$a_k = \frac{1}{6}\left(1 - e^{-j 2\pi \frac{k}{6}(3)}\right)$$

$$= \frac{1}{6}\left(1 - e^{-j\pi k}\right)$$

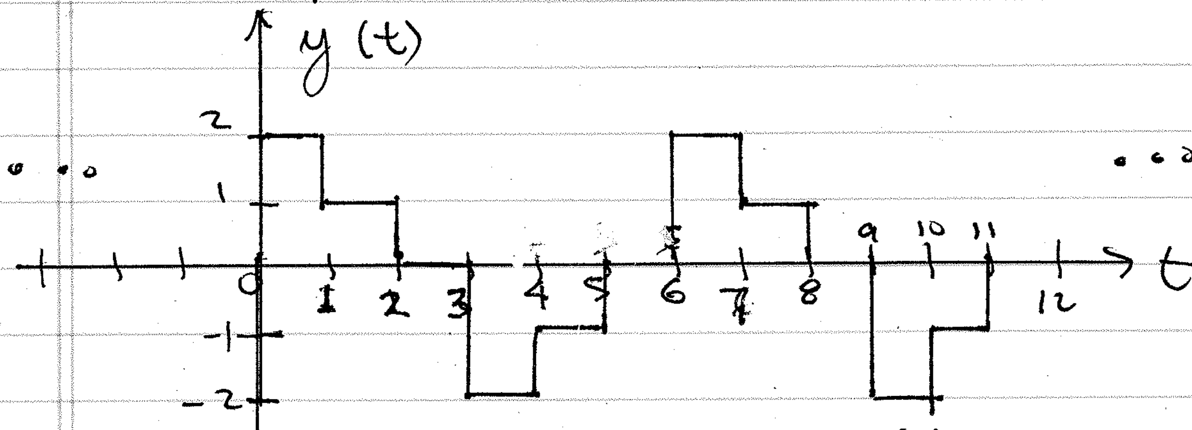
$$= \frac{1}{6}\left(1 - (-1)^k\right)$$

Prob. 1 (g)

(i) $$y(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-k3) * h(t)$$

5 pts.

$$= \sum_{k=-\infty}^{\infty} (-1)^k h(t-k3)$$



(ii) $$b_k = \frac{\sin\left(k\pi \frac{1}{6}\right)}{k\pi} \left\{ 2e^{-j 2\pi \frac{k}{6}\left(\frac{1}{2}\right)} + e^{-j 2\pi \frac{k}{6}\left(\frac{3}{2}\right)} - 2e^{-j 2\pi \frac{k}{6}\left(\frac{7}{2}\right)} - e^{-j 2\pi \frac{k}{6}\left(\frac{9}{2}\right)} \right\}$$

10 pts.

(iii) $$\sum_{k=-\infty}^{\infty} |b_k|^2 = \frac{1}{6} \int_0^6 y^2(t)\, dt$$

5 pts.

$$= \frac{1}{6} \left\{ 2^2(1) + (1)^2(1) + (-2)^2(1) + (-1)^2(1) \right\}$$

$$= \frac{1}{6} \left\{ 4 + 1 + 4 + 1 \right\} = \frac{10}{6} = \frac{5}{3}$$

Problem #2

$$x[n] = e^{j\pi/8 n} + e^{j3\pi/4 n}$$

Periodic? Yes, sum of two periodic signals.

Period? $T_1 = \frac{2\pi}{\pi/8} = 16$, $T_2 = \frac{2\pi}{3\pi/4} = 8/3$ } period = 8

$$T_{overall} = \text{lcm}(16, 8) = 16$$

S_1 : $y[n] = |x[n]|^2$

(a) Linear?

$$\hat{x}[n] = \alpha x_1[n] + \beta x_2[n]$$

$$S_1[\hat{x}[n]] = \hat{y}[n] = |\alpha x_1[n] + \beta x_2[n]|^2$$

$$= (\alpha x_1[n] + \beta x_2[n])(\alpha x_1[n] + \beta x_2[n])^*$$

$$= \alpha^2 |x_1[n]|^2 + \beta^2 |x_2[n]| + \alpha\beta x_1^*[n] x_2[n]$$

$$+ \alpha\beta x_1[n] x_2^*[n]$$

$$\alpha S_1[x_1[n]] + \beta S_1[x_2[n]] = \alpha |x_1[n]|^2 + \beta |x_2[n]|^2$$

$$\neq S_1[\hat{x}[n]].$$ Not Linear!

(b) Time-Invariant?

(1) Delay input: $(x[n] \Rightarrow x[n-k])$

$$|x[n]| \rightarrow |x[n-k]|.$$

(2) Delay output: $(y[n] \rightarrow y[n-k])$

$$y[n-k] = |x[n-k]|$$

(1) = (2) $\Rightarrow$ Time-Invariant.

(c) Output?

$$y[n] = \left| e^{j\pi/8 n} + e^{j3\pi/4 n} \right|^2$$

$$= \left(e^{j\pi/8 n} + e^{j3\pi/4 n}\right)\left(e^{j\pi/8 n} + e^{j3\pi/4 n}\right)^*$$

$$= \left(e^{j\pi/8 n} + e^{j3\pi/4 n}\right)\left(e^{-j\pi/8 n} + e^{-j3\pi/4 n}\right)$$

$$= 1 + e^{j(\pi/8 - 3\pi/4)n} + e^{j(3\pi/4 - \pi/8)n}$$

$$+ 1$$

$$= 2 + e^{-j5\pi/8 n} + e^{j5\pi/8 n}$$

$$= \underline{2 + 2\cos(5\pi/8 n)}.$$

Freq's: $\underline{5\pi/8}$, $-5\pi/8$, 0

S_2: $y[n] = x[4n]$

(a) Linear? $S_2[\alpha x_1[n] + \beta x_2[n]] =$

$$\alpha x_1[4n] + \beta x_2[4n]$$

$$= \alpha S_2[x_1[n]] + \beta S_2[x_2[n]].$$

<u>Linear!</u>

(b) Time-Invariant?

$x[n] = \delta[n] \longrightarrow y[n] = \delta[n]$

$x[n] = \delta[n-4] \longrightarrow y[n] = \delta[n-1]$

<u>Time-Varying!</u>

(c) Output?

$$y[n] = e^{j\pi/8(4n)} + e^{j3\pi/4(4n)}$$

$$= e^{j\pi/2\, n} + e^{j3\pi n}$$

$\hookrightarrow 3\pi - 2\pi = \pi$

$$= \underline{e^{j\pi/2\, n} + e^{j\pi n}}.$$

Freqs: $\underline{\pi/2, \pi}$.

S_3: (a) Linear? $S_3[\alpha x_1[n] + \beta x_2[n]] =$

$$-(\alpha x_1[n-1] + \beta x_2[n-1]) + 2(\alpha x_1[n] + \beta x_2[n]) - (\alpha x_1[n+1] + \beta x_2[n+1])$$

$$= (-\alpha x_1[n-1] + 2\alpha x_1[n] - \alpha x_1[n+1]) + (-\beta x_2[n-1] + 2\beta x_2[n] - \beta x_2[n+1])$$

$$= \alpha(-x_1[n-1] + 2x_1[n] - x_1[n+1]) + \beta(-x_2[n-1] + 2x_2[n] - x_2[n+1]).$$

$$= \alpha S_3[x_1[n]] + \beta S_3[x_2[n]].$$

Linear!

(b) Time-Invariant?

$x[n] \Rightarrow x[n-k]$:

$-x[n-k-1] + 2x[n-k] - x[n-k+1]$

$y[n] \Rightarrow y[n-k]$:

$y[n-k] = -x[n-k-1] + 2x[n-k] + x[n-k+1]$

Equal $\Rightarrow$ $\underline{\text{Time-Invariant!}}$

(e) Output?

$$y[n] = -\left(e^{j\pi/8(n-1)} + e^{j3\pi/4(n-1)}\right) + 2\left(e^{j\pi/8 n} + e^{j3\pi/4 n}\right)$$

$$-\left(e^{j\pi/8(n+1)} + e^{j3\pi/4(n+1)}\right)$$

$$= \left(-e^{-j\pi/8} + 2 - e^{j\pi/8}\right) e^{j\pi/8 n} + \underline{\left(-e^{-j3\pi/4} + 2 - e^{+j3\pi/4}\right) e^{j3\pi/4 n}}.$$

Freq's: $\underline{\pi/8,\ 3\pi/4}$.

S_4: (a) Linear? $y[n] = (j)^n x[n]$

$S_4[\alpha x_1[n] + \beta x_2[n]] =$

$(j)^n (\alpha x_1[n] + \beta x_2[n])$

$= \alpha (j)^n x_1[n] + \beta (j)^n x_2[n]$

$= \alpha S_4[x_1[n]] + \beta S_4[x_2[n]]$.

<u>Linear!</u>

(b) Time-Invariant?

$x[n] \rightarrow x[n-k]$

$(j)^n x[n-k]$

$y[n] \rightarrow y[n-k]$

$y[n-k] = (j)^{n-k} x[n-k]$

Not equal!

<u>Time-Varying!</u>

(c) Output?

$$y[n] = (j)^n \left(e^{j\pi/8 n} + e^{j3\pi/4 n}\right)$$

Note: $j = e^{j\pi/2}$

$$y[n] = e^{j\pi/2 n}\left(e^{j\pi/8 n} + e^{j3\pi/4 n}\right)$$

$$= e^{j5\pi/8 n} + e^{j5\pi/4 n}$$

$$\boxed{5\pi/4 - 2\pi = -3\pi/4}$$

$$= \underline{e^{j5\pi/8 n} + e^{-j3\pi/4 n}}.$$

Freq's: $\underline{5\pi/8, -3\pi/4}$.