

Cover Sheet

Test Duration: 75 minutes.

Coverage: Chaps. 1,2,3

Open Book but Closed Notes.

Calculators NOT allowed.

This test contains **two** problems.

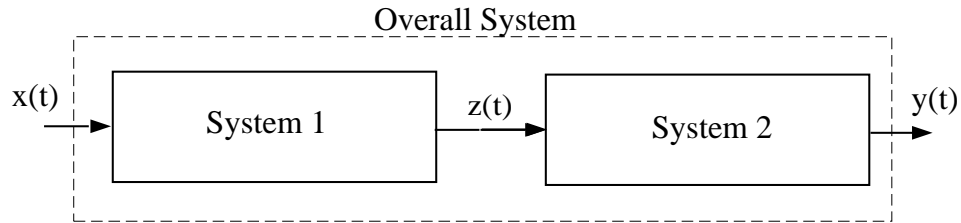
All work should be done in the blue books provided.

You must show all work for each problem to receive full credit.

Do **not** return this test sheet, just return the blue books.

Prob. No.	Topic(s)	Points
1.	Continuous Time Signals and System Properties and CT Fourier Series	50
2.	Discrete Time Signals and System Properties	50

Problem 1. [50 points]

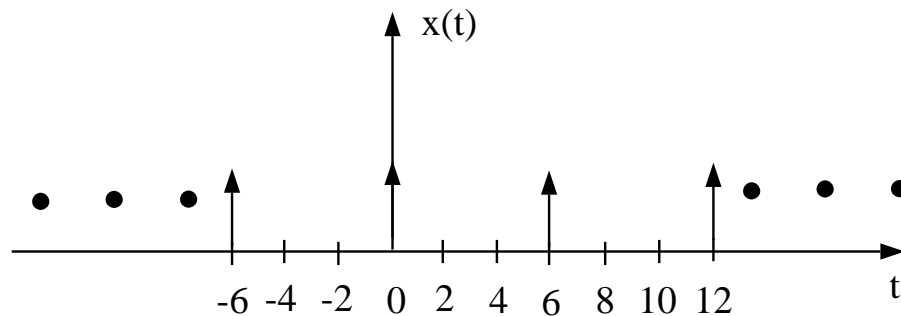


Consider the system above characterized by the following two input-output relationships:

$$\text{System 1: } z(t) = \int_{t-3}^t x(\tau) d\tau$$

$$\text{System 2: } y(t) = \int_{t-3}^t z(\tau) d\tau$$

- (a) Is the overall system linear? Substantiate your answer.
- (b) Is the overall system time-invariant? Substantiate your answer.
- (c) Determine and plot the impulse response of the overall system.
- (d) Is the overall system causal? Use your answer to (c) to justify your answer.
- (e) Is the overall system stable? Use your answer to (c) to justify your answer.
- (f) Consider that the input to this system is the periodic signal $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - k6)$.



Determine the Fourier Series coefficients, denoted a_k , $-\infty < k < \infty$, for $x(t)$. Express your final answer for a_k as a closed-form function of k that works for all k .

- (g) Determine and plot several periods of the output, $y(t)$, of the overall system above to the periodic signal $x(t)$ above. **ALSO:** Determine the Fourier Series coefficients, denoted b_k , $-\infty < k < \infty$, for $y(t)$. Express your final answer for b_k as a closed-form function of k that works for all k .

Problem 2. [50 points] An signal $x[n]$ is a sum of two DT sinewaves with frequencies $\pi/2$ and $\pi/3$, respectively.

$$x[n] = e^{j\frac{\pi}{3}n} + e^{j\frac{\pi}{2}n} \quad (1)$$

Is this signal periodic? if so, what is the period? Consider this signal as the input to each of the four systems described below.

$$\text{System 1: } y[n] = x[n-1] + 2x[n] + x[n+1] \quad (2)$$

$$\text{System 2: } y[n] = x[2n] \quad (3)$$

$$\text{System 3: } y[n] = x^2[n] \quad (4)$$

$$\text{System 4: } y[n] = (-1)^n x[n] \quad (5)$$

$$(6)$$

For EACH of the four systems below, you must answer the following THREE questions:

- (a) Is the system linear? Yes or No: don't need to substantiate your answer.
- (b) Is the system time-invariant? Yes or No: don't need to substantiate your answer.
- (c) Determine the output $y[n]$ of the system given the input in Equation 1 above. Specify the frequencies present in the output. Each answer should be in the range $(-\pi, \pi)$.

For which of the four systems is the set of output frequencies equal to the set of input frequencies? Is that system both linear and time-invariant?