

Cover Sheet

Test Duration: 70 minutes.

Coverage: Chaps. 1,2,3

Open Book but Closed Notes.

Calculators NOT allowed.

This test contains **two** problems.

All work should be done in the blue books provided.

You must show all work for each problem to receive full credit.

Do **not** return this test sheet, just return the blue books.

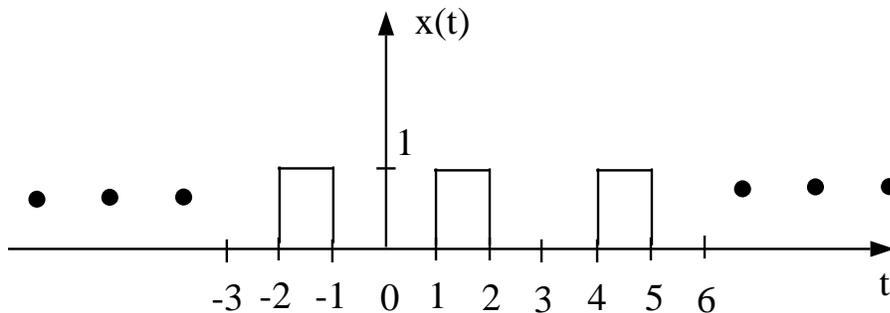
| Prob. No. | Topic(s) | Points |
|-----------|--|--------|
| 1. | Continuous Time System Properties and Continuous Time Fourier Series | 50 |
| 2. | Discrete Time System Properties and Discrete Time Fourier Series | 50 |

Problem 1. [50 points]

Consider the linear and time-invariant system described by the input-output relationship:

$$y(t) = \int_{t-1}^t x(\tau) d\tau$$

- (a) Determine and plot the impulse response $h(t)$ of this system.
- (b) Is this system causal? You must use $h(t)$ to justify your answer.
- (c) Is this system stable? You must use $h(t)$ to justify your answer.
- (d) Consider that the input to this system is the periodic signal $x(t)$ below with period $T = 3$.



Determine the Fourier Series coefficients, denoted a_k , $-\infty < k < \infty$, for $x(t)$. Express your final answer for a_k as a closed-form function of k that works for all k (like we've done in class many times.)

- (e)
 - (i) Plot several periods of the output of the system $y(t) = \int_{t-1}^t x(\tau) d\tau$ to the input signal $x(t)$ above.
 - (ii) Determine the Fourier Series coefficients, denoted b_k , $-\infty < k < \infty$, for $y(t)$. Express answer for b_k as a closed-form function of k .
 - (ii) Is b_k real-valued for all k ? Explain either why they are real-valued OR why they are not real-valued.
- (f) Consider the signal

$$w(t) = x(t) \cos\left(\frac{2\pi}{3}t\right)$$

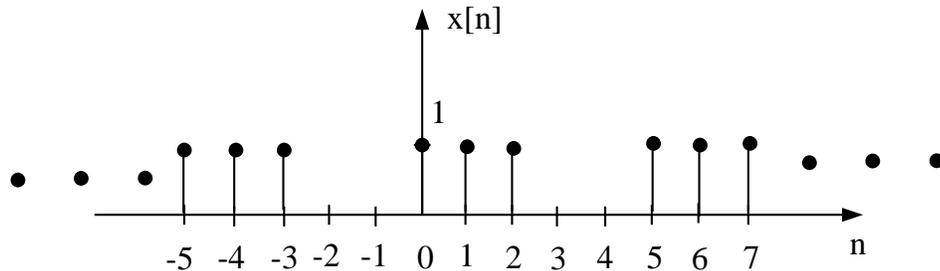
What is the period of $w(t)$? Determine the Fourier Series coefficients, denoted c_k , $-\infty < k < \infty$, for $w(t)$. Express your answer for c_k as a closed-form function of k .

Problem 2. [50 points]

Consider the linear and time-invariant system described by the input-output relationship:

$$y[n] = \sum_{k=n-2}^n x[k]$$

- (a) Determine and plot the impulse response $h[n]$ of this system.
- (b) Is this system causal? Use $h[n]$ to justify your answer.
- (c) Is this system stable? Use $h[n]$ to justify your answer.
- (d) Consider that the input to this system is the periodic signal $x[n]$ below with period $N = 5$.



Determine the Fourier Series coefficients, denoted a_k , $0 < k < 4$, for $x(t)$. Express your final answer for a_k as a closed-form function of k .

- (e)
 - (i) Plot several periods of the output of the system $y[n] = \sum_{k=n-2}^n x[k]$ to the input signal $x[n]$ above. **Hint** : you might want to solve part (f) below first.
 - (ii) Determine the Fourier Series coefficients, denoted b_k , $-\infty < k < \infty$, for $y[n]$. Express answer for b_k as a closed-form function of k .
 - (iii) Is b_k real-valued for all k ? Explain either why they are real-valued OR why they are not real-valued.
- (f) Consider the signal $w[n] = y[n] - y[n - 1]$, where $y[n]$ is the output signal $y[n]$. Determine the Fourier Series coefficients, denoted c_k , $0 \leq k \leq 4$, for $w[n]$. Express your final answer as c_k as a closed-form function of k .