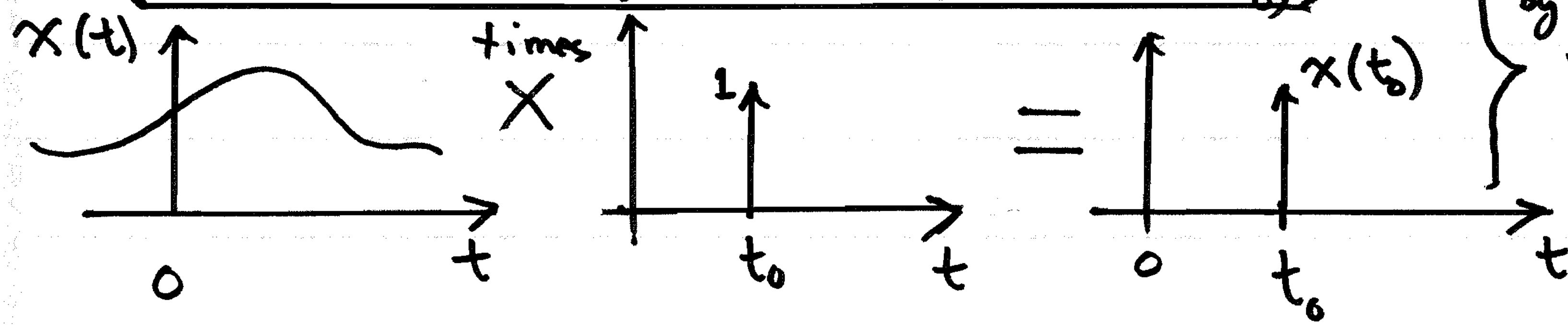
- Recall properties of (Dirac) Delta Function:

<u>Area:</u> $$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad \int_{-\infty}^{\infty} \delta(t-t_0)\,dt = \int_{t_0^-}^{t_0^+} \delta(t-t_0)\,dt = 1$$

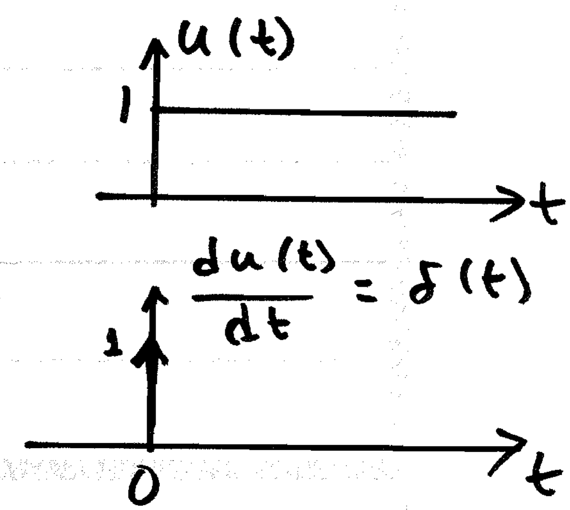
$$\boxed{x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)}$$



Multiplication by a Delta Function

<u>Symmetry:</u> $$\boxed{\delta(-t) = \delta(t)}$$

<u>Derivative at discontinuity</u>: $$\boxed{\delta(t) = \frac{du(t)}{dt}}$$



• Additional Properties of Delta Function:

$$\boxed{x(t) * \delta(t) = x(t)} \qquad \boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

Proof: $x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-\tau)\, d\tau$

• Sifting property dictates

$$= \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-t_0)\, d\tau$$

• $x(t-t_0)$ does not depend on integration variable τ

$$= x(t-t_0) \int_{-\infty}^{\infty} \delta(\tau - t_0)\, d\tau$$

• area under Delta Function is unity

$$= x(t-t_0)$$

Can easily show:

$$\delta(t-t_1) * \delta(t-t_2) = \delta(t-(t_1+t_2))$$