# How to measure the elongation and detect the symmetries of an object in a gray scale image.

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## Introduction

This document presents a few tricks that can be used to identify simple shapes or shape features in gray scale images. My graduate student Shanshan Huang and I develop these tricks in order to address some shape recognition problems that I encountered in some of my other projects, namely

- The Rosetta Phone Project, where we worked to develop a smart phone app to automatically read and translate signs in foreign (Arabic) characters;
- The MERGE Project, where we worked to develop a smart phone app to automatically segment and recognize HAZMAT signs;
- The GARI Project, where we worked to develop an app to automatically recognize gang graffiti.

1. The Pascal Triangle of a gray scale image

1.1 Inspiration

Shanshan Huang and I invented the Pascal Triangle in order to get a geometric interpretation of the content of a gray scale image. We were mostly interested in the geometric features of simple objects, such as

- Dots,
- Straight line segments,
- Loops,
- Crosses
- Etc.

Our inspiration was the concept of principal curvatures in differential geometry, which can be used to classify surfaces. However, we did not want to rely on derivatives, as these are ill-suited for application to discrete images. Rather, we worked with moments, a discrete analogue, which we could define directly in terms of the data given in the image.

1. The Pascal Triangle of a gray scale image

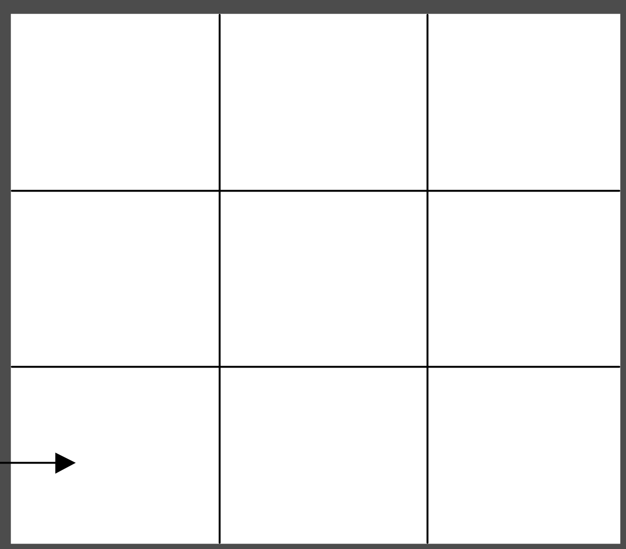
1.2 Definition

Start with a gray scale image with

pixel locations $(x_k, y_k)$

pixel intensity $w_k$

for $k = 1, \ldots, N$



Example: N=9 pixels

$(x_1, y_1) = (1,1)$

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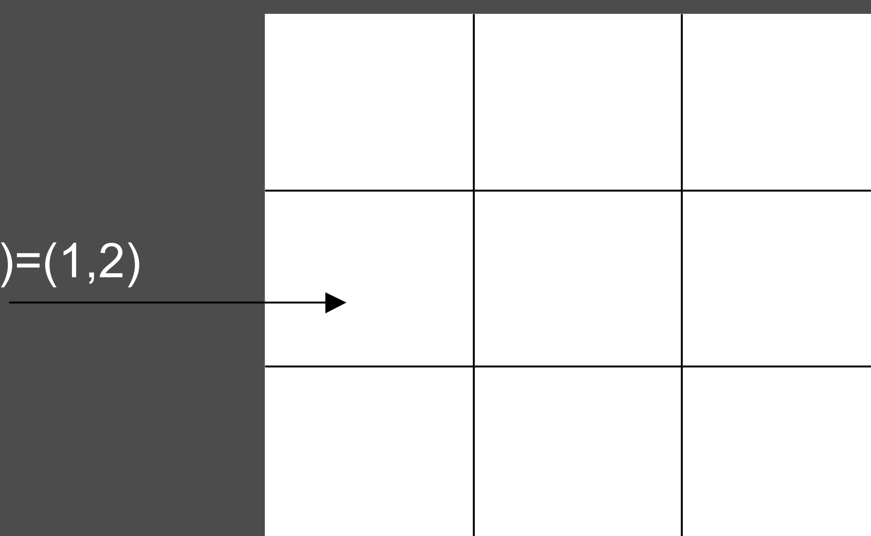

$(x_2,y_2)=(1,2)$

Example: N=9 pixels

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pixel intensity $w_k$

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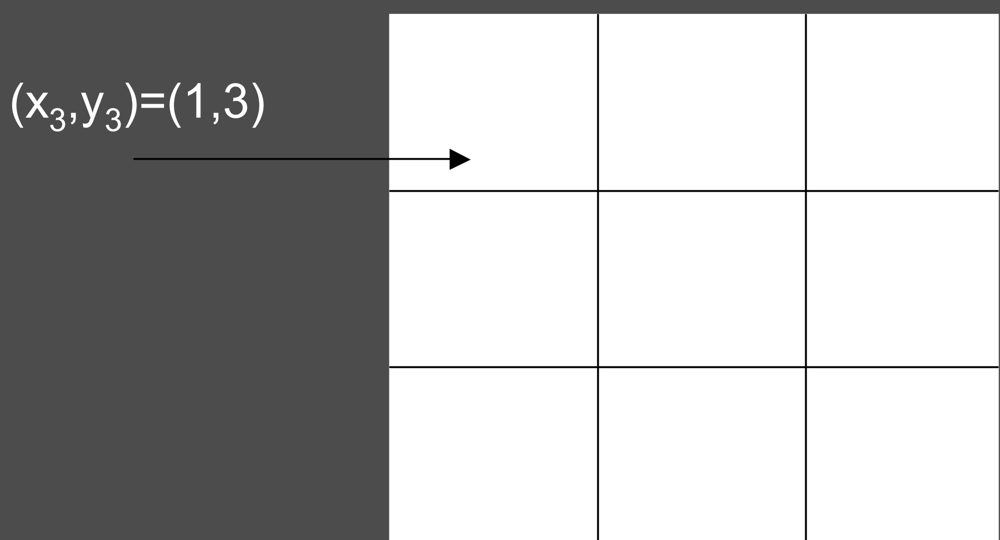


Example: N=9 pixels

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Start with a gray scale image with

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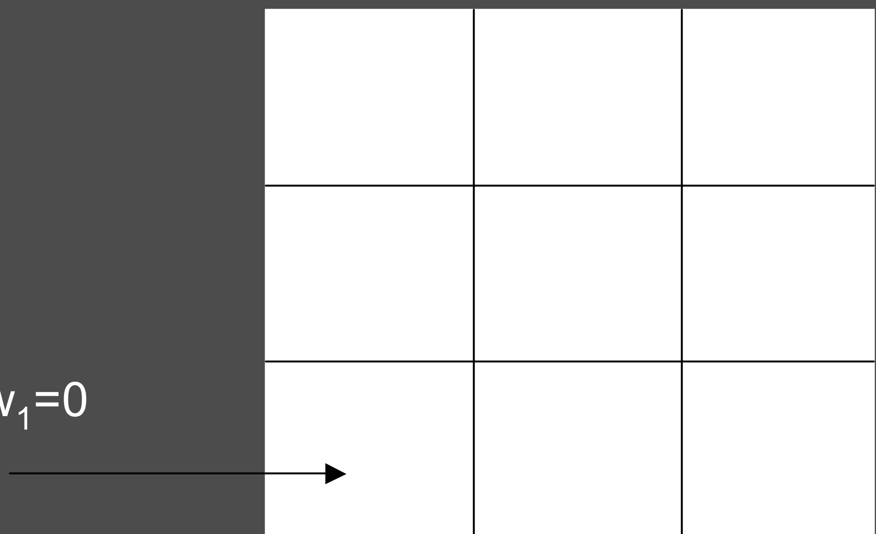
for $k = 1, \ldots, N$

Example: N=9 pixels

$w_1$=0



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## 1.2 Definition

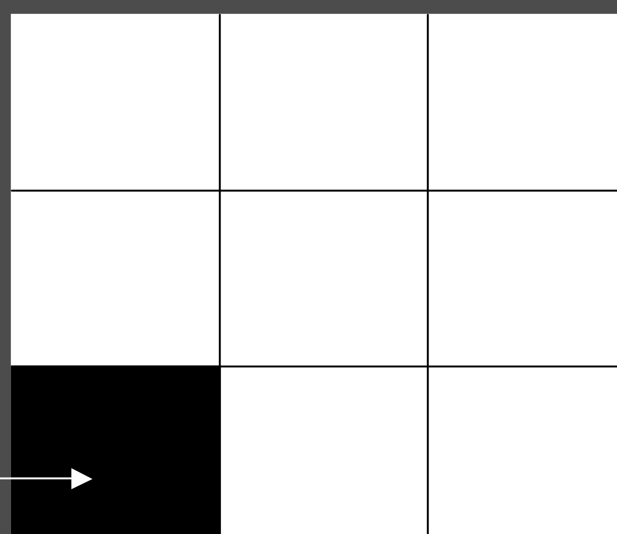
Start with a gray scale image with

pixel locations $(x_k, y_k)$

pixel intensity $w_k$

for $k = 1, \ldots, N$

$w_1$=255



Example: N=9 pixels

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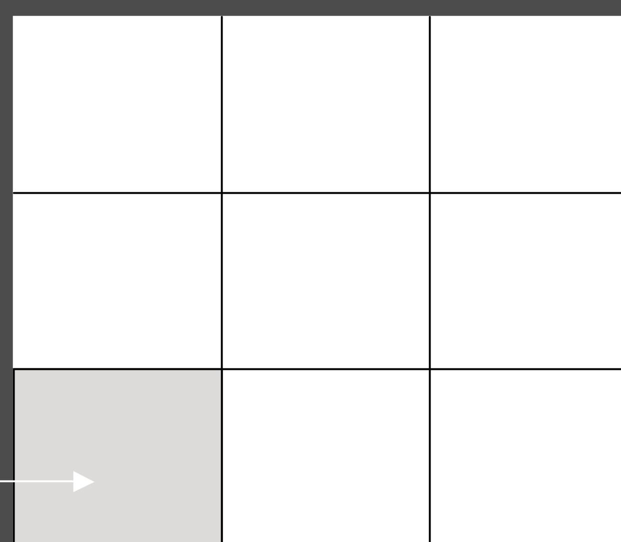
1.2 Definition

Start with a gray scale image with

pixel locations $(x_k, y_k)$

pixel intensity $w_k$

for $k = 1, \ldots, N$



Example: N=9 pixels

$w_1$=53

## 1.2 Definition

Write pixel location using complex numbers

$$(x_k, y_k) \leftrightarrow z_k = x_k + i\, y_k$$

for k = 1, …, N

1. The Pascal Triangle of a gray scale image

1.2 Definition

Write pixel location using complex numbers

$(x_k,y_k) \leftrightarrow z_k = x_k + i\, y_k$

for k = 1, …, N

Compute complex moments

$$\mu_{mn} = \sum_{k=1}^{N} w_k z_k^m \left(\overline{z_k}\right)^n$$

## 1.2 Definition

We form the Pascal triangle of the image by arranging the moments this way:

$$
\begin{array}{c}
\mu_{0,0} \\
\mu_{0,1} \quad \mu_{1,0} \\
\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0} \\
\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0} \\
\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0} \\
\vdots \\
\mu_{0,n} \quad \binom{n}{1}\mu_{1,n-1} \cdots \binom{n}{l}\mu_{l,n-l} \cdots \binom{n}{n-1}\mu_{n-1,1} \quad \mu_{n,0} \\
\vdots
\end{array}
$$

The elongation of the shape contained in the image is quantified by the quantity

$$\left|\frac{\mu_{02}}{\mu_{11}}\right|$$

$$\mu_{0,0}$$

$$\mu_{0,1} \quad \mu_{1,0}$$

$$\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0}$$

$$\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0}$$

$$\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0}$$

$$\vdots$$

$$\mu_{0,n} \quad ({}^{n})\,1\mu_{1,n-1} \quad \cdots \quad ({}^{n})\,l\mu_{l,n-l} \quad \cdots \quad ({}^{n})\,n-1\mu_{n-1,1} \quad \mu_{n,0}$$

$$\vdots$$

1. The Pascal Triangle of a gray scale image

1.3 How to quantify the elongation of an object in an image

The elongation of the shape contained in the image is quantified by the quantity

$$0 \le \left| \frac{\mu_{02}}{\mu_{11}} \right| \le 1$$

$$\begin{array}{c}
\mu_{0,0} \\
\mu_{0,1} \quad \mu_{1,0} \\
\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0} \\
\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0} \\
\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0} \\
\vdots \\
\mu_{0,n} \quad \binom{n}{1}\mu_{1,n-1} \cdots \binom{n}{l}\mu_{l,n-l} \cdots \binom{n}{n-1}\mu_{n-1,1} \quad \mu_{n,0} \\
\vdots
\end{array}$$

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The elongation of the shape contained in the image is quantified by the quantity

$$0 \leq \left| \frac{\mu_{02}}{\mu_{11}} \right| \leq 1$$

Straight lines (Most elongation)

$$\mu_{0,0}$$

$$\mu_{0,1} \quad \mu_{1,0}$$

$$\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0}$$

$$\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0}$$

$$\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0}$$

$$\vdots$$

$$\mu_{0,n} \quad (\,{}^{n}\,)\,1\mu_{1,n-1} \cdots (\,{}^{n}\,)\,l\mu_{l,n-l} \cdots (\,{}^{n}\,)\,n-1\mu_{n-1,1} \quad \mu_{n,0}$$

$$\vdots$$

## 1. The Pascal Triangle of a gray scale image

### 1.3 How to quantify the elongation of an object in an image

The elongation of the shape contained in the image is quantified by the quantity

Least elongation → $$0 \le \left|\frac{\mu_{02}}{\mu_{11}}\right| \le 1$$

$$\begin{array}{c}
\mu_{0,0} \\
\mu_{0,1} \quad \mu_{1,0} \\
\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0} \\
\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0} \\
\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0} \\
\vdots \\
\mu_{0,n} \quad ({}^{n})\,1\mu_{1,n-1} \quad \cdots \quad ({}^{n})\,l\mu_{l,n-l} \quad \cdots \quad ({}^{n})\,n-1\mu_{n-1,1} \quad \mu_{n,0} \\
\vdots
\end{array}$$

1. The Pascal Triangle of a gray scale image

1.3 How to detect reflection symmetry of an object in an image

Assume that center of mass of the image is at the origin.

Then the shape contained in the image is

symmetric with respect to the horizontal axis

if and only if

all the entries of the Pascal triangle $T^{2N-2}(I)$ are real.

1. The Pascal Triangle of a gray scale image
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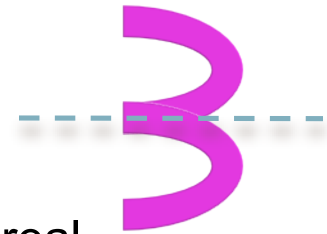


symmetric

$$
\begin{array}{c}
\mu_{0,0} \\
\mu_{0,1} \quad \mu_{1,0} \\
\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0} \\
\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0} \\
\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0} \\
\vdots \\
\mu_{0,n} \quad \binom{n}{1}\mu_{1,n-1} \cdots \binom{n}{l}\mu_{l,n-l} \cdots \binom{n}{n-1}\mu_{n-1,1} \quad \mu_{n,0} \\
\vdots
\end{array}
$$

All real entries

## 1.3 How to detect reflection symmetry of an object in an image

Assume that center of mass of the image is at the origin.

Then the shape contained in the image is
symmetric with respect to the horizontal axis
if and only if
all the entries of the Pascal triangle $T^{2N-2}(I)$ are real.

not symmetric

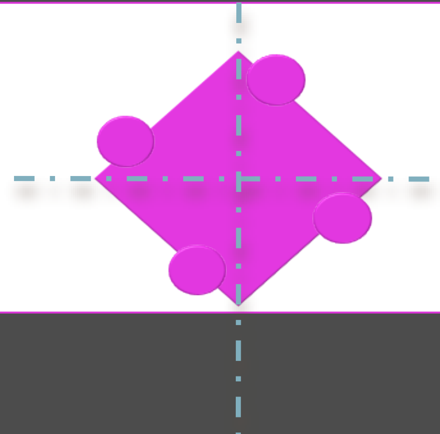
$$
\begin{array}{c}
\mu_{0,0} \\
\mu_{0,1} \quad \mu_{1,0} \\
\mu_{0,2} \quad 2\mu_{1,1} \quad \mu_{2,0} \\
\mu_{0,3} \quad 3\mu_{1,2} \quad 3\mu_{2,1} \quad \mu_{3,0} \\
\mu_{0,4} \quad 4\mu_{1,3} \quad 6\mu_{2,2} \quad 4\mu_{3,1} \quad \mu_{4,0} \\
\vdots \\
\mu_{0,n} \quad \binom{n}{1}\mu_{1,n-1} \cdots \binom{n}{l}\mu_{l,n-l} \cdots \binom{n}{n-1}\mu_{n-1,1} \quad \mu_{n,0} \\
\vdots
\end{array}
$$

Entries not all real

1. The Pascal Triangle of a gray scale image

1.3 How to detect rotationally symmetry of an object in an image

Assume that center of mass of the image is at the origin.
Let R be a positive integer.
Then an image I has an R-fold rotational symmetry
if and only if $\mu_{ij} = 0, \forall \frac{j-i}{R} \notin Z$



Example: R=4

$$
\begin{array}{c}
\tilde{\mu}_{0,0} \\
0 \quad 0 \\
0 \quad 2\tilde{\mu}_{1,1} \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
\tilde{\mu}_{0,4} \quad 0 \quad 6\tilde{\mu}_{2,2} \quad 0 \quad \tilde{\mu}_{4,0} \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 6\tilde{\mu}_{1,5} \quad 0 \quad 20\tilde{\mu}_{3,3} \quad 0 \quad 6\tilde{\mu}_{5,1} \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
\tilde{\mu}_{0,8} \quad 0 \quad 28\tilde{\mu}_{2,6} \quad 0 \quad 70\tilde{\mu}_{4,4} \quad 0 \quad 28\tilde{\mu}_{6,2} \quad 0 \quad \tilde{\mu}_{8,0} \\
\vdots
\end{array}
$$

For more details:

M. Boutin, S. Huang, “The Pascal Triangle of a discrete image: definition, properties, and application to shape analysis,” available at http://arxiv.org/abs/1209.4850

A.W. Haddad, S. Huang, M. Boutin and E. J. Delp, ``Detection of symmetric shapes on a mobile device with applications to automatic sign interpretation,'' Proc. of SPIE-IS&T Electronic Imaging 8304, p. 83040G, 2012.

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