Formation Control of the MAXI M L₂ Libration Orbit Mission

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Agenda

- MAXIM Introduction
- MAXIM Formation
- Formation Assumptions
- Formation Definition
- Control – Discrete and Continuous
- Results
- Summary
MAXIM Overview

- The MAXIM concept for NASA's Black Hole Imager mission utilizes interferometric techniques at the short wavelengths of X-rays
- Very long optical baselines are needed to achieve high-precision angular resolution images
Multiple free-flying spacecraft comprise a sparse aperture providing collecting area of ~ 1000cm².

Images are generated through interference patterns gathered from the multiple satellites housing the optical elements that form the aperture.

The interference patterns or fringes are observed only if the path lengths are controlled to great precision.

The challenge is to control this path length in the presence of environmental and spacecraft disturbances driving the need for active control systems.

We focus on the dynamics and control of formation flight in a full ephemeris modeling of the libration orbit to incorporate all gravitational perturbations and solar radiation pressure.

Analysis focuses on amount and duration of the control effort versus science observation requirements as measured in the formation optics plane.
MAXIM Formation Assumptions

- MAXIM formation components;
  Hub (1.3 x 2 meters, 331kg), Freeflyer (periscope) (1.3 x 2 meters, 304kg), and the Detector (varying area 1.9 m² to 5.6 m², 619kg)

- Optics Plane:
  - Hub and Freeflyers form a physical configuration perpendicular to detector-hub line of sight (LOS) to a target.
  - Associates physical configuration to science requirements derived from a Fourier transform of the image plane, the UV plane.

- Observation duration is 100,000 secs

- Controller options:
  - Off during observation and on to realign and maintain the formation
  - Continuously on during observations

- Inertial target of 45⁰ elevation and 45⁰ azimuth

- Tolerance of radial distance of a Freeflyer from Hub less than 5 microns

- Detector at 20,000km, six freeflyers at the maximum nominal radial distance of 500 meters from the Hub.
• MAXIM L₂ libration orbit is a typical mission
• A_y = 700,000 km and A_z = 200,000 km
• Halo orbit computed with a full Ephemeris model
  ✓ Sun, Earth, Moon point mass
  ✓ Solar Radiation Pressure

• Hub follows Halo orbit

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MAXIM Frame Definition

The MAXIM hub spacecraft is located at the $X,Y,Z$ origin and the angles $\alpha, \delta$ provide the alignment toward the target.

$$\hat{w} = C_\alpha C_\delta \hat{X} + S_\alpha C_\delta \hat{Y} + S_\delta \hat{Z}$$

$$\hat{u} = \frac{\hat{Z} \times \hat{w}}{|\hat{Z} \times \hat{w}|}$$

$$\hat{v} = \hat{w} \times \hat{u}$$

Direction Cosines for conversion between Optics frame and Inertial Frame

$$I^C U = \begin{bmatrix} -S_\alpha & -C_\alpha S_\delta & C_\alpha C_\delta \\ C_\alpha & -S_\alpha S_\delta & S_\alpha C_\delta \\ 0 & C_\delta & S_\delta \end{bmatrix}$$
MAXIM Control Strategies

- Our investigation takes a global view of the large-scale formation flying problem.

- Previous Research:
  - Near Earth - minimized gravitational perturbation - no close tracking of a reference solution - or use of non-linear (adaptive) 2-body problems
  - Multi-body systems - CRTBP only or controller effectiveness is demonstrated relative to the linear dynamics, not the full nonlinear system - Evolution approximated from the linear dynamics of the integrated lissajous trajectory
  - Naturally occurring formations derived from center manifold analysis, as well as a discrete impulsive control approach to maintain a prescribed formation plane

- Continuous control approach
  Obtain a rough analytical approximation of center manifold motion and determine how continuous optimal control and exact feedback linearization compares, in terms of cost, to the discrete station-keeping approach.
MAXIM Control Strategies

• Previous work demonstrates the efficiency and cost effectiveness of both input feedback linearization (IFL) and output feedback linearization (OFL) methods for formation control in the CRTBP.

• A linear quadratic regulator (LQR), derived from optimal control theory, yields essentially an identical error response and control acceleration history as the IFL approach.

• IFL controller is computationally much less intensive and, by comparison, conceptually simple.

• We address the properties of the IFL controller in defining the MAXIM formation control

• Analysis of position deviation of freeflyer or detector wrt Hub

• For a comparison, a discrete stationkeeping control approach is devised to force the orientation of the formation plane to remain fixed inertially.
MAXIM Discrete Control

- Accuracy of formation maintenance
- Simple DC can maintain formation
- Discrete LQR yields optimal magnitude of differential control impulse
- Simple: Target the end state
  \( \Phi = STM \)
  \( \delta = \) state perturbation
  \( \Delta v_0 = \) Impulsive \( \Delta V \) at beginning
- Discrete Optimal Control:
  \( (Q_m) \) Weighted quadratic of end state error
  \( (Q) \) Weighted quadratic of state deviation along path
- Simple has greatest error along path

\[
\begin{bmatrix}
\delta \bar{r}_1 \\
\delta \bar{v}_1
\end{bmatrix}
= \Phi(t_1, t_0)
\begin{bmatrix}
\delta \bar{r}_0 \\
\delta \bar{v}_0^+
\end{bmatrix}
= \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
\delta \bar{r}_0 \\
\delta \bar{v}_0^- + \Delta \bar{v}_0
\end{bmatrix}
\]

\( \Delta \bar{v}_0 = B^{-1}(\delta \bar{r}_1 - A \delta \bar{r}_0) - \delta \bar{v}_0^- \)
**MAXIM Nominal Motion and Determination of Vehicle Position Relative to Optics-Frame**

The nominal motion is in the local (spherical) coordinates while the control effort is formulated in the inertial focal frame.

Freeflyer / Detector Kinematics are written as

\[ \ddot{r}^{HD} = r\dot{d}_1 \]
\[ U\ddot{r}^{HD} = \dot{r}\dot{d}_1 + r\dot{v}C_v\dot{d}_2 + r\dot{\epsilon}\dot{d}_3 \]
\[ \dot{d}_1 = C_\epsilon C_v\dot{u} + C_\epsilon S_v\dot{v} + S_\epsilon\dot{w} \]

Cartesian coordinates to spherical:

\[ \tilde{x} = rC_vC_\epsilon \]
\[ \tilde{y} = rS_vC_\epsilon \]
\[ \tilde{z} = rS_\epsilon \]
\[ \dot{\tilde{x}} = rC_vC_\epsilon - r\dot{v}S_vC_\epsilon - r\dot{\epsilon}C_vS_\epsilon \]
\[ \dot{\tilde{y}} = rS_vC_\epsilon + r\dot{v}C_vC_\epsilon - r\dot{\epsilon}S_vS_\epsilon \]
\[ \dot{\tilde{z}} = r\dot{S}_\epsilon + r\dot{\epsilon}C_\epsilon \]

\[ \nu = \tan^{-1}\left(\frac{\tilde{z}}{\tilde{y}}\right) \]
\[ \epsilon = \tan^{-1}\left(\frac{\tilde{x}}{\sqrt{\tilde{y}^2 + \tilde{z}^2}}\right) \]

**NOMINAL MOTION:**

- **FF 1:** \( \nu^* = 0^\circ, \ \epsilon^* = 0^\circ, \ |\bar{r}_w| = 500 \text{ m} \)
- **FF 2:** \( \nu^* = 60^\circ, \ \epsilon^* = 0^\circ, \ |\bar{r}_w| = 500 \text{ m} \)
- **FF 3:** \( \nu^* = 120^\circ, \ \epsilon^* = 0^\circ, \ |\bar{r}_w| = 500 \text{ m} \)
- **FF 4:** \( \nu^* = 180^\circ, \ \epsilon^* = 0^\circ, \ |\bar{r}_w| = 500 \text{ m} \)
- **FF 5:** \( \nu^* = 240^\circ, \ \epsilon^* = 0^\circ, \ |\bar{r}_w| = 500 \text{ m} \)
- **FF 6:** \( \nu^* = 300^\circ, \ \epsilon^* = 0^\circ, \ |\bar{r}_w| = 500 \text{ m} \)
- **Detector:** \( \nu^* = 0^\circ, \ \epsilon^* = -90^\circ, \ |\bar{r}_w| = 20,000 \text{ km} \)

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MAXIMUM IFL Controller Development

Control of Equations of Motion (EOM) in Ephemeris Frame Wrt Earth (P₂)

- EOM for Freeflyer/detector
  \[ \ddot{r}^{P_2D_i}_I = f \left( r^{P_2D_i}_I, \dot{r}^{P_2D_i}_I \right) + \ddot{u}^{(D_i)}_I(t) \]

- EOM for Hub
  \[ \ddot{r}^{P_2H}_I = f \left( r^{P_2H}_I, \dot{r}^{P_2H}_I \right) \]

- Controller is selected as type of response as a critical damped

- Control in the local frame
  \[ U \ddot{r}_{UHD_i} = \left\{ U C^I \right\} \Delta f_I + \left\{ U C^I \right\} \ddot{u}^{(D_i)}_I(t) = \left\{ U C^I \right\} \Delta f_I + \ddot{u}^{(D_i)}_I(t) \]

- Controller eliminates system dynamics terms yields critical response control
  \[ \ddot{u}^{(D_i)}(t) = -\left\{ U C^I \right\} \Delta \dot{r}^{(D_i)}_I - 2\omega_n \left( U \ddot{r}_{UHD_i} - \dot{r}^* \right) - \omega_n^2 \left( r_{UHD_i} - \dot{r}^* \right) \]

- Once control determined in optics frame, rotate into inertial frame for controller
  \[ \ddot{u}^{(D_i)}_I(t) = \left\{ I C^U \right\} \ddot{u}^{(D_i)}(t) \]

(Note: Full state feedback for IFL and no constraints)
MAXIM Freeflyer Placement

Freeflyers at a maximum 500 meters from hub evenly spaced in azimuth at 60 degrees

Optics Plane View

Inertial View

UV–Plane View

Inertial Frame View ($\alpha=45^\circ, \delta=45^\circ$)
MAXIM Maintenance – Thrust Profiles

Detector < 7 mN

• 180 day IFL continuous control

Freeflyer ~ tenths of μN

Thrust Profiles proportional to spacecraft mass, e.g. 2:1

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MAXI M Maintenance and Recovery

- Maintenance for 1 day
- Control off during observation of 100,000 seconds
- Increase in radial errors of detector and freeflyer
- Recovery back to original positions in ½ day

✓ Error growth is not linear

✓ Peak error of 15 km for detector

✓ Peak errors range from 300mm to 550mm for freeflyer
MAXIM Maintenance and Recovery

Deviation in the Optics Plane During Observation With Control Off

Detector
Vertical Scale: u: 15 km to 0 km
v: +/- 5 km
w: +/- 5 km

Freeflyer
Vertical Scale: +/- 400 mm
In all 3 components (u,v,w)
Freeflyer Errors As Pointing Errors (Arc-seconds)

- Azimuthal angle ($\nu$) maximum $\sim 120$
- Out-of-plane ($\varepsilon$) Maximum $\sim 120$
MAXIM Maintenance, Observation, and Recovery

Three day simulation with maintenance 1 day, 100,000 sec observation, and ½ days recovery

Maintenance:
Detector required $3 \cdot 10^{-3}$ N  
Freeflyers required $< 0.05 \mu N$

Recovery:
Detector required 1N  
Freeflyers required $< 15 \mu N$

Detector

Recovery Profile

Freeflyers

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MAXIMUM Reorientation

- 90 degrees rotation about the z-axis
- Target initially along the inertial x-axis
- x-axis reoriented into y-axis direction
- Elevation angle set to zero

**Initial Orientation of Optics Plane**

**Final Orientation of Optics Plane**

**NOMINAL MOTION:**

- \( \nu' = 0^\circ, \varepsilon' = 0^\circ, |\vec{p}_w| = 500 \text{ m} \)
- \( \nu' = 60^\circ, \varepsilon' = 0^\circ, |\vec{p}_w| = 500 \text{ m} \)
- \( \nu' = 120^\circ, \varepsilon' = 0^\circ, |\vec{p}_w| = 500 \text{ m} \)
- \( \nu' = 180^\circ, \varepsilon' = 0^\circ, |\vec{p}_w| = 500 \text{ m} \)
- \( \nu' = 240^\circ, \varepsilon' = 0^\circ, |\vec{p}_w| = 500 \text{ m} \)
- \( \nu' = 300^\circ, \varepsilon' = 0^\circ, |\vec{p}_w| = 500 \text{ m} \)

Detector: \( \nu' = 0^\circ, \varepsilon' = -90^\circ, |\vec{p}_w| = 20,000 \text{ km} \)

Initial Formation Orientation: \( (\alpha,\delta) = (0^\circ,0^\circ) \)

Target Formation Orientation: \( (\alpha,\delta) = (90^\circ,0^\circ) \)
MAXIM Reorientation

- 7 day Simulation
- Detector ~ 1.5 N
- Freeflyer ~ 2.5 μN

**Thrust Levels**

**Freeflyer Displacement in Inertial Frame**
Vertical Scale +/- 0.5 Km

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\begin{align*}
\dot{w} & \parallel \hat{X} \\
\text{Target: } \dot{w} & \parallel \hat{Y} \\
\text{Reconfiguration Time Increased to } & \text{ 7 days to reduce Detector S/C Control Thrust}
\end{align*}
```
Summary

• Two Approaches, Discrete and Continuous, Were Investigated for the Control of the Maxim Formation.

• Simple or Optimal Discrete or by Input Feedback Linearization (IFL) Control.
  ✓ Discrete Control Approaches Continuous Time Interval Effort.
  ✓ IFL Continuous Control Combines the Effect of Annihilating the Environmental Dynamics While Adding a Specific User-defined Critically Damped Response

• The Total Maintenance Control Effort Requires
  ✓ Detector Thrust Level that Ranges From 4 mN to 7 mN
  ✓ Freeflyer Thrust Levels of 0.1 μN to 0.3 μN.

• Formation Recovery
  ✓ Detector Thrust Less than 1 N
  ✓ Freeflyers Less than 15 μN

• These Efforts Do Not Include Navigation or Maneuver Errors or Navigation Measurement Updates.

• The Challenge Is Propulsion System Implementation and Required Power Levels as Current Propulsion Technology Can Meet Minimum Thrust Levels