Geometry of Optimal Coverage for Space-based Targets with Visibility Constraints

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Problem Statement

• Define algorithm for maximizing coverage of space based targets within the bounds of a pre-specified altitude band.

• Assumptions:
  – No visibility below given altitude (tangent height)
  – Focus only on space based targets
  – Sensor range pre-defined
BTH Coverage Problem

MAX BTH COVERAGE = MAX COVERAGE ANGLE
Optimal Satellite Height for Maximum ATH Coverage

\[ h_s = \sqrt{(R_e + h_{low})^2 + R^2 - 2R(R_e + h_{low})\cos\phi} - R_e \]

where

- \( h_s \) is the optimal satellite height
- \( R \) is the satellite effective range (constant)
- \( R_{\text{tar}} = R_e + h_{\text{tar}} \) is the target radius (constant)
- \( h_{\text{tan}} \) is the tangent height (constant)

\[ \phi = \sin^{-1}\left(\frac{R_e + h_t}{R_e + h_{low}}\right) \]
Maximizing Visibility within a Bounded Altitude Range

• Goal: To maximize the area of intersection between the following curves (2D) or surfaces (3D)
  – UTAS → Upper Target Altitude Shell
  – LTAS → Lower Target Altitude Shell
  – RS → Range Shell
  – TL → Tangent Line
Factors Influencing Area Calculation

- Satellite Altitude
- Separation of UTAS and LTAS
- Size of RS and where it intersects the TL
- Intersections of TL with UTAS and LTAS
Step 1: Formulate Conditions & Eqns. For Curve Intersections
Step 2: Computing the Coverage Area as a function of Satellite Altitude

- Initially, if coverage exists at all, the area of coverage can be thought of as $\pi R^2 - (\text{area outside UTAS}) - (\text{area below LTAS}) - (\text{area below TL})$
- Each of these three terms depend on the size of the RS and the separation between the UTAS and the LTAS
- There is no single equation that generally describes the coverage area. Thus, all special cases must be identified a priori
- The area may be represented as a piecewise continuous function, but it is a highly nonlinear function.
- Identifying the optimal height is best accomplished by understanding the geometrical structure of the problem and through adequate numerical analysis.
Step 3: Identify Special Cases
Depending on Location of Critical Intersections

[Diagram with points labeled: S, U₂B, U₁B, B₂, T₂, T₁, B₁, U₂A, L₂B, L₁B, L₂A, L₁A, U₁A]
Step 4: Identify Simplest Form of Area Equation for each Possible Case

• There are multiple ways of formulating the same area equation, some more difficult than others.
• Divide area calculation into basic shapes
  – Triangles
  – Arc segments
  – Circular Sectors
• Computation depends only on Cartesian coordinates of Primary and Secondary Intersections
• Composite area equation depends only on elementary components
Constrained Search Space

- Satellite MUST be located:
  - Above the THS
  - Below no-coverage altitude:

\[ r_{s3} = \sqrt{\left( R + \sqrt{r_u^2 - r_t^2} \right)^2 + r_t^2} \]
Intersections of RS with the U/LTAS

\[
\begin{align*}
  x_{B_1}^2 + (y_{B_1} - y_s)^2 &= R^2 \\
  x_{B_1}^2 + y_{B_1}^2 &= r_u^2
\end{align*}
\]

\[
\begin{align*}
  y_{B_1} &= y_{B_2} = \frac{r_u^2 + r_s^2 - R^2}{2r_s} \\
  x_{B_1} &= -x_{B_2} = \sqrt{r_u^2 - y_{B_2}^2}
\end{align*}
\]

\[
\begin{align*}
  x_{A_1}^2 + (y_{A_1} - y_s)^2 &= R^2 \\
  x_{A_1}^2 + y_{A_1}^2 &= r_l^2
\end{align*}
\]

\[
\begin{align*}
  y_{A_1} &= y_{A_2} = \frac{r_l^2 + r_s^2 - R^2}{2r_s} \\
  x_{A_1} &= -x_{A_2} = \sqrt{r_l^2 - y_{A_2}^2}
\end{align*}
\]
Intersection of the TL with the LTAS

The equation for the TL that connects the satellite to the THS is given by,

\[ y = m(x - x_s) + y_s \]

Where \( m \) denotes the slope of the line,

\[ m = \frac{y_t - y_s}{x_t - x_s} \]

and

\[ x_t = r_t \sin \theta_t, \quad y_t = r_t \cos \theta_t, \quad \text{and} \quad \theta_t = \cos^{-1} \left( \frac{r_t}{r_s} \right). \]

The intersection of the TL with the LTAS is identified from the solution to the following system of equations:

\[
\begin{align*}
x_{L_{1/A/B}}^2 + y_{L_{1/A/B}}^2 &= r_t^2 \\
y_{L_{1/A/B}} &= mx_{L_{1/A/B}} + r_s
\end{align*}
\]

\[
\begin{align*}
x_{L_{2/A/B}} &= -x_{L_{2/A/B}} = \frac{-2mr_s \pm \sqrt{4m^2r_s^2 - 4(1 + m^2)(r_s^2 - r_t^2)}}{2(1 + m^2)} \\
y_{L_{2/A/B}} &= y_{L_{2/A/B}} = mx_{L_{2/A/B}} + r_s
\end{align*}
\]
Intersection of the TL with the UTAS

The intersections of the TL with the UTAS are similarly identified through the solution to the following system of equations,

\[
x_{U_{1A/B}}^2 + y_{U_{1A/B}}^2 = r_u^2
\]

\[
y_{U_{1A/B}} = mx_{U_{1A/B}} + r_s
\]

The solution is subsequently identified as,

\[
x_{U_{1A/B}} = -x_{U_{2A/B}} = \frac{-2mr_s \pm \sqrt{4m^2r_s^2 - 4(1+m^2)(r_s^2 - r_u^2)}}{2(1+m^2)}
\]

\[
y_{U_{1A/B}} = y_{U_{2A/B}} = mx_{U_{1A/B}} + r_s
\]
Intersection of the TL with the RS

The intersection of the TL with the RS is identified from the solution to the following system of equations,

\[ x_{T_1}^2 + \left(y_{T_1} - y_s\right)^2 = R^2, \]
\[ y_{T_1} = mx_{T_1} + y_s. \]

The solution to the above system is given by,

\[ x_{T_1} = -x_{T_2} = \frac{R}{\sqrt{1+m^2}}, \]
\[ y_{T_1} = y_{T_2} = mx_{T_1} + r_s. \]
Sample Area Calculation

Coverage Area Ratio = 0.063899

- Satellite Altitude (km): 2441.4
- Sensor Range (km): 15000
- Tangent Height (km): 100.0
- Upper Target Altitude (km): 3000
- Lower Target Altitude (km): 1000
- Planet Radius (km): 6378.14

Autoscale:
- xmin: -23819.54
- ymin: -23819.54
- xmax: 23819.54
- ymax: 23819.54
Geometrical Components
Triangle Area and Semiperimeter

- Δ’s are a large component of the coverage area geometry.
- Define the area of a Δ as a function of the semiperimeter, “s”, and the sides of the Δ; “a”, “b”, and “c”:

\[ s = \frac{(a + b + c)}{2}, \]

- “s” easily computed from available shell intersections
- Subsequently, the area of a triangular section is given by:

\[ A_\Delta (a, b, c) = \sqrt{s(s-a)(s-b)(s-c)} \]
Arc Segments

\[ A_\Sigma \left( r_u, c_{T_3} \right) = \frac{1}{2} \phi r_u^2 - \frac{c_{T_3} r_u^2}{2} \cos \frac{\phi}{2} \]

\[ \phi = 2 \sin^{-1} \left( \frac{c_{T_3}}{2 r_u} \right) \]
Area of Intersection Between Two Circles

The example to the left focuses on the Intersection of the RS with the LTAS. Note, in each case, the area of intersection is given by the sum of the area of two arc segments. However, that equation can vary by a constant factor depending on the geometry of the intersection (4 types).
### Area of Intersection of RS with L/UTAS

<table>
<thead>
<tr>
<th>Condition</th>
<th>Area of Intersection</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_i &gt; R \quad \text{and} \quad r_s &gt; \sqrt{r_i^2 - R^2} ) or ( r_i \leq R \quad \text{and} \quad r_s &gt; \sqrt{R^2 - r_i^2} )</td>
<td>( A_{RS \cap LTAS} = A_\Sigma \left( R,</td>
</tr>
<tr>
<td>( r_i &gt; R \quad \text{and} \quad r_s \leq \sqrt{r_i^2 - R^2} )</td>
<td>( A_{RS \cap LTAS} = \pi R^2 - A_\Sigma \left( R,</td>
</tr>
<tr>
<td>( r_i \leq R \quad \text{and} \quad r_s \leq \sqrt{R^2 - r_i^2} )</td>
<td>( A_{RS \cap LTAS} = \pi r_i^2 - A_\Sigma \left( r_i,</td>
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</tbody>
</table>
Composite Triangles: Type 1

\[ A_{\Lambda_1} \left( r_i, R, |B_2T_2|, |B_2L_2|, |T_2L_2| \right) = A_{\triangle} \left( |B_2T_2|, |B_2L_2|, |T_2L_2| \right) - A_{\Sigma} \left( r_i, |T_2L_2| \right) + A_{\Sigma} \left( R, |B_2T_2| \right) \]
Composite Triangle: Type 2

\[ A_{\Lambda_2} \left( r_i, \left| L_2 S \right|, \left| L_1 L_2 \right|, \left| L_1 S \right| \right) = A_{\Delta} \left( \left| L_2 S \right|, \left| L_1 L_2 \right|, \left| L_1 S \right| \right) - A_\Sigma \left( r_i, \left| L_1 L_2 \right| \right) \]
"Teardrop" Sections

\[
\mathbf{A}_{\pi_2}(r, R, \overline{P_1P_2}) = \begin{cases} 
\mathbf{A}_\Delta(R, \lvert \overline{P_1P_2} \rvert, R) + \mathbf{A}_\Sigma(r, \lvert \overline{P_1P_2} \rvert); & R < \sqrt{r_s^2 + r^2} \\
\mathbf{A}_\Delta(R, \lvert \overline{P_1P_2} \rvert, R) + \tilde{\mathbf{A}}_\Sigma(r, \lvert \overline{P_1P_2} \rvert); & R \geq \sqrt{r_s^2 + r^2} 
\end{cases}
\]
Summary of Special Cases

• Primary cases:
  - $R_t \leq R_s < R_l$
  - $R_l \leq R_s < R_u$
  - $R_u \leq R_s < R_{s3}$

• Subcases due to existence of intersections

  $|A_1A_2| = 0$ (entry/exit)
  $|B_1B_2| = 0$ (entry/exit)

• Subcases due to RS Size

  $|T_2S| < |U_{2b}S|$
  $|U_{2b}S| \leq |T_2S| < |L_{2b}S|$
  $|L_{2b}S| \leq |T_2S| < |L_{2a}S|$
  $|L_{2a}S| \leq |T_2S| < |U_{2a}S|$
  $|L_{2a}S| \leq |T_2S|$
## Area Geometry: Satellite Below UTAS

### Coverage Area Subcases for $r_t \leq r_s < r_l$

<table>
<thead>
<tr>
<th>Condition</th>
<th>Subcase</th>
<th>Condition</th>
<th>Subcase</th>
</tr>
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<tbody>
<tr>
<td>$T_2S &lt; L_{2A}S$</td>
<td>1(a)</td>
<td>$L_{2A}S \leq T_2S &lt; U_{2A}S$</td>
<td>1 (b.i): $[A_1A_2] \neq \emptyset$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 (c.i): $[A_1A_2] \neq \emptyset$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$U_{2A}S \leq T_2S$</td>
<td>1 (b.ii): $[A_1A_2] = \emptyset$</td>
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<td></td>
<td>1 (c.ii): $[A_1A_2] = \emptyset$</td>
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</tbody>
</table>

### Coverage Area Subcases for $r_l \leq r_s < r_u$

<table>
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<tr>
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<th>Subcase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_2S &lt; L_{2B}S$</td>
<td>2(a)</td>
<td>$L_{2B}S \leq T_2S &lt; L_{2A}S$</td>
<td>2(c.i): $[A_1A_2] \neq \emptyset$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_{2A}S \leq T_2S &lt; U_{2A}S$</td>
<td>2(d.i): $[A_1A_2] \neq \emptyset$</td>
</tr>
<tr>
<td></td>
<td>2(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$U_{2A}S \leq T_2S$</td>
<td>2(c.ii): $[A_1A_2] = \emptyset$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2(d.ii): $[A_1A_2] = \emptyset$</td>
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</tbody>
</table>
## Area Geometry: Satellite Above UTAS

### Coverage Area Subcases for $r_u \leq r_s < r_i$

<table>
<thead>
<tr>
<th>$T_2 S &lt; U_{2B} S$</th>
<th>$U_{2B} S \leq T_2 S &lt; L_{2B} S$</th>
<th>$L_{2B} S \leq T_2 S &lt; L_{2A} S$</th>
<th>$L_{2A} S \leq T_2 S &lt; U_{2A} S$</th>
<th>$U_{2A} S \leq T_2 S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(a)</td>
<td>3(b)</td>
<td>3(c)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3(d.i): $A_1 \cap A_2 \neq \emptyset$</th>
<th>3(e.i): $A_1 \cap A_2 \neq \emptyset$</th>
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<tr>
<td>3(d.ii): $A_1 \cap A_2 = \emptyset$</td>
<td>3(e.ii): $A_1 \cap A_2 = \emptyset$</td>
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**Coverage Area for** \( r_i \leq r_z < r_i \)

<table>
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<tr>
<th>Case</th>
<th>Area</th>
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<tbody>
<tr>
<td>1(a)</td>
<td>( A = A_{\text{UTAS} \cap RS} - A_{\text{LTAS} \cap RS} )</td>
</tr>
</tbody>
</table>
| 1(b.i) | \( A = A_{\text{UTAS} \cap RS} - A_{\text{LTAS} \cap RS} \)  
\[ A = A - 2A_{\alpha_1} \left( R, r_i, \overline{T_2 I_{2,4}}, \overline{T_2 T_2}, \overline{A_2 L_{2,4}} \right) \] |
| 1(b.ii) | \( A = A_{\text{UTAS} \cap RS} - \pi r_i^2 \)  
\[ A = A - A_{\alpha_1} \left( R, \overline{T_1 T_2} \right) \]  
\[ A = A + A_{\alpha_2} \left( r_i, R, \overline{L_{1,4} L_{2,4}} \right) \] |
| 1 (c.i) | \( A = A_{\text{UTAS} \cap RS} - A_{\text{LTAS} \cap RS} \)  
\[ A = A - 2A_{\alpha_1} \left( r_i, R, \overline{T_2 I_{2,4}}, \overline{T_2 A_2}, \overline{A_2 L_{2,4}} \right) \]  
\[ A = A + 2A_{\alpha_1} \left( r_i, R, \overline{T_2 U_{2,4}}, \overline{T_2 B_2}, \overline{B_2 U_{2,4}} \right) \] |
| 1 (c.ii) | \( A = \pi r_i^2 - \pi r_z^2 \)  
\[ A = A - A_{\alpha_2} \left( r_i, R, \overline{U_{1,4} U_{2,4}} \right) \]  
\[ A = A + A_{\alpha_2} \left( r_i, R, \overline{L_{1,4} L_{2,4}} \right) \] |
Coverage Area for $r_2 \leq r_2 < r_u$

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>2(a)</td>
<td>$\mathbf{A} = -\mathbf{A}<em>{UTAS-RS} - \mathbf{A}</em>{A_2} \left( r_1, \left</td>
</tr>
</tbody>
</table>
| 2(b) | $\mathbf{A} = \mathbf{A}_{UTAS-RS} - \mathbf{A}_{UTAS-RS}$  
$\mathbf{A} = -\mathbf{A}_{A_2} \left( r_1, \left| \begin{array}{c} L_{12}^1 S \end{array} \right|, \left| \begin{array}{c} L_{12} L_{22} \end{array} \right|, \left| \begin{array}{c} L_{22} S \end{array} \right| \right) $  
$\mathbf{A} = -2\mathbf{A}_{A_4} \left( r_1, R, \left| \begin{array}{c} T_2 L_{22} \end{array} \right|, \left| \begin{array}{c} T_2 A_1 \end{array} \right|, \left| \begin{array}{c} A_2 L_{22} \end{array} \right| \right) $ |
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$\mathbf{A} = -\mathbf{A}_{A_1} \left( R, \left| \begin{array}{c} T_2 T_2 \end{array} \right| \right) $  
$\mathbf{A} = \mathbf{A} + \mathbf{A}_{A_3} \left( r_1, R, \left| \begin{array}{c} T_2 L_{22} \end{array} \right| \right) $ |
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$\mathbf{A} = \mathbf{A} + 2\mathbf{A}_{A_4} \left( r_1, R, \left| \begin{array}{c} T_2 U_2 \end{array} \right|, \left| \begin{array}{c} T_2 B_2 \end{array} \right|, \left| \begin{array}{c} B_2 U_2 \end{array} \right| \right) $ |
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| 2(d.ii) | $\mathbf{A} = \pi r_2^2 - \pi r_1^2$  
$\mathbf{A} = -\mathbf{A}_{A_2} \left( r_1, R, \left| \begin{array}{c} U_1 U_2 \end{array} \right| \right) $  
$\mathbf{A} = \mathbf{A} + \mathbf{A}_{A_3} \left( r_1, R, \left| \begin{array}{c} L_1 L_2 \end{array} \right| \right) $  
$\mathbf{A} = -\mathbf{A}_{A_2} \left( r_1, \left| \begin{array}{c} L_{12}^1 S \end{array} \right|, \left| \begin{array}{c} L_{12} L_{22} \end{array} \right|, \left| \begin{array}{c} L_{22} S \end{array} \right| \right) $ |
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<td>3(a)</td>
<td>$A = 0$</td>
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<td>$A = A_{1,2, 3} - A_{h_1} (R,</td>
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<tr>
<td></td>
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<tr>
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<td>$A = A - 2A_{h_1} (r,</td>
</tr>
<tr>
<td></td>
<td>$A = A + 2A_{h_1} (r, \bar{I}_{12},</td>
</tr>
<tr>
<td>3(e.ii)</td>
<td>$A = \pi r_1^2 - \pi r_2^2$</td>
</tr>
<tr>
<td></td>
<td>$A = A - A_{h_1} (r, \bar{I}<em>{12}, \bar{I}</em>{12}, \bar{I}_{23})$</td>
</tr>
<tr>
<td></td>
<td>$A = A + A_{h_1} (r, \bar{I}<em>{12}, \bar{I}</em>{12}, \bar{I}_{23})$</td>
</tr>
<tr>
<td></td>
<td>$A = A + A_{h_1} (r, \bar{I}<em>{12}, \bar{I}</em>{12}, \bar{I}_{23})$</td>
</tr>
</tbody>
</table>
Coverage Area Analysis Tool
\[ R = 5000 \text{ km}, \ h_l = 1000 \text{ km}, \ h_u = 5000 \text{ km}, \text{ and } h_t = 100 \text{ km}. \]
\(h_u = 5600 \text{ km}, \ h_l = 600 \text{ km} \text{ (5000x5000 grid)}\)
Optimal Altitude Space
Conclusions

• Optimal satellite altitude non-intuitive
• Graphical tools helpful in design process
• Ongoing work
  – Multi-objective optimization for constellation design with applications to constrained ATH coverage problem.
• The results of the current investigation represent useful startup solutions for numerical optimization process.
• Results also provide physical insight into the expected trends.