Applications of Artificial Potential Function Methods to Autonomous Space Flight

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Artificial Potential Function (APF) Methods

- Extensive use in path planning applications
- Global minimum at goal, peaks at constraints
- Vehicle follows steepest descent of potential
- Extendable to more general trajectory planning
Modification for General Trajectory Design

- Discrete control parameter ($\Delta v$) vs. continuous control
- Formation flight problem - time derivative of $\Phi$ to define switching time
- Minimum of $\Phi$ $\rightarrow$ lowest available maneuver cost
- Potential as a function of velocity error $^1$ (i.e. $\Delta v$)
- Dynamical model to calculate desired velocity field

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Potential Function Construction

- Desired state/desired orbit (transfer time constraint)
- Two distinct cases: (1) intersecting orbits, and (2) non-intersecting orbits
- Initial orbit intersects target orbit $\rightarrow$ maneuver at intersection point:
  \[
  \Phi_{int} = (\mathbf{r}_{intersect} - \mathbf{r}_0)^T(\mathbf{r}_{intersect} - \mathbf{r}_0),
  \]
- For non-intersecting orbits, $\Phi = \Phi(\Delta v)$:
  \[
  \Phi_{vel} = \Delta v^2,
  \]
APF Maneuver Planning

- Initialize
- Is the vehicle on the desired orbit?
  - NO
  - Maximum number of maneuvers reached?
    - NO
    - Propagate current state
      - NO
      - \( \phi = \phi_{rel} \)
    - YES
    - \( \phi = \phi_{int} \)
  - YES
  - Apply computed \( \Delta v \)
    - YES
    - Apply computed \( \Delta v \)
- YES
- STOP
The Desired Velocity Field

- Desired velocity depends on target point $r_f$ along final orbit
- Target point assumed to be
  - apoapsis of the transfer orbit, if transferring to higher altitude
  - periapsis of the transfer orbit, if decreasing altitude
- Gives eccentricity vector direction, leads to desired velocity vector:

$$r_f \rightarrow \hat{e}_t \rightarrow e_t \rightarrow v_t$$
Coplanar Transfer: Desired Velocity Field

- Target point: 180° from current position

\[ \hat{r}_f = -\frac{r_0}{r_0}. \]

Figure: Coplanar Velocity Field and Potential Function
Coplanar Transfer

- Total $\Delta v$: 1.25 km/s
- Doubly cotangential $180^\circ$ transfer - matches analytical optimal result

Table: Coplanar Transfer
Initial and Target States

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial</th>
<th>Target</th>
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</thead>
<tbody>
<tr>
<td>$x$ (km)</td>
<td>-6478.145</td>
<td>0.000</td>
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<tr>
<td>$y$ (km)</td>
<td>0.000</td>
<td>12587.983</td>
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<tr>
<td>$z$ (km)</td>
<td>0.000</td>
<td>0.00000</td>
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<tr>
<td>$v_x$ (km/s)</td>
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<tr>
<td>$v_y$ (km/s)</td>
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<tr>
<td>$v_z$ (km/s)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Figure: Coplanar Transfer
The Desired Velocity Field: Inclined Transfer

- Target point: intersection of initial and desired orbit planes
- Higher altitude solution to minimize cost of the plane change

![Velocity Field](figure1a.png)

![Resulting Potential](figure1b.png)

**Figure:** Non-Coplanar Velocity Field and Potential Function
Inclined Transfer

- Total $\Delta v$: 3.46 km/s $\rightarrow$ Compare to 6.11 km/s for Lambert solution
- Consider three-maneuver sequence to reduce plane change

Table: Inclined Transfer
Initial and Target States

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial</th>
<th>Target</th>
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</thead>
<tbody>
<tr>
<td>$x$ (km)</td>
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<td>$y$ (km)</td>
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<td>$v_y$ (km/s)</td>
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<td>$v_z$ (km/s)</td>
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<td>0.410</td>
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Figure: Non-Coplanar Transfer
Lunar Example (1/3)

- More complex test of APF algorithm
- Specific time needed at target state; not guaranteed by APF method as-is
- Offset targeting to do timing match, typically converges in 3-4 iterations

Table: Initial Conditions

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<thead>
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<tbody>
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<td>y (km)</td>
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<td>z (km)</td>
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Table: Estimated Arrival Conditions

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<td>Longitude (deg)</td>
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<td>Geocentric Latitude</td>
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<td>Geocentric Azimuth (deg)</td>
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<td>Geocentric Flight Path Angle (deg)</td>
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</tbody>
</table>
Lunar Example (2/3)

- Total $\Delta v$: 1.9483 km/s

Figure: APF Lunar Return, 1.9483 km/s
Lunar Example (3/3)

- Investigate phasing effects on total cost
- Shift departure/arrival epoch by $n$ revolutions

**Figure:** $\Delta v$ of Time-Shifted Transfers vs. # of Revolutions
Conclusions

- Preliminary exploration of artificial potential function methods as a design tool for generating startup arcs
- Candidate potential function construction presented
- Method for calculating a desired velocity field developed based on two-body analysis
- APF trajectory design algorithm developed and tested
- APF method is promising, but room for improvement in design of potential field
- Future work will focus on constructing more complex potentials for use in multi-body regimes