Optimal Constellation Design for Space Based Situational Awareness Applications
A Numerical Approach

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Coverage Regions

Above vs. Below the Horizon

Above the Horizon (ATH)

Above the Horizon (ATH)

Below the Horizon (BTH)

Local Horizon

Tangent Height (i.e. edge of atmosphere)

Volumetric to Planar

Sensor Regions

Coverage Above the Horizon

Target Regions
Boolean Operations: $1 \times$ Coverage

\[
RS_{E_1} \cup RS_{E_2} = T_1
\]

\[
T_1 \cup RS_{E_3} = T_2
\]
Boolean Operations: $1 \times$ Coverage (cont.)

$T_4 \cup RS_{E_6} = RS_{TE}$

$RS_{TE} \cap AS = C_{1 \times}$
Boolean Operations: $2 \times$ Coverage

$RS_{E_1} \cap RS_{E_2} = T_1$

$RS_{E_2} \cap RS_{E_3} = T_2$
Boolean Operations: $2 \times$ Coverage (cont.)

$T_1 \cup T_2 = T_3$

$RS_{TE} \cap AS = C_{2\times}$
Boolean Operation Sequences

Region of $1 \times$ coverage inside target region:

\[
C_{1\times} = \left( \bigcup_{i=1}^{n} R_{SE_i} \right) \cap AS
\]

\[
\text{total region of } 1 \times \text{ coverage}
\]

Region of $2 \times$ coverage inside target region:

\[
C_{2\times} = \left( \bigcup_{i_1=1}^{n-1} \bigcup_{i_2=i_1+1}^{n} \left( R_{SE_{i_1}} \cap R_{SE_{i_2}} \right) \right) \cap AS
\]

\[
\text{total region of } 2 \times \text{ coverage}
\]
Region of $p \times$ coverage inside target region:

$$C_{p \times} = \left( \bigcup_{i_1=1}^{n-p+1} \bigcup_{i_2=i_1+1}^{n-p+2} \ldots \bigcup_{i_{p-1}=i_{p-2}+1}^{n-1} \bigcup_{i_p=i_{p-1}+1}^{n} \bigcap_{i_1}^{i_p} (RSE_{i_1} \cap \cdots \cap RSE_{i_p}) \right) \cap AS$$

total region of $p \times$ coverage

region of $p \times$ coverage between sensors $i_1, i_2, \ldots, i_p$
System Classification

**Time-invariant:** satellite/target region separations fixed
- Can solve analytically
- Occurs under special conditions

**Time-varying:** satellite/target region separations variable
- Numerical approach:
  - Approximate solutions
  - Easier to implement a wide variety of analyses
- Fewer necessary assumptions
Assumptions:

- Single circular orbit
- $n$ satellites equally distributed
- Omni-directional sensor range
- Annular target region
Time-Invariant: Example 1

**Objective:** Fewest satellites for full $1 \times$, $2 \times$, $3 \times$ ATH coverage

![Graph showing minimum constellation population vs. altitude](image)

Minimum Constellation Population vs. Altitude
Objective: Fewest satellites for full $1 \times$ ATH coverage

Min. Constellation Population vs. Altitude & Sensor Range
Time-Varying Coverage Analysis
Assumptions:

- Annular target region (prescribed)
- 2 elliptical orbits, \( n_1 \) & \( n_2 \) sats (variables)
- Opposite periapsis directions
- Satellites equal distributed in \( M \)
- Phasing between orbits (variable)
- Same semi-major axis, eccentricity (variables)
- Dissimilar sensor performance between orbits

Approach:

- MINLP problem – MIDACO – Ant colony optimization
Time-Varying: Example 3

**Objective:** Fewest satellites for continuous ATH coverage
Time-Varying: Example 4

Assumptions:
- GEO Belt target: ±1000 km alt., 148°-61°W longitude
- $n$ satellites equally distributed across 4 orbits
- 4 orbits equally spaced in periapsis direction
- Satellites grouped in orbits – spread by $\Delta M$ (variable)
- Same eccentricity between orbits (variable)
- Omni-directional sensor range (variable)
- Fixed semi-major axis (1/2 day period)
- Synchronized for group apogee during target region passage

Approach:
- NLP problem – $fmincon$ in MATLAB
**Objective:** Shortest sensor range for continuous ATH coverage
Conclusions

A new model for SBSA applications is developed

- Elements from computational geometry used to compute constellation ATH coverage
- Numerical approach:
  - Versatile, wide range of scenarios require no rederivation
  - Suitable for time-invariant and time-varying cases

A new approach to optimal constellation design

- Model is used as a metric, considering actual ATH coverage provided by a constellation
- Can use traditional parameter optimization techniques
Current Approach (1/2)

- Begin by considering planar coverage only (i.e. cross-section of 3D coverage volume)
- In the plane of motion, define sensor profiles and regions of interest as polygons, discretized via a series of points.

Low PPC
Accuracy of coverage area calculation will depend on specified number of PPC

High PPC
Current Approach (2/2)

- Use sequences of Boolean operations to identify regions of desired coverage multiplicity. Implementation employs the general **polygon clipping** library (GPL),\(^1\) based on Vatti’s Method.\(^2\)

- Algorithm allows for systematic calculation of coverage area as a function of satellite position and sensor orientation
  - Facilitates numerical optimization process
    - Minimize # of satellites to provide desired level of coverage
    - Maximize ATH coverage subject to other constraints

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Polygon Clipping

• Polygon clipping is essentially the act of ‘cutting’ one polygon (the subject polygon) with another (the clip polygon).
• By analyzing the interaction of the two shapes, and identifying the location of all intersections, various Boolean operations can be carried out to determine a result polygon.
• In the context of the ATH constellation coverage problem, unions, intersections, and differences are of primary interest in constructing the result polygon that represents the coverage area.
• Related concepts from computer graphics
  • In 1980, Weiler was first to develop an algorithm that could handle non-convex clip and subject polygons, and polygons with self-intersections.
  • More efficient algorithms were later developed by Vatti (1992), Greiner and Hormann (1998), and Martinez and Rueda, et al (2009).
Algorithm Implementation

- The set notation expressions illustrated on the previous slides are implemented in a generalized algorithm that performs the necessary sequences of Boolean operations for any desired coverage multiplicity
  - MATLAB and C++ Implementations
  - Currently uses the GPC polygon clipping library\(^1\), based on Vatti’s method\(^2\), to perform Boolean operations
  - Can be directly used to handle the volumetric case (future work, the sequence of operations remains unchanged)
  - Extensive analysis done into performance and approximation error relative to recently developed analytical coverage models.

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Locating Intersections

- All polygon clipping methods require knowledge of **polygon edge intersections** – in general, they form the necessary corners of result polygons where the subject and clip polygon boundaries intersect.

- Determination of edge intersections must be addressed in order to achieve accurate **clipping solutions**, which directly relates in the present case to the calculation of coverage area.
Plane-sweep method more intelligently performs intersection checks based on what is geometrically feasible.

The endpoints of each line segment are inserted into an ordered data structure (such as a binary search tree), ordered in one primary geometric direction, i.e. increasing in \( x \).

Each endpoint is an ‘event,’ and the ‘sweep line’ moves from one event to the next taking advantage of the sorted order to make only one ‘sweep’ across the coordinate plane.
Checking/Locating Intersections (Bentley-Ottman, 1979) (2/2)

- Associated with the sweep line is another ordered data structure (linked list or binary search tree) that keeps track of the ‘active’ line segments, i.e. the line segments that intersect the sweep line at a particular instant.

- When the sweep line encounters a left endpoint, a segment pointer is added to the sweep line structure. These pointers are sorted in order of their segment intersection with the sweep line, i.e. bottom to top.

- When the sweep line encounters a right endpoint of one of these segments, the segment pointer is deleted from the sweep line data structure.
Constructing the Result Polygon: (Vatti’s Polygon Clipping Method)

- Vatti’s polygon clipping method performs some additional tasks that differentiate it from the simple edge intersection detection algorithm by Bentley and Ottmann.
- Because the clipping process is not only interested in edge intersections, but also in producing a result polygon, as line segments are removed from the sweep line, they are considered for inclusion in the result polygon.
- Thus, the result polygon is sequentially constructed during the sweep.
- The algorithm is considered to be robust, once all degeneracies are adequately addressed.
Simple Polygons
(i.e. not self-intersecting)

- Convex polygon
  - All internal angles < 180 degrees.
- Concave polygon
  - Always has an interior angle with a measure > 180 degrees.
Area of an m-polygon

- The area of a simple polygon with \( m \) unique vertices is:

\[
A_{P_1P_2\ldots P_m} = \frac{1}{2} \left( \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix} + \ldots + \begin{vmatrix} x_m & x_1 \\ y_m & y_1 \end{vmatrix} \right)
\]

where \( x_i, y_i \) denote the planar coordinates of the \( i^{th} \) vertex.

- This is valid for all convex and concave simple polygons.
Example:
Impact of PPC on Area Calculation

\[ A_{\text{err}} = \frac{A_{\text{sector}} - A_\Delta}{A_{\text{sector}}} = 1 - \frac{n}{2\pi} \sin \frac{2\pi}{n} \]
Single ATH Coverage: Boolean Operations

- The sequence of Boolean operations necessary to evaluate single coverage is illustrated over the next two slides.
- All $n$ sensor regions (themselves regions of single coverage) are joined by $n-1$ union operations.
- Resulting total effective sensor region, $RS_{TE}$, is intersected with the region of interest, $AS$.
- Thus, region $C_{1X}$ represents the region of single coverage in the region of interest.
Double Coverage Boolean Operations

- The sequence of Boolean operations necessary to evaluate double coverage is illustrated over the next two slides.
  - Unique pairs between the n satellites are intersected to form individual regions of double coverage
  - These regions of double coverage are joined by union operations
  - Resulting total effective sensor region, $R_{SE}$, is intersected with the region of interest, $A_S$
  - Thus, region $C_{2X}$ represents the region of double coverage in the region of interest.