Actuator Constrained Optimal Control of Formations Near the Libration Points

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Outline

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  Dynamic Sensitivities and Control Limitations

Transcription Formulations
  Direct Collocation
  Multiple Segment Formulations
  Switching Segments and Time

Applications
  Costs and Constraints
  Initial Guess
  Sample Solution
Formation Limitations for Deep-Space Imaging Formations

- Fixed size, shape, and orientation of the formation
- Fixed orientation of each member of the formation (deputy spacecraft)

Figure: Formation Pointing
Dynamical Sensitivities Near the Libration Points

- Previous investigations have focused on unconstrained continuous control solutions
  - Linear and nonlinear; feasible and optimal solutions
  - Non-natural formations require extremely precise control ($< \text{nm/s}^2$ accelerations)
- These controls are impossible to implement with existing actuator technology

- Cannot reproduce the fidelity of continuous control
- Continuous control may even be smaller than minimum thrust bound

Figure: Implementing a Continuous Control Solution
Control Limitations for Deep-Space Imaging Formations

- Fixed thruster location on each spacecraft body
- Specified thrust acceleration magnitude
  - Based on actuator performance capability

\[ T^* \]

\[ \uparrow \]

\[ \downarrow \]

\[ T^* \]

\[ \rightarrow T^* \]

\[ \leftarrow T^* \]

Figure: Spacecraft Body
Transcription Methods for Highly Constrained Problems

- The libration point formation problem motivates a unique solution method
  - Direct optimization methods serve as the foundation
  - Modifications allow for creative treatment of difficult constraints

*Solution methods are generalized for any number of dynamical models.*
Optimization via Direct Transcription

- Define a parameter vector consisting of state and control values at nodes (discrete points in time)

\[ \mathbf{x} = [\ldots, y^T(t_j), \ldots, u^T(t_j), \ldots, t_0, t_f]^T \]

- Convert the Optimal Control Problem into a Parameter Optimization Problem

Minimize

\[ J = \phi(t_0, y_0, t_f, y_f) + \int_{t_0}^{t_f} L(t, y, u) \, dt \]

subject to

\[ \dot{y} = f(t, y, u) \]
\[ 0 = \psi_0(t_0, y_0) \]
\[ 0 = \psi_f(t_f, y_f) \]
\[ 0 = \beta(t, y, u) \]

Minimize

\[ F(\mathbf{x}) \]

subject to

\[ c(\mathbf{x}) = \left[ c_{\psi_0}^T(\mathbf{x}) \ c_{\psi_f}^T(\mathbf{x}) \ c_\beta^T(\mathbf{x}) \ c_y^T(\mathbf{x}) \right]^T = 0 \]

- Solve the resulting optimization problem with a standard Nonlinear Programming (NLP) algorithm
Multiple Segment Formulations

- Account for state or control discontinuities by dividing the problem into segments
  - Ideal treatment for finite burn control solutions
- Enforce appropriate constraints at the knots (segment boundaries)
- Include knot times or segment durations in parameter vector

Figure: An Example of Segments and Knots
Impacts of Fixed Spacecraft Orientation

- A traditional finite burn formulation specifies thrust (acceleration) magnitude, but not direction
  - Assumes spacecraft can re-orient to deliver required thrust vector
  - Control space $\mathcal{U}_1$: $u^T u = (T^*)^2$
- If spacecraft orientation is predetermined (according to other mission requirements)
  - Actuator configuration must provide 3-axis maneuverability
    - Assume thrusters are located on principal axes of body frame $\mathcal{B} \equiv \{\hat{x}_B, \hat{y}_B, \hat{z}_B\}$
    - Control space $\mathcal{U}_2$: $u_i(u_i - T^*)(u_i + T^*) = 0, i = \hat{x}_B, \ldots, \hat{z}_B$

Fixed spacecraft orientation leads to discrete optimization, which gradient-type NLP algorithms cannot support.

Figure: Control Spaces (a) $\mathcal{U}_1$ (Orientation Free), and (b) $\mathcal{U}_2$ (Orientation Fixed)
Managing Fixed Spacecraft Orientation

- Instead of optimizing control values (i.e. \(-T^*, 0, T^*)\), . . .

  \textit{Prespecify control values by segment and optimize switching times}

- Knots are used to designate switching times in each control axis
- Segments are bounded by switches in any control
- The chronological ordering of knots changes at each iteration of the optimization

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{control_profile.png}
\caption{Conceptual Control Profile with Segment Divisions}
\end{figure}
Costs and Constraints

- **Constraints**
  - Initial time and states specified
  - Final time and formation size and plane specified
    - $r_{cd}^* = 1 \text{ km distance between chief and deputy, } r_{dd}^* = 1.73 \text{ km distance between deputies}$
    - Specified pointing $r_{cs}^* = [1 \ 0 \ 0]$
  - State continuity (differential constraints) by segment
  - State equality across segments (at knots)

- **Weighted Costs**
  - Minimize thrust
  - Minimize formation size deviations along trajectory
  - Minimize formation plane deviations along trajectory

\[
J = w_1 J_1 + w_2 J_2 + w_3 J_3
\]
\[
F(x) = w_1 F_1(x) + w_2 F_2(x) + w_3 F_3(x)
\]
Baseline Initial Guess

Trajectory Legend
- Deputy 1 Trajectory
- Deputy 2 Trajectory
- Deputy 3 Trajectory

Control Legend
- Axis 1 Control ($u_x$)
- Axis 2 Control ($u_y$)
- Axis 3 Control ($u_z$)
Baseline Solution

**Trajectory Legend**
- Deputy 1 Trajectory
- Deputy 2 Trajectory
- Deputy 3 Trajectory

**Control Legend**
- Axis 1 Control ($u_x$)
- Axis 2 Control ($u_y$)
- Axis 3 Control ($u_z$)

**Spacecraft Positions - Inertial Frame**

**Control Accelerations - Body Frame**
Conclusions

- A modified collocation method with a segment-time switching algorithm leads to highly constrained control solutions
- Generalized formulation allows users to input
  - formation configuration, size, orientation, and rotation rate
  - thruster capability and placement
  - dynamic model and reference trajectory
  - initial and terminal conditions
- Suited to aid in establishing requirements and capabilities for highly constrained formations
References I


References II


References III


References IV


Varying Parameters to Obtain Different Solutions

- Final time
- Number of nodes and knots
- Initial guess
- Thrust magnitude

Table: Comparison of Solutions with Various Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>$t_f$</th>
<th>$n_k, t_f$</th>
<th>Feasible Guess</th>
<th>Thrust</th>
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<td>$n_n$</td>
<td>4</td>
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<td>6</td>
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<td>$n_k$</td>
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<td>10</td>
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<td>10</td>
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<td>$t_f$ ($10^6$ sec)</td>
<td>5.1183</td>
<td>10.2366</td>
<td>10.2366</td>
<td>5.1183</td>
<td>5.1183</td>
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<td>Baseline</td>
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<td>Baseline</td>
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<td>Baseline</td>
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<td>$w_{Thrust}$</td>
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<td>$\frac{1}{400}$</td>
<td>$\frac{1}{400}$</td>
<td>$\frac{1}{400}$</td>
<td>$\frac{1}{1600}$</td>
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<tr>
<td>$w_{Distance}$</td>
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<td>0.1</td>
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<td>$w_{Plane}$</td>
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<td>1</td>
<td>1</td>
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<td>1</td>
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<tr>
<td>Thrust (km/s$^2$)</td>
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<td>2.0e-12</td>
<td>2.0e-12</td>
<td>2.0e-12</td>
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<td>Total Cost</td>
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