CHAPTER 7
HIGHER ORDER RECONSTRUCTION ALGORITHMS

7.1 Introduction

Reconstructions based on the theory in the first five chapters of this work are based on first order approximations to the scattered field. In other words for both the Born and the Rytov approximations it is necessary to assume that the field inside the object is equal to the incident field and then it is possible to derive a simple (linear) expression for the scattered field as a function of the object. Finding a better estimate for the field inside the object is the central problem in improving diffraction reconstructions.

The reconstruction problem is more difficult than the forward problem discussed in Chapter 6 because now both the object and the field inside the object are unknown. This means that is is necessary to design a procedure that simultaneously estimates both the object and the field inside the object. This procedure is made more difficult because the reconstruction is formed by illuminating the object by a number of different fields and the exact field inside the object must be calculated for each view.

Three approaches to the inverse problem will be described here. Most general and therefore computationally most expensive is to write a system of equations that describes both the field and the object and then find the solution vector that gives the smallest error. Unfortunately the system of equations is non-linear and some sort of search procedure must be used to find the best solution. This approach to the problem was first discussed by Johnson et al[Joh83].

Computationally less demanding solutions to the problem are based on iterative algorithms. A perturbational approach much like that used to generate the Born or the Rytov series was first proposed by Jost and Kahn [Jos52] and later extended by Moses [Mos56] and Prosser [Pro69, Pro76]. This approach was first developed for quantum scattering problems but is equally valid for the electromagnetic and acoustic wave equations.

A hybrid approach to the problem was first proposed by Johnson [Joh83] and is a two step iteration procedure. It is based on the idea that is is only necessary to calculate the object since for any given object and incident field it is possible to calculate the exact field inside the object. Thus the reconstruction procedure first estimates the object's refractive index distribution and then an estimate of the field inside the object can be calculated using any one of the procedures described in Chapter 6. The key to this procedure is then to calculate a better estimate of the object given a better estimate of the actual field inside the object. This approach will be described in section 7.4 as an example of a fixed point iteration.

7.2 Non Linear Approach

The most general approach to estimate the object given the scattered field is to define a solution space that includes both the refractive index of the object and the exact field inside the object for each of the views. Both the non-linear equations and the number of unknowns combine to make this a difficult problem. The non-linear nature of the problem means that a search procedure must be used to find the best solution and unlike fixed point or perturbation methods it is not possible to say a priori how fast the search will reduce the error or whether it will ever converge.

The unknowns in this problem are defined over an NxN grid and consist of the object and the exact scattered field from N_ϕ views. If the number of views, N_ϕ, is on the same order as N then there are a total of N^3 unknowns and thus at least N^3 defining equations are needed for a well behaved solution [Sar81].

For each view N_M measurements of the scattered field are taken and this defines $N_\phi N_M$ equations of the form

$$u_{M,\phi}(\vec{r}) = \int u_{t,\phi}(\vec{r}\,')o(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' \tag{7.1}$$

where $u_{M,\phi}$ is the measured scattered field for a view at angle ϕ and position $\vec{r}$. Both $u_{t,\phi}(\vec{r})$, the total field inside the object for the view at angle ϕ, and $o(\vec{r})$, the object's refractive index, are unknown. Since the unknowns are calculated over a discrete NxN grid the integral above becomes a summation or

$$u_{M,\phi}(\vec{r}_j) = T^2\sum_i u_{t,\phi}(\vec{r}\,'_i)o(\vec{r}\,'_i)g(\vec{r}-\vec{r}\,'_i). \tag{7.2}$$

Unfortunately the measured points only contribute $N_\phi N_M$ equations therefore there are more unknowns than equations by a factor of approximately N (again assuming that N_ϕ and N_M are on the same order as N). The additional equations are defined by noting that the field inside the object must also satisfy the Helmholtz equation. There are $N_\phi N^2$ equations of the form

$$u_{t,\phi}(\vec{r}) = u_{0,\phi}(\vec{r}) + \int u_{t,\phi}(\vec{r}\,')o(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' \tag{7.3}$$

or in discrete form

$$u_{t,\phi}(\vec{r}_i) = u_{0,\phi}(\vec{r}_i) + T^2 \sum u_{t,\phi}(\vec{r}\,'_j)o(\vec{r}\,'_j)g(\vec{r}_j - \vec{r}\,'_j). \tag{7.4}$$

Thus the combination of equations (7.2) and (7.4) define a total of $N_\phi N_M + N_\phi N^2$ equations which must be solved for both $u_{t,\phi}(\vec{r})$ and $o(\vec{r})$.

The difficulty caused by the large number of unknowns is compounded by the non linearity in the equations. In both equations (7.2) and (7.4) the product of the two unknowns, the field and the object, is convolved with the Green's function and this product means that the Kaczmarz algorithm as used in Chapter 6 is no longer applicable.

The usual approach to solve a system of non-linear equations is to define the error as a function of the difference between the left and right side of the equations. An optimum solution is then formed by a search procedure that looks for a minimum in the error function. One implementation of this algorithm reported by Tracy et al [Tra83] took 7 hours of computer time on a small minicomputer to find the object over an 11x11 grid. Calculating the object over a larger grid (at least 128x128 is probably needed for medical imaging) would be prohibitively expensive.

7.3 Perturbation Algorithms

As already described in Chapter 6 perturbation algorithms are an important technique for solving the scattering problem. This technique was first used to solve the inverse scattering equation by Jost and Kahn and the general techniques are described in more detail in the books by Nayfeh [Nay73, Nay81]. A discrete version of the work by Jost and Kahn was first reported by Devaney in [Dev82].

A perturbational expansion of the forward scattering problem is found by letting the object function be written in terms of a small perturbation parameter ϵ or

$$o(\vec{r}) = \epsilon X(\vec{r}) \tag{7.5}$$

and the total field written as a polynomial in terms of the same perturbation parameter or

$$u(\vec{r}) = \sum_{i=0}^{\infty} \epsilon^i u_i(\vec{r}). \qquad (7.6)$$

Now both the object and the field are expressed as a function of the free variable ϵ. At first glance the problem is made more difficult by the addition of the extra perturbation parameter but by gathering together the powers of ϵ the problem can be easily solved.

The equation for the total field

$$u(\vec{r}) = u_{inc}(\vec{r}) + \int u(\vec{r}\,')o(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' \qquad (7.7)$$

is now written as a function of ϵ or

$$\sum_{i=0}^{\infty} \epsilon^i u_i(\vec{r}) = u_{inc} + \int \sum_{i=0}^{\infty} \epsilon^i u_i(\vec{r}\,')\epsilon X(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,'. \qquad (7.8)$$

Rearranging this equation as a polynomial function of ϵ both the scattered field and the object satisfy an equation of the form

$$\begin{aligned} 0 = {} & [u_0(\vec{r})-u_{inc}] + \qquad (7.9) \\ & \epsilon[u_1(\vec{r})-\int u_0(\vec{r}\,')X(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,'] + \\ & \epsilon^2[u_1(\vec{r})-\int u_0(\vec{r}\,')X(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,'] + \\ & \epsilon^3[u_1(\vec{r})-\int u_0(\vec{r}\,')X(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,'] + \cdots. \end{aligned}$$

In order for this equation to be valid each coefficient of ϵ in the series expansion must be identically equal to zero. This requirement is all that is necessary to solve the more general problem for u_i as a function of the perturbation parameter ϵ, but in the scattering problem only the solution for $\epsilon=1$ is interesting. Therefore by setting the coefficient of each power of ϵ equal to zero and then setting the value of ϵ equal to one the following equations result

$$u_0 = u_{inc} \qquad (7.10)$$

$$u_1(\vec{r}) = \int u_0(\vec{r}\,')X(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' \qquad (7.11)$$

and in general

$$u_i(\vec{r}) = \int u_{i-1}(\vec{r}\,')X(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' \qquad i\geq 1. \qquad (7.12)$$

This is the same system of equations defined as the Born series in Chapter 6 therefore the same conditions define the region of convergence for this perturbation solution. This method of analysis was also applied to the scattering problem by Keller [Kel69] and by Oristaglio [Ori85].

The forward scattering problem represents a relatively simple example of the perturbation method. The inverse problem is solved by assuming that the Born series converges and writing the total scattered field as

$$u_s(\vec{r}) = \int u_0(\vec{r}\,')o(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' + \tag{7.13}$$
$$\int\int u_0(\vec{r}\,')o(\vec{r}\,')g(\vec{r}\,''-\vec{r}\,')d\vec{r}\,'\,o(\vec{r}\,'')g(\vec{r}-\vec{r}\,'')d\vec{r}\,'' +$$
$$\int\int\int u_0(\vec{r}\,')o(\vec{r}\,')g(\vec{r}\,''-\vec{r}\,')d\vec{r}\,'\,o(\vec{r}\,'')g(\vec{r}\,'''-\vec{r}\,'')d\vec{r}\,''$$
$$o(\vec{r}\,''')g(\vec{r}-\vec{r}\,''')d\vec{r}\,''' + \cdots .$$

Now replace the scattered field by

$$u_s(\vec{r}) = \epsilon\psi(\vec{r}) \tag{7.14}$$

and express the object as a polynomial in ϵ or

$$o(\vec{r}) = \sum_{i=0}^{\infty} \epsilon^i o_i(\vec{r}). \tag{7.15}$$

The scattered field is now written

$$\epsilon\psi = \int u_0(\vec{r}\,')\sum_i \epsilon^i o_i(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,' + \tag{7.16}$$
$$\int\int u_0(\vec{r}\,')\sum_i \epsilon^i o_i(\vec{r}\,')g(\vec{r}\,''-\vec{r}\,')d\vec{r}\,' \sum_j \epsilon^j o_j(\vec{r}\,'')g(\vec{r}-\vec{r}\,'')d\vec{r}\,'' +$$
$$\int\int\int u_0(\vec{r}\,')\sum_i \epsilon^i o_i(\vec{r}\,')g(\vec{r}\,''-\vec{r}\,')d\vec{r}\,' \sum_j \epsilon^j o_j(\vec{r}\,'')g(\vec{r}\,'''-\vec{r}\,'')d\vec{r}\,''$$
$$\sum_k \epsilon^k o_k(\vec{r}\,''')g(\vec{r}-\vec{r}\,''')d\vec{r}\,''' + \cdots$$

where S_i is used to denote scattering by object u_i.

This expression can be simplified by defining an integral operator S that maps an incident field into the field scattered by the object. If an incident field, $u_0(\vec{r})$, is scattered by an object, $o(\vec{r})$, then the scattering operator is defined by the following integral

$$u_s(\vec{r}) = S(u_0) \quad \leftrightarrow \quad \int u_0(\vec{r}\,')o(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,'. \tag{7.17}$$

Now the Born series in equation (7.13) can be written

$$u_s(\vec{r}) = S(u_0) + S^2(u_0) + S^3(u_0) + \cdots \tag{7.18}$$

where

$$S^i(u_0) = S\left[S^{i-1}(u_0)\right]. \tag{7.19}$$

Using this notation (7.16) can be written

$$\epsilon\psi = \sum_i \epsilon^i S_i(u_0) + \tag{7.20}$$

$$\sum_i \sum_j \epsilon^i \epsilon^j S_j(S_i(u_0)) +$$

$$\sum_i \sum_j \sum_k \epsilon^i \epsilon^j \epsilon^k S_k(S_j(S_i(u_0))) + \cdots .$$

This expression can be simplified even further by denoting the iterated kernel $S_j(S_i(u_0))$ by the expression $S_{ji}(u_0)$. Now the Born series is written

$$\epsilon\psi = \sum_i \epsilon^i S_i(u_0) + \tag{7.21}$$

$$\sum_i \sum_j \epsilon^i \epsilon^j S_{ji}(u_0)) +$$

$$\sum_i \sum_j \sum_k \epsilon^i \epsilon^j \epsilon^k S_{kji}(u_0))) + \cdots .$$

Notice that the first summation above represents first order scattering from a number of different objects while the second set of summations represents all possible second order scatterings from the same ensemble of objects.

A series solution for the object function is found by gathering together the coefficients of like powers of ϵ. This gives the following polynomial in ϵ

$$\epsilon\psi(\vec{r}) = \epsilon S_1(u_0) + \epsilon^2[S_2(u_0) + S_{11}(u_0)] + \tag{7.22}$$

$$\epsilon^3[S_3(u_0) + S_{21}(u_0) + S_{12}(u_0) + S_{111}(u_0)] + \cdots .$$

Just as was done in deriving the Born series each power of ϵ is independent and therefore the following equalities can be written

$$\psi = S_1(u_0) \tag{7.23}$$

$$S_2(u_0) = -S_{11}(u_0) \tag{7.24}$$

$$S_3(u_0) = -S_{21}(u_0) - S_{12}(u_0) - S_{111}(u_0) \tag{7.25}$$

and in general

$$S_i(u_0) = - \sum_{i_1+i_2+\cdots+i_n=i} S_{i_1 i_2 \cdots i_n}(u_0) \qquad n \geq 2. \tag{7.26}$$

Consider first the equation for ψ. The solution for the object functions is only interesting when $\epsilon=1$ therefore $\psi=u_s$ and the first equality above can be written

$$u_s(\vec{r}) = \int u_0(\vec{r}\,')o_1(\vec{r}\,')g(\vec{r}-\vec{r}\,')d\vec{r}\,'. \tag{7.27}$$

This equation represents the measured scattered field as the first order scattered field from the object o_1 and by the Fourier Diffraction Theorem this equation can be solved exactly for o_1. Because the object is illuminated with the incident field this is true for all experiments, regardless of the size of the object and its refractive index. This contrasts with first order diffraction tomography where the object is modulated by the total field and thus the Fourier Diffraction Tomography is only valid when the total field can be approximated by the incident field.

While equation (7.23) only expresses the scattered field from one view of the object it is possible to combine the scattered field from a number of different views and then use the first order reconstruction algorithm described in Chapter 4. Thus the result of the first order reconstruction algorithms described in Chapter 4 and 5 is exactly equal to o_1.

The second order object is slightly more difficult to compute. From equation (7.23) the first order scattering from the second order object is given by

$$S_2(u_0) = -S_{11}(u_0). \tag{7.28}$$

The expression $S_{11}(u_0)$ represents the second order scattering from the first order object and is easily calculated because o_1 has already been computed. The higher order terms follow in a similar fashion except a number of partial scattered fields are summed to find the first order scattered field from o_i.

The procedure used to calculate the higher order object is slightly different from that of the first order object because of the location of the receiver line. The first order object is a function of the scattered field and the placement of the receiver line is limited by experimental constraints. On the other hand the higher order fields are defined on a rectangular grid since an FFT based implementation of the Born integral is the most efficient procedure. Thus for the higher order terms the field are calculated over the entire grid and then only the field along one side of the grid is used as input to the reconstruction procedure.

In summary the algorithm for reconstructing the object using the higher order Born series is

- Use the measured fields and the first order Born reconstruction algorithm to compute o_1.
- For each $i>1$ do the following.
  - Calculate the higher order scattered field from each of the already computed objects (See equation (7.26)).
  - Use the first order inversion algorithm to invert S_i and find o_i.
- Sum up each of the o_i to get the object reconstruction.

Notice that this algorithm is "exact." Except for the numerical approximations needed for the reconstruction procedure there are no mathematical approximations to limit the quality of the reconstruction.

The most expensive part of this algorithm is not doing the reconstructions but instead in computing the higher order scattered field, S_i. While it is easy to implement a fast algorithm to compute each partial field in S_i the total number of integrals increases rapidly with each succeeding iteration. Table 7.1 shows the number of partial fields and the total number of integrals needed for each of the first twenty iterations. Since each integral takes a constant amount of CPU time, no matter how it is implemented, the practical limit of this algorithm with today's computers is certainly under ten iterations.

The convergence of this series is dependent on the convergence of both the forward Born series shown in equation (7.13) and the object series shown in equation (7.15). Thus if either series is divergent then this reconstruction procedure will also diverge and produce an undefined answer.

The convergence of the forward series was discussed in Chapter 6. This, for example, showed that for an object of radius 2λ the scattered fields could be calculated using the Born series only for objects with a refractive index of less than about 11%. This puts a severe limitation on the type of objects that can be reconstructed with the higher order Born series. For objects that do fall within the allowable range the reconstruction with the higher order Born series should be quantitatively more accurate than that done with a first order algorithm.

Table 7.1. The number of partial field terms and integrals needed to calculate each iteration of the inverse higher order Born series.

Iteration	Terms	Integrals
1	0	0
2	1	2
3	3	7
4	7	19
5	15	47
6	31	111
7	63	255
8	127	575
9	255	1279
10	511	2815
11	1023	6143
12	2047	13311
13	4095	28671
14	8191	61439
15	16383	131071
16	32767	278527
17	65535	589823
18	131071	1245183
19	262143	2621439

The seriousness of this limitation is further seen by recalling that the higher order Born series only converges when the first order field closely approximates the total scattered field. This is the same condition that determines the accuracy of the first order reconstruction algorithms so the ultimate improvement is limited by the quality of first order reconstructions. Thus it will not be possible to image any object with a larger refractive index or radius than those in Figure 7.1 using an algorithm based on the Born series.

The convergence of the object series has yet to be determined. Jost and Kahn in [Jos52] report the convergence of the higher order Born inversion procedure for two quantum mechanical scattering experiments.

While the above derivation of the perturbation approach has expanded the object as a perturbation about zero it is also possible to consider the object to be a small perturbation of a known object. This generalization is known as the Distorted Wave Born Approximation (DWBA) and is described in [Tay83, New66, Dev83 and Bey85]. This procedure is made more difficult because the Green's function used in the integral now represents the scattered field from a point source with the effect of the known object included. The convergence of this procedure is not known.

7.4 Fixed Point Algorithms

A third untested approach to solve the inverse diffraction problem is to use a fixed point algorithm. While the overall algorithm is relatively straightforward it is necessary to perform a first order reconstruction of the object when illuminated by an arbitrary field. This is much more difficult than the first order reconstruction algorithms based on plane wave illumination described in Chapter 4. (The synthetic aperture approach does use point sources, but since a different phase is added to the scattered field for each transmitter position a plane wave is synthesized.)

A fixed point algorithm for calculating the object that scattered a measured field is based on the equation

$$o = f(o) \qquad (7.29)$$

where o is the desired object function. Within the limits of convergence of the series, an initial guess o_{i-1} can be improved upon by iterating the equation

$$o_i = f(o_{i-1}). \qquad (7.30)$$

The exact form of the iteration function, f, can take a number of different forms. Given only the scattered fields then either the Born or the Rytov

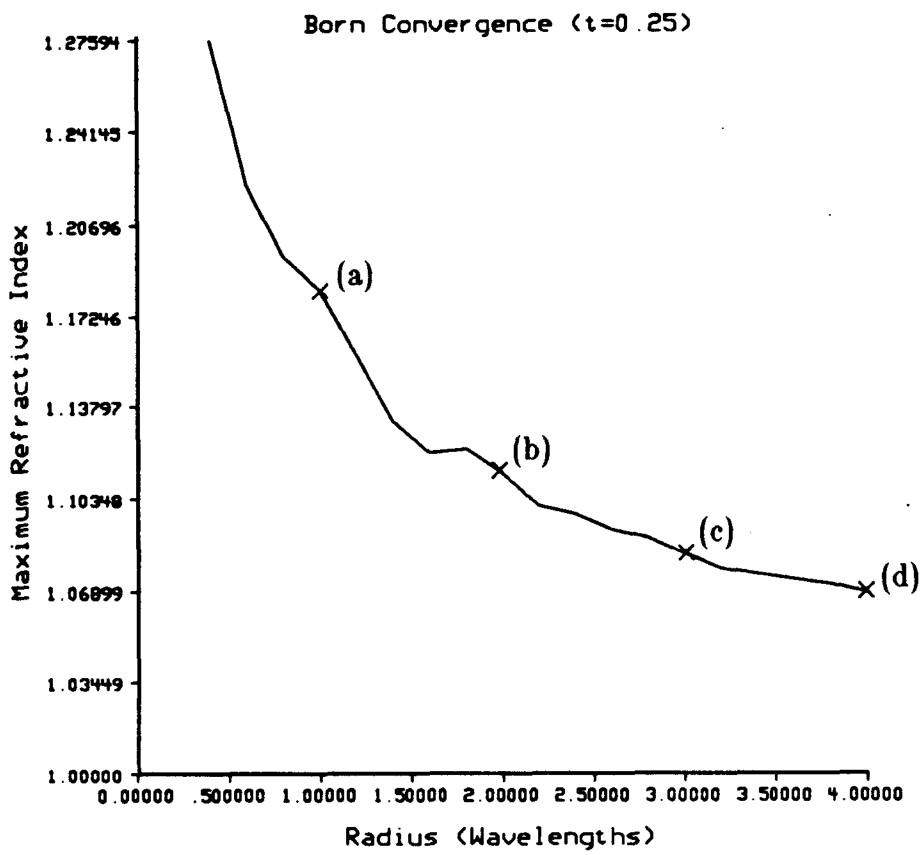
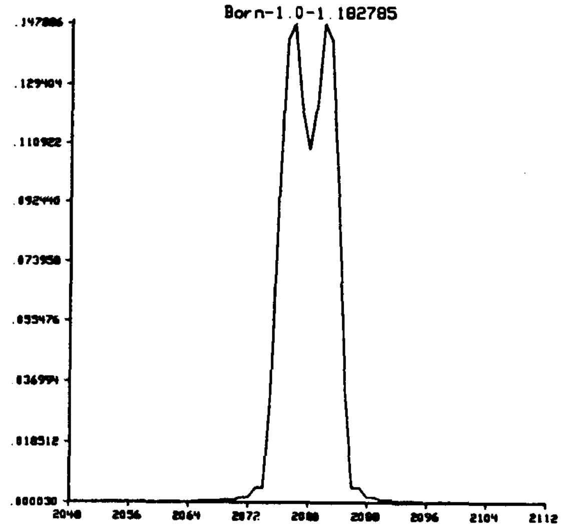
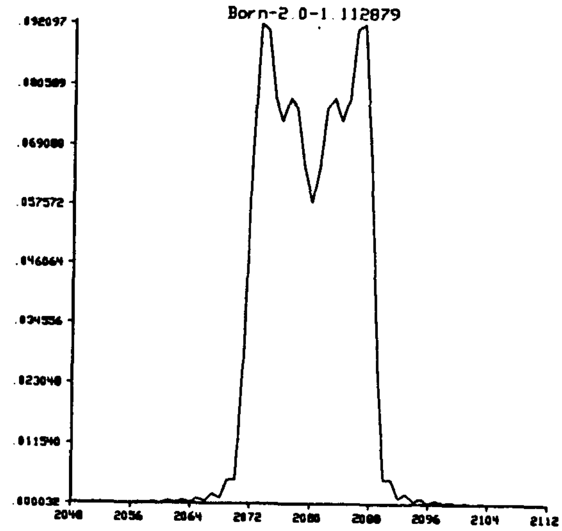


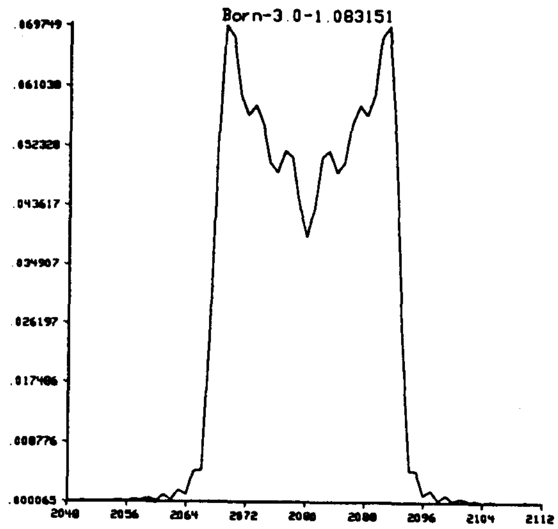
Figure 7.1 First order reconstructions of four objects at the limit of the Born series are shown here. Objects with a reconstruction worse then these can not be improved by an inversion procedure based on the Born series because the Born series will not converge.



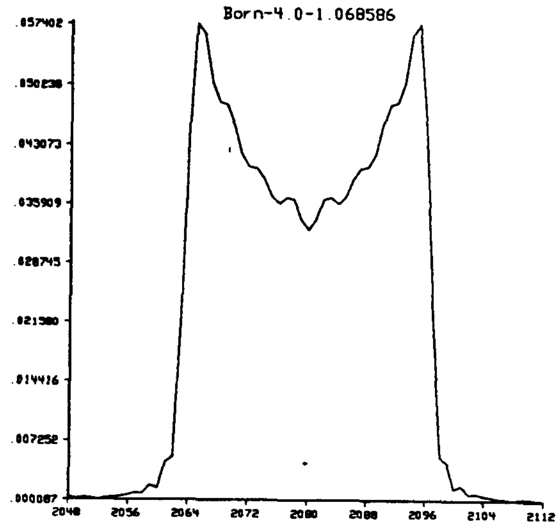
(a)



(b)



(c)



(d)

Figure 7.1 Continued.

approximation can be used to make an initial guess for the object. Both of these approximations assume that the field inside the object can be approximated by the incident field and this is the major source of error in the first order reconstruction procedures.

It seems reasonable that a better estimate of the object function could be found if the field inside the object is known. While it is not possible to know this without knowing the object first, a good estimate of the field should be possible given an estimate of the object. If the initial estimate of the object is "good enough" then a calculation of the scattered field given this estimate of the object should be more accurate then just using the incident field.

Using this new estimate for the field inside the object a better estimate of the object function should be possible. The general iteration formula can now be written

$$o_i = \text{Reconstruct}(\text{Estimate Field}(o_{i-1})) \tag{7.31}$$

Here the function labeled "Estimate Field" consists of estimating the total fields inside the object o_{i-1} and the function labeled "Reconstruct" consists of estimating the object given the total field in the object. The first iteration is the simplest since the estimate of the total field in the object is simply the incident field. Higher order iterations are made even more difficult since it is necessary to estimate the fields inside the object for each of the N_ϕ views.

Any number of means can be used to implement the two different steps in the algorithm. Estimating the field inside the object can be done with any of the procedures described in Chapter 6, including the Born and Rytov series or the algebraic approach. The procedure to use would depend on whether the object falls in the algorithm's region of convergence and on the efficiency of the algorithm with the available hardware.

Inverting the total fields to get an estimate for the object is the most difficult part of the algorithm. The Fourier Diffraction Theorem only applies to objects illuminated with a plane wave so a more general approach is needed. One solution to this problem was proposed by Vezzetti and Aks [Vez79]. Their work still assumes plane waves inside the object but now the field inside the object is modified by the average refractive index. With this approach they do show an improvement in the quality of the reconstruction but it is doubtful whether this approach would be accurate enough for a fixed point algorithm.

A complete solution for the object given an arbitrary set of illuminating fields would undoubtably be based on a least squared approach. While there are enough equations, given the field everywhere, to determine the object the system of equations would be very unstable because the Green's function only samples one arc of the scattering potential's Fourier transform. This means that if the field is known over an NxN grid and it is desired to calculate the object over the same grid then there would be a total of N^2N_ϕ equations defining the N^2 unknowns. Thus the system of equations to determine the object is overdetermined and any error in the field estimates will lead to an inconsistent set of equations. A least squared approach could then be used to find the solution vector that best satisfies the defining equations.

The convergence of this method is unknown. Like the fixed point methods discussed in Chapter 6 it is necessary for the "derivative" of the function be less than one in some region for the algorithm to converge. It probably isn't unreasonable to assume that this method will only converge when the first order estimate of the object is "good."

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