CHAPTER 1
INTRODUCTION

The word tomography comes from the Greek words tomo, meaning sectional, and graphy, meaning representation. Thus a tomographic image is a cross sectional image of an object. As the term is used today tomography refers to a procedure to collect data about the internal structure of an object and then mathematically generate an image of some otherwise hidden property of the object.

Diffraction, on the other hand, describes the spreading of acoustic and electromagnetic waves as they propagate through space and around objects. While conventional tomography uses x-rays to generate an image of the object's x-ray attenuation other sources of energy can also be used. Thus diffraction tomography uses diffracting energy sources to illuminate the object and then generates a cross sectional image of the object. Since ultrasound and microwaves diffract and refract as they pass through most objects they require more sophisticated algorithms then the ones used for x-ray tomography. These new algorithms for diffraction tomography are the subject of this work.

Tomography first became practical only a few years ago with the invention of the CAT (Computer Assisted Tomography) scanner [Hou72]. Hounsfield implemented a machine that illuminated an object with x-rays and measured the proportion of energy that passed through the object. Then by inverting a large system of equations he was able to generate an accurate estimate of the spatial variation of x-ray attenuation in the object.

The ability to generate a tomographic image of an object has revolutionized the medical field. For the first time it was possible to get a clear image of the internal morphology of a patient without the use of surgery. Now, x-ray CAT scanners are routinely built with resolutions of less than a millimeter and images with more than 512x512 pixels [Kak85, Her80, Mac83, Bar81].

While medical CAT scanners often generate an image of an object's x-ray attenuation there are limitations to this procedure. Foremost is the fact that not all types of soft tissue are differentiated by their x-ray attenuation. Thus x-ray CAT scans have wide use for orthopedic medicine but are of limited use, for example, in diagnosing malignant vs. benign tumors. In addition x-rays are an ionizing radiation and thus there is a small chance of cancer with each use. This prevents, for example, the use of x-ray CAT scanners for mass screening of female patients for breast cancer.

X-ray tomography is based on the Fourier Slice Theorem. Consider the experiment shown on the left side of Figure 1.1. Here a projection is shown that represents the attenuation of the object along each of the indicated lines. The Fourier Slice Theorem states that the Fourier transform of the projection is equal to the values of the two dimensional Fourier transform of the object along a radial line. An estimate of the object can then be formed by measuring projections at a number of angles and then simply inverting the Fourier data.

Conventional tomography is based on the idea that x-rays travel in straight lines through the object and a projection measures the total x-ray attenuation of the object along straight lines. When the object is relatively large and has a small refractive index it is possible to use other types of energy, for example microwaves, seismic and ultrasound, to image the object. With a small refractive inex the energy doesn't bend as it goes through the object and thus it is possible to measure the attenuation of the object along straight lines. This is the only requirement needed to use the Fourier Slice Theorem and form an image of the object's acoustic or microwave attenuation [Gre74, Gre75, Car76, Jak76, Glo77 and Cra82].

Since microwaves and acoustic waves are easier to generate and measure than x-rays it is also possible to generate images of the object's refractive index. As was mentioned earlier it is necessary to assume that the refractive index change is small so that the energy doesn't bend as it travels through the object. If a projection is measured representing the delay encountered by the energy as it travels through different parts of the object then an image is formed of the object's acoustic or electromagnetic refractive index. This extra information can often make it easier to characterize the object.

Two methods have been used to form images when the energy no longer travels through the object in a straight line. Perhaps the most straightforward approach is to simply model the flow of energy through the object as a ray and calculate its location based on the refractive index of the object [And82, Her76, Her73]. Unfortunately these algorithms can only be used when the refractive index change is less than 10 or 20 percent and most of the energy is refracted instead of diffracted. Thus this approach is only valid when the wavelength of

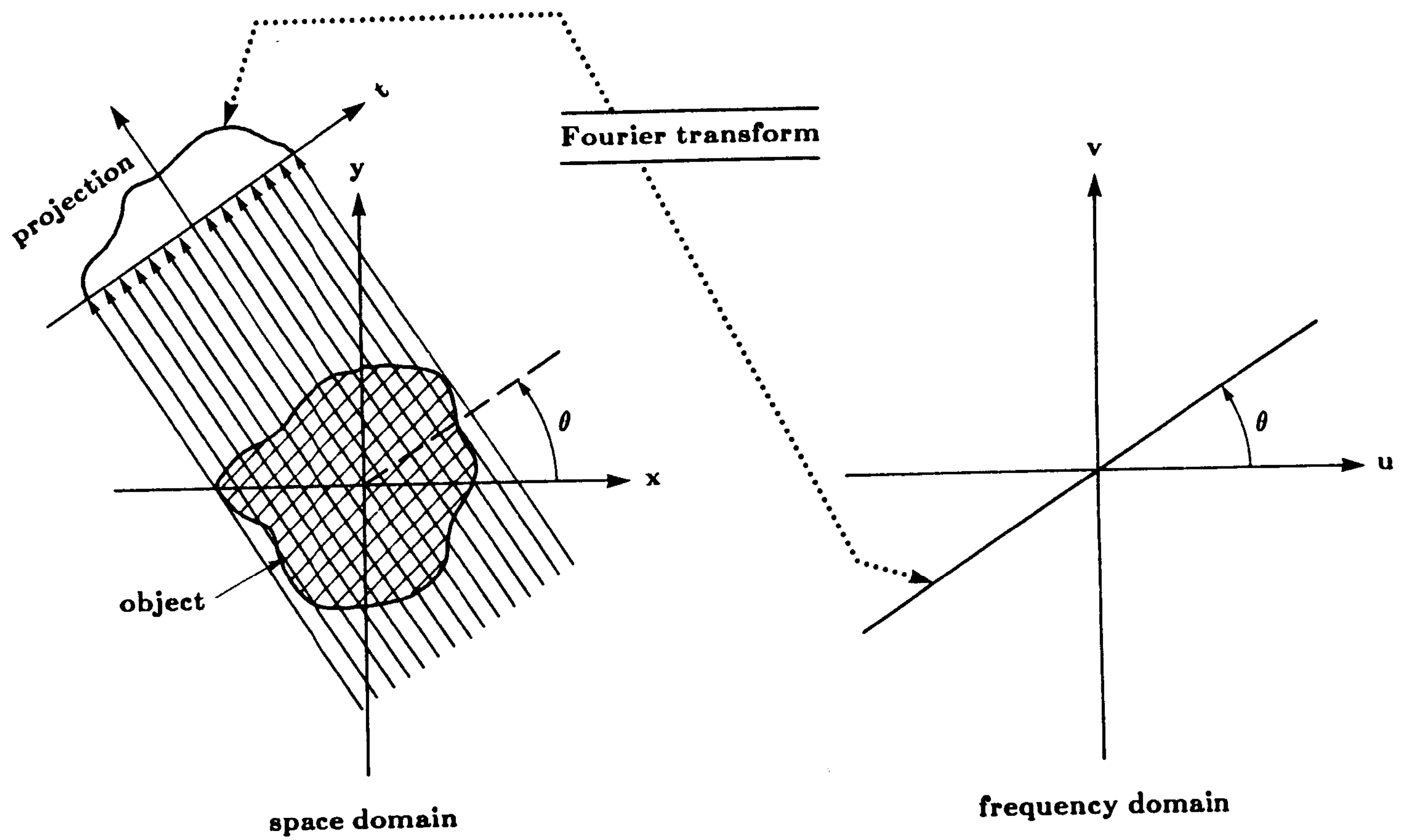

Figure 1.1 The Fourier transform of a projection is equal to the two dimensional Fourier transform of the object along a radial line.

the energy is much smaller than any details of the object [And84].

A second approach is to model the flow of energy through the object with the wave equation. While this approach is more accurate then other approaches it is not always possible to invert the resulting system of equations and find a closed form solution. This is the core of the problem for successful diffraction tomographic images.

A simple approach to solve the wave equation is to linearize it [Ish78, Che60, Sla84, Mue79, Wol69]. This is usually done by assuming that the object represents a very small perturbation to the field. Only the linear terms are retained and all higher order terms are simply ignored. Unfortunately this approach is also limited to those object that satisfy the constraints of the approximations. As will be shown later in this work linearizing the wave equation greatly limits the objects that can be imaged.

Finally in the past years work has been done on iterative techniques to solve the wave equation. Most of the work was originally applied to the inverse scattering problem of high energy physics [Bal78, New66, Tay83] and only recently applied to the diffraction tomography problem.

This work is in three parts: the derivation of the wave equation and first order reconstruction algorithms, limitations of first order algorithms and finally a summary of iterative techniques that can be applied to the diffraction tomography problem.

First in Chapter 2 the wave equation is defined for both acoustic and electromagnetic experiments. This scalar equation is valid for both types of energy and forms the basis of all work to be described here. In addition the Born and Rytov approximations are introduced and a linearized model for the scattered field as a function of the object is derived.

In Chapter 3 the linearized wave equation is inverted to find an expression for the object given the scattered field. This leads to the Fourier Diffraction Theorem which is fundamental to diffraction tomography. Finally several experimental procedures are described that generate enough data to uniquely determine the object.

Chapter 4 is a discussion of the numerical algorithms to invert the scattered data. Both of these algorithms are computationally very expensive and the algorithm used will depend on the architecture of the available computer resources. In addition some of the signal processing issues will be discussed and simulation results presented.

The limitations of the first order algorithms are presented in Chapter 5. Both the mathematical approximations and the experimental limitations contribute to the error in the final image but in different ways. The mathematical approximations are only valid for a small range of object and if these limits are exceeded then no amount of data will improve the reconstruction. The experimental limitations, on the other hand, are entirely caused by the ability to only collect a finite amount of data. The experimental errors can always be reduced by using more data or more accurate signal processing algorithms.

The severe limitations of first order diffraction algorithms is addressed in Chapters 6 and 7. The major problem in diffraction tomography is to find a method to invert the wave equation. In Chapters 2, 3 and 4 of this work this is done by linearizing the wave equation but as seen this severly limits the objects that can be imaged.

Chapter 6, therefore, discusses two approaches to model the scattered field given the (complex) refractive index of the object. This is the forward problem and both approaches are iterative. The simpler of the two approaches includes more than just the linear terms in the perturbation approach described in Chapter 2. This gives a series solution for the scattered field and simulations studying the type of objects for which these series converge will be presented. The second approach to solve the forward problem exploits the simple structure of the problem to compute a brute force solution. Objects with large refractive indices eventually cause this algorithm to converge too slowly for the method to be practical.

Finally Chapter 7 presents a survey of several approaches that have been proposed as better solutions for the inverse problem. Each of these algorithms has limitations and some of these limitations and computational aspects will be discussed.

References

[And82] A. H. Andersen and A. C. Kak, "Digital ray tracing in two-dimensional refractive fields," *The Journal of the Acoustical Society of America,* Vol. 72, November 1982, pp. 1593-1606.

[And84] A. H. Anderson and A. C. Kak, *The application of ray tracing towards a correction for refracting effects in computed tomography with diffracting sources,* TR-EE 84-14, School of Electrical Engineering, Purdue University, 1984.

[Bal78] H. P. Baltes (editor), *Inverse Source Problems in Optics,* Springer-Verlag, 1978.

[Bar81] H. H. Barrett and W. Swindell, *Radiological Imaging: The Theory of Image Formation, Detection and Processing,* Academic Press, New York, 1981.

[Car76] P. L. Carson, T. V. Oughton, and W. R. Hendee, "Ultrasound transaxial tomography by reconstruction," in *Ultrasound in Medicine II,* D. N. White and R. W. Barnes, eds., Plenum Press, 1976, pp. 391-400.

[Che60] L. A. Chernov, *Wave Propagation in a Random Medium,* McGraw Hill Book Company, New York, 1960.

[Glo77] G. H. Glover and J. L. Sharp, "Reconstruction of ultrasound propagation speed distribution in soft tissue: Time-of-flight tomography," *IEEE Trans. Sonics and Ultrasonics,* Vol. SU-24, July 1977, pp. 229-234.

[Gre74] J. F. Greenleaf, S. A. Johnson, S. L. Lee, G. T. Herman, and E. H. Wood, "Algebraic reconstruction of spatial distributions of acoustic absorption within tissue from their two dimensional acoustic projections," in *Acoustical Holography,* Plenum Press, 1974, pp. 591-603.

[Gre75] J. F. Greenleaf, S. A. Johnson, W. F. Wamoya, and F. A. Duck, "Algebraic reconstruction of spatial distributions of acoustic velocities in tissue from their time-of-flight profiles," in *Acoustical Holography,* Plenum Press, 1975, pp. 71-90.

[Her73] G. T. Herman, A. Lent, and S. Rowland, "ART: Mathematics and applications: A report on the mathematical foundations and on applicability to real data of the algebraic reconstruction techniques," *J. Theor. Biol.,* Vol. 43, 1973, pp. 1-32.

[Her76] G. T. Herman, A. V. Lakshminarayanan, A. Naparstek, E. L. Ritman, R. A. Robb, and E. H. Wood, "Rapid computerized tomography," *Med. Data Process.,* 1976, pp. 582-598.

[Her80] G. T. Herman, *Image Reconstructions from Projections,* Academic Press, New York, 1980.

[Hou72] G. N. Hounsfield, *A method of apparatus for examination of a body by radiation such as x-ray or gamma radiation,* Patent Specification 1283915, The Patent Office, 1972.

[Ish78] A. Ishimaru, *Wave Propagation and Scattering in Random Media,* Academic Press, New York, 1978.

[Jak76] C. V. Jakowatz Jr. and A. C. Kak, *Computerized tomography using x-rays and ultrasound,* Research Report TR-EE 76-26, School of Electrical Engineering, Purdue University, 1976.

[Kak85] A. C. Kak and B. Roberts, "Image reconstruction from projections," in *Handbook of Image Processing,* T. Y. Young and K. S. Fu, eds., Academic Press, 1985.

[Mac83] Albert Macovski, *Medical Imaging Systems,* Prentice Hall, Englewood Cliffs, NJ, 1983.

[Mue79] R. K. Mueller, M. Kaveh, and G. Wade, "Reconstructive tomography and applications to ultrasonics," *Proceedings of the IEEE,* Vol. 67, 1979, pp. 567-587.

[New66] R. G. Newton, *Scattering Theory of Waves and Particles,* McGraw-Hill Book Company, New York, 1966.

[Sla84] Malcolm Slaney, A. C. Kak, and L. E. Larsen, "Limitations of imaging with first order diffraction tomography," *IEEE Transactions on Microwave Theory and Techniques,* August 1984, pp. 860-873.

[Tay83] J. R. Taylor, *Scattering Theory: The Quantum Theory of Nonrelativistic Collisions,* Robert E. Krieger Publishing Company, Malabar, FL, 1983.

[Wol69] E. Wolf, "Three-dimensional structure determination of semi-transparent objects from holographic data," *Optics Communications*, Vol. 1, 1969, pp. 153-156.